The Multi-Period Service Territory Design Problem

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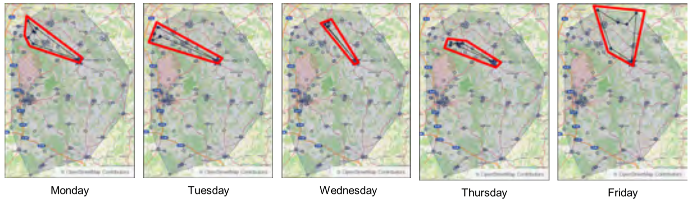
INSTITUTE OF OPERATIONS RESEARCH (IOR)
DISCRETE OPTIMIZATION AND LOGISTICS



INTRODUCTION TO DISTRICTING

Districting in General

Planning task

Group small geographic units (basic areas) into larger cluster (districts or territories) such that some relevant planning criteria are satisfied.

Typical planning criteria:

- Compactness
- Contiguity
- Balance

Classification of Districting Literature

Political districting

- Has attracted the attention of many researchers since the 1960s
- Goals:
  - Prevent gerrymandering
  - Ensure that each vote has the same power

Design of service territories

Services at fixed locations

- „Customer comes to the service."
- Examples: School districts, districts for social facilities
- Goals:
