The Periodic Rural Postman Problem with Irregular Services on Mixed Graphs

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Outline

Preliminary Issues

The aim of the **Routing Problems** is to design a number of vehicle routes that visit a given set of customers in order to service them.
In the **Periodic Routing Problems**:

- the service occurs in some periods (days) of a given time horizon. Each customer has to be serviced in different days, frequency, ...
- usually the customers are identified as nodes of a graph representing the streets network (**Periodic Vehicle Routing Problems**); when the requirement of service can be identified with the arcs or edges of the graph, we talk about **Periodic Arc Routing Problems**.

Scientific Literature: Some Pertinent Works

⇒ **Periodic Vehicle Routing Problem**: It has been studied extensively!

- A.M. Campbell and J.H. Wilson, Forty years of periodic vehicle routing, Networks 63 (2014), 2–15.

⇒ **Periodic Arc Routing Problem**: The literature is still scarce!

- G. Ghiani, R. Musmanno, G. Paletta, and C. Triki, A heuristic for the periodic rural postman problem, Computers & Operations Research 32 (2005) 219–228.
- F. Chu, N. Labadi, and C. Prins, A Scatter Search for the periodic capacitated arc routing problem, European Journal of Operational Research 169 (2006), 586–605.
- P. Lacomme, C. Prins, and W. Ramdane-Chérif, Evolutionary algorithms for periodic arc routing problems, European Journal of Operational Research 165 (2005), 535–553.
- I.M. Monroy, C.A. Amaya, and A. Langevin, The periodic capacitated arc routing problem with irregular services, Discrete Applied Mathematics 161 (2013), 691–701.

The Periodic Rural Postman Problem with Irregular Services (PRPP-IS) belongs to the class of Periodic Arc Routing Problems. To the best of our knowledge, it has not yet been studied.

Problem Description

PRPP-IS:

- There is a single vehicle (no capacity restriction).
  A route has to be designed for each day of the time horizon in order to meet the service requirements.
- The required elements correspond to links of a graph $G = (V, E, A)$. In particular, $E_R \subseteq E$ is the set of the required edges, $A_R \subseteq A$ the set of required arcs. $L_R = A_R \cup E_R$ denotes the set of required links. Vertex $1 \in V$ represents the depot.
- Every required link has its own service plan that identifies the number of visits needed in sub–periods of the time-horizon. This number is called frequency.
  The service requirements can be irregular (different sizes of the sub-periods, different frequencies, etc.).
- The goal is to minimize the total cost over the time horizon.

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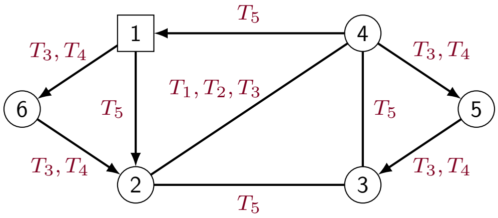
A simple example

⇒ Horizon includes 7 days (a week): 7 routes to be constructed

⇒ All links are required ($L_R = A \cup E$). The costs of traversing/servicing the links are not shown.

⇒ There are 5 sub–periods (i.e., subsets of the time horizon):

$$T_1 = \{Mon, Tue\},$$
$$T_2 = \{Wed, Thu\},$$
$$T_3 = \{Fri, Sat, Sun\},$$
$$T_4 = \{Mon, Tue, Wed, Thu\},$$
$$T_5 = \{Mon, Tue, Wed, Thu, Fri, Sat, Sun\}.$$



A simple example

$T_1 = \{Mon, Tue\}$,
$T_2 = \{Wed, Thu\}$,
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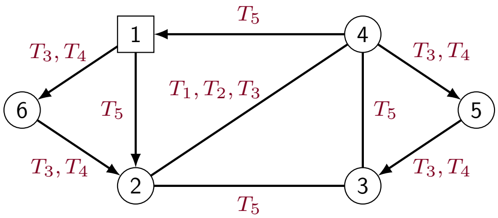


If we assume that the frequencies for each sub–period and for each link are equal to 1, then:
edge (2,4) must be serviced once over the first sub–period (i.e., Monday and Tuesday), once over the second sub-period (i.e., Wednesday and Thursday), once over the third sub–period (i.e., Friday, Saturday, and Sunday), and so on for the other links.

Applications

The PRPP-IS is a natural extension of the rural postman problem when a single route must be planned not just for a day, but for a set of days. Typically, the required elements correspond to streets and do not require to be serviced every day. There are various applications: waste collection, inspection of electric power lines, mowing vegetation on roads, etc.

$\Rightarrow$ The assumption of irregularity makes the problem more general!

Specific Example: Monitoring Activities

Monroy, Amaya, and Langevin (2013)

Road network monitoring is carried out periodically. Each road has different surveillance requirements (number of passages) during sub–periods over the time horizon.

Notation

Parameters

- H: finite and discrete time horizon; it is a set of days;
- P_l: set of disjoint sub–periods for required link l (a sub–period is a subset of H);
- f_T^l: frequency, i.e. number of services needed in sub–period $T \in P_l$ by link l ($f_T^l \leq |T| \rightarrow$ at most one visit per day);
- $c_l = c_{ij}$: traversal cost of link $l = (i, j)$;
- $c_l^s = c_{ij}^s$: service cost of required link $l = (i, j)$, with $c_{ij}^s \geq c_{ij}$;
- K: set of all route indices (since a route index corresponds to a day, and vice versa, a sub–period T can also denote a subset of K).

Decision Variables

- $x_l^k = x_{ij}^k$: number of times that link $l = (i, j)$ is traversed in route k from i to j;
- y_l^k: binary variable that takes value 1 if link l is serviced in route k, and value 0 otherwise.

Notation

Additional Notation

- $A(S:S')$: set of arcs from $S \subset V$ to $S' \subset V$;
- $E(S:S')$: set of edges between $S \subset V$ and $S' \subset V$;
- $(S:S') = E(S:S') \cup A(S:S') \cup A(S':S)$;
- $A^+(S) = A(S:V \backslash S)$;
- $A^-(S) = A(V \backslash S:S)$;
- $A(S) = A^+(S) \cup A^-(S)$;
- $E(S) = E(S:V \backslash S)$;
- $\delta(S) = E(S) \cup A(S)$ (cutset);
- $\gamma(S)$: set of links with both endpoints in $S \subset V$.

These sets are defined in a similar way with respect only to the required links (subscript R): $A_R^+(S)$, $\delta_R(S)$, etc.

In addition, $S = \{i\}$ is represented by simply i.

Mathematical Formulation

$$\text{Min} \sum_{k\in K}\sum_{l\in L_R}(c_l^s - c_l)y_l^k + \sum_{k\in K}\sum_{(i,j)\in E} c_{ij}\left(x_{ij}^k + x_{ji}^k\right) + \sum_{k\in K}\sum_{(i,j)\in A} c_{ij}x_{ij}^k \tag{1}$$

$$\sum_{(i,j)\in A^+(i)} x_{ij}^k + \sum_{j:(i,j)\in E(i)} x_{ij}^k = \sum_{(j,i)\in A^-(i)} x_{ji}^k + \sum_{j:(i,j)\in E(i)} x_{ji}^k \quad \forall\, i\in V, \forall\, k\in K, \tag{2}$$

$$\sum_{(i,j)\in A^-(S)} x_{ij}^k + \sum_{(i,j)\in E: i\in V\setminus S, j\in S} x_{ij}^k \geq y_l^k \quad \forall\, S\subseteq V\setminus\{1\}, \forall\, l\in \gamma_R(S), \forall\, k\in K \tag{3}$$

$$x_{ij}^k \geq y_{ij}^k \quad \forall\, (i,j)\in A_R, \forall\, k\in K \tag{4}$$

$$x_{ij}^k + x_{ji}^k \geq y_{ij}^k \quad \forall\, (i,j)\in E_R, \forall\, k\in K \tag{5}$$

$$\sum_{k\in T} y_l^k = f_T^l \quad \forall\, l\in L_R, \forall\, T\in P_l \tag{6}$$

$$y_l^k \in \{0,1\} \quad \forall\, l\in L_R, \forall\, k\in K \tag{7}$$

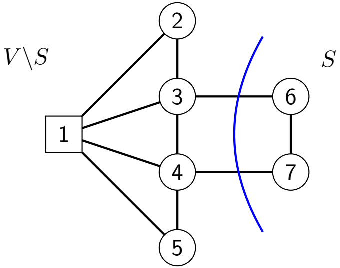
$$x_{ij}^k \geq 0 \text{ and integer } \quad \forall\, (i,j)\in A, \forall\, k\in K \tag{8}$$

$$x_{ij}^k, x_{ji}^k \geq 0 \text{ and integer } \quad \forall\, (i,j)\in E, \forall\, k\in K. \tag{9}$$

Connectivity Constraints (3)

For $k = 1$; $S = \{6, 7\}$; $l = (6, 7) \in \gamma_R(S)$:

$$\sum_{(i,j)\in A^-(S)} x_{ij}^1 + \sum_{(i,j)\in E: i\in V\setminus S, j\in S} x_{ij}^1 \geq y_l^1$$



$$x_{36}^1 + x_{47}^1 \geq y_{67}^1$$

If the route associated with day 1 does not service edge (6,7), then $y_{67}^1 = 0$ and the inequality is trivially satisfied; otherwise: $x_{36}^1 + x_{47}^1 \geq 1$.

Valid Inequalities

We propose several valid inequalities for the PRPP-IS . They are used to strengthen the linear relaxation. Let $S \subset V$ be a vertex subset and $\delta(S)$ a link cutset.

We consider: $x^k(\delta(S)) = \sum_{(i,j)\in\delta(S)\cap A} x_{ij}^k + \sum_{(i,j)\in\delta(S)\cap E}(x_{ij}^k + x_{ji}^k)$.

Sub–Period Aggregate Parity Inequalities

Let T be a sub-period and $\delta_R(S)$ a required link cutset such that $f(\delta_R(S), T) = \sum_{l\in\delta_R(S):T\in P_l} f_T^l$ is odd:

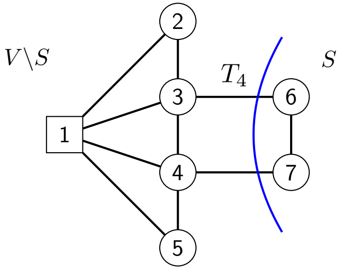
$$\sum_{k\in T} x^k(\delta(S)) \geq f(\delta_R(S), T) + 1. \qquad (10)$$

Sub–Period Aggregate Parity Inequalities

For $T = T_4 = \{Mon, Tue, Wed, Thu\}$; $S = \{6, 7\}$;
$(3, 6) \in \delta_R(S)$ and $f_{T_4}^{(3,6)} = 3 \rightarrow$ odd:

$$x_{36}^1 + x_{63}^1 + x_{47}^1 + x_{74}^1 + x_{36}^2 + x_{63}^2 + x_{47}^2 + x_{74}^2 +$$

$$+ x_{36}^3 + x_{63}^3 + x_{47}^3 + x_{74}^3 + x_{36}^4 + x_{63}^4 + x_{47}^4 + x_{74}^4 \geq 3 + 1$$



Valid Inequalities

Disaggregate Parity Inequalities

Let F be a subset of $\delta_R(S)$ such that $|F|$ is odd:

$$x^k(\delta(S)) \geq 2y^k(F) - |F| + 1, \quad (11)$$

where $y^k(F) = \sum_{l \in F} y_l^k$.

P-Aggregate Parity Inequalities

In addition, let $\bar{K} = \{k_1, k_2, \ldots, k_P\}$ be a subset of $P < |K|$ routes such that $\bar{f}(F, \bar{K}) = \sum_{l \in F} \sum_{T \in P_l} \min(f_T^l, |T \cap \bar{K}|)$ is odd:

$$\sum_{k \in \bar{K}} x^k(\delta(S)) \geq 2 \sum_{k \in \bar{K}} y^k(F) - \bar{f}(F, \bar{K}) + 1, \quad (12)$$

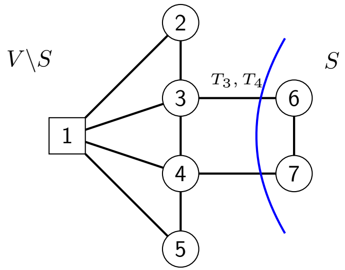
For $\bar{K} = \{Mon, Tue, Sat, Sun\}$ and $F = \{(3,6)\}$ with $P_{(3,6)} = \{T_4 = \{Mon, Tue, Wed, Thu\}; T_3 = \{Fri, Sat, Sun\}\}$, $f_{T_4}^{(3,6)} = 3$ and $f_{T_3}^{(3,6)} = 1$:
$\bar{f}(F, \bar{K}) = \min(f_{T_4}^{(3,6)}, |T_4 \cap \bar{K}|) + \min(f_{T_3}^{(3,6)}, |T_3 \cap \bar{K}|) = 2 + 1.$

Valid Inequalities

P-Aggregate Parity Inequalities

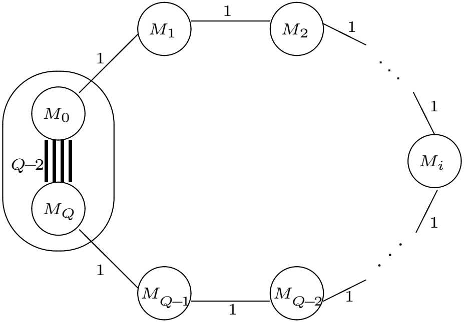
For $S = \{6,7\}$, $\bar{K} = \{Mon, Tue, Sat, Sun\}$ and $F = \{(3,6)\}$ with $P_{(3,6)} = \{T_4 = \{Mon, Tue, Wed, Thu\}; T_3 = \{Fri, Sat, Sun\}\}$, $f_{T_4}^{(3,6)} = 3$ and $f_{T_3}^{(3,6)} = 1$: $\bar{f}(F,\bar{K}) = \min(f_{T_4}^{(3,6)}, |T_4 \cap \bar{K}|) + \min(f_{T_3}^{(3,6)}, |T_3 \cap \bar{K}|) = 2 + 1.$

$$\sum_{k \in \bar{K}} x^k(\delta(S)) \geq 2 \sum_{k \in \bar{K}} y^k(F) - 3 + 1$$



K-C Inequalities

They refer to a partition of V into $\{M_0, M_1, \ldots, M_Q\}$ such that each subset of vertices associated with the connected components induced by the required links (R-set) is contained in one of the sets $M_0 \cup M_Q$, $M_1, \ldots, M_{Q-1}$, each M_i contains at least one R-set, the induced subgraphs $G(M_i)$ are connected, and $(M_0 : M_Q)$ contains a positive and even number of required links.



$\Rightarrow$ Set $F(x^k) = \sum_{(i,j)\in A} a_{ij}x_{ij}^k + \sum_{(i,j)\in E} \left(a_{ij}x_{ij}^k + a_{ji}x_{ji}^k\right)$.

Valid Inequalities

Disaggregate K-C Inequalities

Let $\{M_0, M_1, M_2, \ldots, M_{Q-1}, M_Q\}$ be a partition of V where each subgraph G(M_i) is connected. We assume that there is a required link subset $D \subseteq (M_0 : M_Q)_R$ such that $|D|$ is positive and even and another required subset $Z = \{l_1, l_2, \ldots, l_{Q-1}\} \subset L_R$ such that each $l_i \in \gamma_R(M_i)$.

If $1 \in M_0 \cup M_Q$:

$$F(x^k) \geq 2y^k(Z) + (Q-2)\big(2y^k(D) - |D|\big). \tag{13}$$

If $1 \in M_i$ with $i \notin \{0, Q\}$:

$$F(x^k) \geq 2 + 2y^k(Z) + (Q-2)\big(2y^k(D) - |D|\big). \tag{14}$$

Sub–period aggregate K-C Inequalities

Assume that $\{M_0, M_1, M_2, \ldots, M_{Q-1}, M_Q\}$ is a partition of V, and let $D \subseteq (M_0 : M_Q)_R$ and $Z = \{l_1, l_2, \ldots, l_{Q-1}\} \subset L_R$, with $l_i \in \gamma_R(M_i)$, such that all the links in $D \cup Z$ have to be serviced at least once in sub–period T, i.e. $T \in P_l$ and $f_T^l > 0$, for all $l \in D \cup Z$. Consider $l_{i^*} \in \gamma_R(M_{i^*})$such that $f_T^{l_{i^*}} = max\{f_T^{l_j} : l_j \in Z\}$ and assume that the depot is located in $M_0 \cup M_Q$.

If $f_T^{l_{i^*}} = 1$:

$$\sum_{k \in T} F(x^k) \geq 2(Q-1) + (Q-2) f(D,T), \tag{15}$$

where $f(D,T)$ ($=\sum_{l \in D : T \in P_l} f_T^l$) has to be an even number.

If $f_T^{l_{i^*}} \geq 2$:

$$\sum_{k \in T} F(x^k) \geq 2(Q-1) + 2 + 2(f_T^{l_{i^*}} - 2) d(M_0 \cup M_Q, M_{i^*}) + (Q-2) f(D,T), \tag{16}$$

where $d(M_0 \cup M_Q, M_{i^*})$= min $\{i^*, Q - i^*\}$, and $f(D,T)$ has to be an even number.

Sub–period aggregate K-C Inequalities

If the depot is located in M_r, then $r \notin \{0, Q\}$, we choose the link $l_{i^*} \in \gamma(M_{i^*})$, $i^* \notin \{0, Q, r\}$, as the one satisfying $f_T^{l_{i^*}} = max\{f_T^{l_j} : l_j \in \gamma(M_j), j \neq r\}$.

If $f_T^{l_{i^*}} \geq 2$:

$$\sum_{k \in T} F(x^k) \geq 2(Q-1) + 2 + 2(f_T^{l_{i^*}} - 2)d(M_r, M_{i^*}) + (Q-2)f(D,T), \qquad (17)$$

where $d(M_r, M_{i^*}) = \min\{|r - i^*|, r + Q - i^*, i^* + Q - r\}$, and $f(D,T)$ has to be an even number.

Proposition

Sub–period aggregate K-C inequalities (15), (16), and (17) are valid for the **PRPP-IS** *if $f(D,T)$ is an even number.*

A Branch-and-Cut Algorithm: Simplified Outline

Step 0. Let $LB = 0$ be the lower bound and $UB = +\infty$ the upper bound.
(Note: the value of LB is opportunely updated during the search)

Step 1. Define a relaxed problem by eliminating connectivity constraints (3) and integer conditions and insert it into a list Θ (first problem in the list).

Step 2. If Θ is empty or $LB = UB$, then STOP. Otherwise, extract a problem from Θ.

Step 3. Solve the current problem. Let OBJ be the solution value. If $OBJ \geq UB$ then go to Step 2.

Step 4. Identify inequalities (3) and other valid inequalities for the PRPP-IS by using adequate separation procedures.

Step 5. If some violated inequalities have been identified, add these inequalities to the problem and go back to Step 3. Otherwise, if the current solution is feasible, set $UB = OBJ$ and go back to Step 2.

Step 6. If the current solution is not integer, generate two subproblems by branching on a fractional variable. Insert the subproblems into Θ and go back to Step 2.

Separation Procedures

Some inequalities are separated at each node of the search tree. Other valid inequalities are separated at the root node solely.

- Connectivity Constraints: exact and heuristic procedures described by Benavent, Corberán, Plana, and Sanchis (2009) for the Min-Max K-vehicles Windy Rural Postman Problem.
- Sub–period aggregate parity inequalities: heuristic procedure commonly used in the literature for the separation of the aggregate parity inequalities.
- Disaggregate and P-Aggregate Parity Inequalities: heuristic procedure described by Ghiani and Laporte (2000) for the Undirected Rural Postman Problem.
- Disaggregate and Sub–Period Aggregate K-C Inequalities: three-phase heuristic procedure inspired by the one introduced by Corberán, Letchford, and Sanchis (2001) for the General Routing Problem.

Computational Experiments

Instances

Instances derived from `mval` datasets designed for the *Mixed Capacitated Arc Routing Problem* by Belenguer, Benavent, Lacomme and Prins (2006).

- `pcp-mval` (periodic Chinese postman): the nature of the `mval` instances has been kept, i.e., all links of the graph are required. For each required link l, a set P_l of sub-periods has been generated. The number of times that required link l must be serviced in each sub–period $T \in P_l$ was uniformly generated in $[1, \ldots, \lceil |T|/2 \rceil]$.

  Horizon $=$ 7 days. 5 different sub–periods. 1 - 3 sub–periods assigned to each link
- `spcp-mval` (simplified periodic Chinese postman): `pcp-mval` instances have been simplified by "switching off" two sub–periods (i.e., by setting to zero the frequencies concerning these sub–periods).
- `prp-mval` (periodic rural postman): `pcp-mval` instances have been transformed into periodic rural postman instances by declaring some links as non–required.
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Computational Experiments

Workstation and Software

- Intel(R)Core(TM) i7-3630QM CPU running at 2.40 GHz; 32 GB of memory.
- C++ for Linux and ILOG CPLEX Library 12.6.

Column Headings

FILE	instance name
$\|V\|$,$\|A\|$,$\|E\|$	number of vertices, arcs and edges
$\|A_R\|$,$\|E_R\|$	number of required arcs and edges
LB, UB	final lower bound and upper bound
GAP	percentage gap (from CPLEX)
SEC	computation time in seconds (TL: time limit $\rightarrow$ 6 hours)
CON	number of connectivity inequalities
DIP,SAP,PAP	number of disaggregate, sub–period aggregate and P-aggregate parity inequalities
DKC,SKC	number of disaggregate and sub–period aggregate K-C inequalities
NOD	number of nodes processed in the search (apart from the root)

Numerical Results: `pcp-mval` dataset

	Instance Features					*Main Results*				*Other Results*						
FILE	$\|V\|$	$\|A\|$	$\|E\|$	$\|A_R\|$	$\|E_R\|$	LB	UB	GAP	SEC	CON	DIP	SAP	PAP	DKC	SKC	NOD
pcp-mval1A	24	35	20	35	20	546.5	556	1.71	TL	3072	315	26	563	346	1	7601
pcp-mval1B	24	38	13	38	13	692	692*	0.00	112.21	51	14	3	65	0	0	0
pcp-mval1C	24	36	17	36	17	636	639	0.47	TL	1615	179	22	324	163	0	11072
pcp-mval2A	24	28	16	28	16	818	818*	0.00	99.77	223	49	7	107	3	0	0
pcp-mval2B	24	40	12	40	12	868	868*	0.00	374.53	519	181	14	321	44	0	42
pcp-mval2C	24	35	14	35	14	774.533	776	0.19	TL	1947	177	15	344	285	0	17115
pcp-mval3A	24	33	15	33	15	297	297*	0.00	7139.65	911	25	10	63	82	0	7847
pcp-mval3B	24	29	16	29	16	355	355*	0.00	434.03	522	124	11	259	36	0	54
pcp-mval3C	24	25	18	25	18	225.519	229	1.52	TL	638	190	17	359	68	0	15678
pcp-mval4A	41	69	26	69	26	1433.842	1476	2.86	TL	2924	145	15	280	231	0	2656
pcp-mval4B	41	83	19	83	19	1472.429	1481	0.58	TL	1445	136	6	174	116	0	443
pcp-mval4C	41	82	21	82	21	1503.586	1528	1.60	TL	3754	288	11	483	273	0	2622
pcp-mval4D	41	83	21	83	21	1576	1590	0.88	TL	2815	238	13	375	261	0	2563
pcp-mval5A	34	74	22	74	22	1454	1484	2.02	TL	3222	291	10	499	248	0	3394
pcp-mval5B	34	56	35	56	35	1387.784	1441	3.69	TL	7683	323	28	761	413	0	2635
pcp-mval5C	34	81	17	81	17	1697	1699	0.12	TL	1917	30	5	63	71	0	3910
pcp-mval5D	34	63	29	63	29	1439.562	1449	0.65	TL	4068	278	13	562	389	0	2446
pcp-mval6A	31	47	22	47	22	770	770*	0.00	519.59	330	194	12	297	9	0	10
pcp-mval6B	31	44	22	44	22	792	794	0.25	TL	1529	338	7	419	124	0	7806
pcp-mval6C	31	45	23	45	23	778	778*	0.00	14224.47	2364	183	8	306	224	0	1540
pcp-mval7A	40	50	36	50	36	932	942	1.06	TL	2712	193	28	571	78	0	3877
pcp-mval7B	40	66	25	66	25	1084	1087	0.28	TL	2568	75	13	187	76	0	3384
pcp-mval7C	40	62	28	62	28	964.095	975	1.12	TL	3121	124	23	342	467	0	3786
pcp-mval8A	30	76	20	76	20	1428	1439	0.76	TL	1691	312	10	463	143	0	3478
pcp-mval8B	30	64	27	64	27	1320	1345	1.86	TL	2730	453	11	647	164	0	3610
pcp-mval8C	30	55	28	55	28	1333	1336	0.23	TL	1658	233	19	447	76	0	5400
pcp-mval9A	50	100	32	100	32	1147	1156	0.78	TL	4893	322	7	468	271	0	1580
pcp-mval9B	50	76	44	76	44	1059.875	1074	1.32	TL	2403	210	27	689	94	0	1816
pcp-mval9C	50	83	42	83	42	1032	1053	1.99	TL	4115	263	31	619	207	0	1179
pcp-mval9D	50	93	38	93	38	1139	1163	2.06	TL	3560	301	23	569	277	2	1425
pcp-mval10A	50	106	32	106	32	1611	1626	0.92	TL	3450	322	23	550	69	0	1370
pcp-mval10B	50	101	33	101	33	1597.25	1638	2.49	TL	2899	288	22	488	62	0	1659
pcp-mval10C	50	100	36	100	36	1494	1542	3.11	TL	6206	286	24	668	462	0	780
pcp-mval10D	50	87	42	87	42	1368	1410	2.98	TL	4540	261	30	581	241	0	1609
average gap %								**1.10**								
maximum gap %								**3.69**								
# optima							**7**									

Numerical Results: spcp-mval dataset

FILE	Instance Features $\|V\|$	$\|A\|$	$\|E\|$	$\|A_R\|$	$\|E_R\|$	Main Results LB	UB	GAP	SEC	Other Results CON	DIP	SAP	PAP	DKC	SKC	NOD
spcp-mval1A	24	35	20	35	20	443	448	1.12	TL	3536	182	20	522	601	1	8101
spcp-mval1B	24	38	13	38	13	447	447*	0.00	56.38	93	21	2	129	0	0	0
spcp-mval1C	24	36	17	36	17	510	510*	0.00	131.77	298	61	7	233	8	0	28
spcp-mval2A	24	28	16	28	16	652	652*	0.00	157.55	260	60	6	140	8	0	8
spcp-mval2B	24	40	12	40	12	693	693*	0.00	261.15	424	54	5	174	23	0	84
spcp-mval2C	24	35	14	35	14	647	647*	0.00	124.81	364	57	10	210	18	2	22
spcp-mval3A	24	33	15	33	15	249	249*	0.00	200.15	317	57	5	138	19	0	27
spcp-mval3B	24	29	16	29	16	263	263*	0.00	548.48	371	60	4	304	11	0	136
spcp-mval3C	24	25	18	25	18	184	184*	0.00	5774.56	582	141	7	391	99	1	7399
spcp-mval4A	41	69	26	69	26	1117	1126	0.80	TL	1999	124	7	288	218	0	3186
spcp-mval4B	41	83	19	83	19	1256	1261	0.40	TL	2333	89	5	236	237	0	2692
spcp-mval4C	41	82	21	82	21	1177.497	1190	1.05	TL	3770	145	5	342	296	0	2460
spcp-mval4D	41	83	21	83	21	1163	1165	0.17	TL	1970	54	3	147	168	0	4924
spcp-mval5A	34	74	22	74	22	1160	1161	0.09	TL	2895	90	2	209	150	0	1973
spcp-mval5B	34	56	35	56	35	1074.03	1109	3.15	TL	5090	197	12	756	117	0	4028
spcp-mval5C	34	81	17	81	17	1281	1285	0.31	TL	2525	35	6	77	63	0	3398
spcp-mval5D	34	63	29	63	29	1021.5	1053	2.99	TL	3595	243	13	616	250	0	1064
spcp-mval6A	31	47	22	47	22	624	634	1.58	TL	1505	124	9	321	135	0	2752
spcp-mval6B	31	44	22	44	22	566	566*	0.00	15483.68	987	116	6	476	80	0	9719
spcp-mval6C	31	45	23	45	23	629	629*	0.00	9306.72	222	59	6	186	19	0	4
spcp-mval7A	40	50	36	50	36	756	757	0.13	TL	1509	110	11	511	34	1	4390
spcp-mval7B	40	66	25	66	25	872	872*	0.00	11818.95	1638	77	3	226	68	0	1095
spcp-mval7C	40	62	28	62	28	797	798	0.13	TL	1713	138	9	330	156	1	7180
spcp-mval8A	30	76	20	76	20	1139.5	1149	0.83	TL	2262	92	3	173	84	0	2162
spcp-mval8B	30	64	27	64	27	1121	1136	1.32	TL	2899	269	12	540	146	0	862
spcp-mval8C	30	55	28	55	28	982	992	1.01	TL	2368	124	9	532	149	0	2513
spcp-mval9A	50	100	32	100	32	967	967*	0.00	1543.96	930	166	3	328	64	0	31
spcp-mval9B	50	76	44	76	44	842.833	860	2.00	TL	3293	191	14	751	140	0	736
spcp-mval9C	50	83	42	83	42	836	842	0.71	TL	2456	182	10	631	153	0	771
spcp-mval9D	50	93	38	93	38	830.636	842	1.35	TL	7456	150	14	482	286	0	1380
spcp-mval10A	50	106	32	106	32	1238	1268	2.37	TL	2549	128	5	278	158	0	514
spcp-mval10B	50	101	33	101	33	1195.233	1214	1.55	TL	6800	132	12	453	528	0	898
spcp-mval10C	50	100	36	100	36	1094.577	1130	3.14	TL	4212	166	12	505	492	0	278
spcp-mval10D	50	87	42	87	42	1129	1146	1.48	TL	3316	178	12	433	260	0	1450
average gap %								**0.81**								
maximum gap %								**3.15**								
# optima							**12**									

Numerical Results: prp-mval dataset

FILE	Instance Features					Main Results				Other Results						
	$\|V\|$	$\|A\|$	$\|E\|$	$\|A_R\|$	$\|E_R\|$	LB	UB	GAP	SEC	CON	DIP	SAP	PAP	DKC	SKC	NOD
prp-mval1A	24	35	20	21	13	380	380*	0.00	1381.38	403	65	10	156	54	1	1514
prp-mval1B	24	38	13	25	9	564	564*	0.00	37.80	98	12	2	20	0	0	0
prp-mval1C	24	36	17	23	10	506	508	0.39	TL	699	30	7	87	27	0	32602
prp-mval2A	24	28	16	14	10	634	634*	0.00	171.36	345	17	3	47	2	0	210
prp-mval2B	24	40	12	26	7	639	639*	0.00	164.61	326	37	11	99	11	1	77
prp-mval2C	24	35	14	22	8	535	535*	0.00	246.58	305	86	4	148	14	0	124
prp-mval3A	24	33	15	22	8	237	237*	0.00	104.12	221	43	2	68	9	0	23
prp-mval3B	24	29	16	18	6	233	233*	0.00	56.23	166	25	4	93	1	0	19
prp-mval3C	24	25	18	15	11	183	183*	0.00	259.21	209	87	10	245	21	6	159
prp-mval4A	41	69	26	41	22	1023	1031	0.78	TL	3713	181	8	284	355	0	4390
prp-mval4B	41	83	19	52	12	1018	1018*	0.00	1750.82	1267	48	12	78	110	0	400
prp-mval4C	41	82	21	50	14	1043	1043*	0.00	8909.32	2263	103	15	226	232	0	3486
prp-mval4D	41	83	21	48	7	1017	1017*	0.00	6724.42	1886	24	6	71	90	1	2834
prp-mval5A	34	74	22	47	15	1104.343	1119	1.31	TL	4781	164	15	291	172	0	4796
prp-mval5B	34	56	35	39	22	1067.115	1076	0.83	TL	4819	195	27	483	375	0	3586
prp-mval5C	34	81	17	51	11	1267	1267*	0.00	4646.25	1655	63	7	110	49	0	2471
prp-mval5D	34	63	29	34	19	966	969	0.31	TL	1723	38	18	122	71	0	7690
prp-mval6A	31	47	22	30	15	551.5	558	1.17	TL	936	70	11	149	60	0	6833
prp-mval6B	31	44	22	26	12	543	543*	0.00	695.23	402	14	6	79	20	0	539
prp-mval6C	31	45	23	28	16	566	566*	0.00	6305.10	1037	68	10	162	120	0	6054
prp-mval7A	40	50	36	28	20	615	615*	0.00	308.53	407	44	18	298	9	0	24
prp-mval7B	40	66	25	43	18	816	816*	0.00	1628.29	1060	49	7	164	58	0	381
prp-mval7C	40	62	28	37	19	746.548	756	1.25	TL	1136	142	19	346	140	0	5862
prp-mval8A	30	76	20	45	8	867	867*	0.00	491.93	666	13	2	23	29	0	105
prp-mval8B	30	64	27	33	16	821.11	825	0.47	TL	1519	179	16	421	85	0	10543
prp-mval8C	30	55	28	35	17	956	959	0.31	TL	2197	100	8	249	53	0	9410
prp-mval9A	50	100	32	65	20	835.5	864	3.30	TL	3267	156	17	253	295	0	2086
prp-mval9B	50	76	44	48	27	755	769	1.82	TL	3891	201	22	455	247	0	2373
prp-mval9C	50	83	42	57	27	784	788	0.51	TL	4329	177	23	445	311	0	2726
prp-mval9D	50	93	38	60	23	823	845	2.60	TL	4439	213	11	305	431	1	1213
prp-mval10A	50	106	32	58	23	1047.926	1049	0.10	TL	2554	97	19	291	85	0	2248
prp-mval10B	50	101	33	63	20	968	1022	5.28	TL	4152	142	19	373	490	0	1466
prp-mval10C	50	100	36	63	21	1008	1013	0.49	TL	4703	129	11	240	414	1	1709
prp-mval10D	50	87	42	58	28	988	1023	3.42	TL	4071	223	32	383	285	0	1950
average gap %								**0.72**								
maximum gap %								**5.28**								
# optima							**17**									

Numerical Results: `sprp-mval` dataset

FILE	Instance Features					Main Results				Other Results						
	$\|V\|$	$\|A\|$	$\|E\|$	$\|A_R\|$	$\|E_R\|$	LB	UB	GAP	SEC	CON	DIP	SAP	PAP	DKC	SKC	NOD
sprp-mval1A	24	35	20	21	13	317	317*	0.00	226.26	245	22	7	158	8	0	95
sprp-mval1B	24	38	13	25	9	349	349*	0.00	46.08	119	4	4	48	0	0	13
sprp-mval1C	24	36	17	23	10	419	419*	0.00	107.38	204	28	3	71	8	0	6
sprp-mval2A	24	28	16	14	10	482	482*	0.00	1310.76	341	35	5	85	13	0	3514
sprp-mval2B	24	40	12	26	7	505	505*	0.00	68.62	308	20	8	171	17	0	5
sprp-mval2C	24	35	14	22	8	480	480*	0.00	112.85	292	27	4	118	23	0	55
sprp-mval3A	24	33	15	22	8	206	206*	0.00	87.09	174	27	2	45	4	0	3
sprp-mval3B	24	29	16	18	6	175	175*	0.00	228.57	207	33	2	93	8	0	361
sprp-mval3C	24	25	18	15	11	145	145*	0.00	437.61	234	59	5	208	16	0	784
sprp-mval4A	41	69	26	41	22	805	805*	0.00	602.00	959	45	5	136	21	0	29
sprp-mval4B	41	83	19	52	12	880	880*	0.00	6479.35	957	15	6	60	79	0	2237
sprp-mval4C	41	82	21	50	14	828.857	835	0.74	TL	2056	44	9	229	487	0	4077
sprp-mval4D	41	83	21	48	7	819.692	825	0.64	TL	2217	40	7	85	185	0	5089
sprp-mval5A	34	74	22	47	15	849	865	1.85	TL	2678	101	5	385	294	0	4735
sprp-mval5B	34	56	35	39	22	811.197	824	1.55	TL	4103	83	18	584	270	2	4840
sprp-mval5C	34	81	17	51	11	989	989*	0.00	363.71	574	59	4	109	23	0	13
sprp-mval5D	34	63	29	34	19	694	694*	0.00	3806.35	1204	33	13	338	75	0	1261
sprp-mval6A	31	47	22	30	15	482	482*	0.00	16218.55	715	34	7	175	43	0	18745
sprp-mval6B	31	44	22	26	12	420	420*	0.00	493.21	333	25	2	260	24	0	409
sprp-mval6C	31	45	23	28	16	471	471*	0.00	236.32	366	13	4	140	16	0	75
sprp-mval7A	40	50	36	28	20	506.143	508	0.37	TL	1084	78	11	535	19	4	6680
sprp-mval7B	40	66	25	43	18	650	650*	0.00	92.08	155	28	0	52	1	0	0
sprp-mval7C	40	62	28	37	19	599	599*	0.00	2292.64	924	91	5	371	61	0	442
sprp-mval8A	30	76	20	45	8	746	746*	0.00	267.56	490	5	2	24	27	0	5
sprp-mval8B	30	64	27	33	16	711.333	713	0.23	TL	1519	82	11	373	108	0	9558
sprp-mval8C	30	55	28	35	17	747	747*	0.00	2959.65	1124	79	1	210	32	0	1764
sprp-mval9A	50	100	32	65	20	712	718	0.84	TL	2843	33	10	249	210	0	1787
sprp-mval9B	50	76	44	48	27	615.875	632	2.55	TL	3514	124	15	550	469	0	2513
sprp-mval9C	50	83	42	57	27	650.476	668	2.62	TL	4261	74	14	427	353	0	1863
sprp-mval9D	50	93	38	60	23	629	632	0.48	TL	2817	84	9	277	210	0	2520
sprp-mval10A	50	106	32	58	23	865	867	0.23	TL	2433	84	9	336	67	0	1030
sprp-mval10B	50	101	33	63	20	772.5	794	2.71	TL	4535	89	10	345	386	0	1700
sprp-mval10C	50	100	36	63	21	791	791*	0.00	6648.30	2337	95	9	320	167	1	698
sprp-mval10D	50	87	42	58	28	820.625	837	1.96	TL	2488	129	11	434	264	1	1886
average gap %								**0.49**								
maximum gap %								**2.71**								
# optima							**21**									

Final Remarks

- The PRPP-IS has been introduced and studied.
- Computational results confirm that our branch-and-cut algorithm is an effective solution approach to the problem.
- Details on procedures, applications and results will be provided in a paper next to submission

E. Benavent, Á. Corberán, D. Laganà, F. Vocaturo (2017). Exact Solution of the Periodic Rural Postman Problem with Irregular Services on Mixed Graphs.

THANK YOU!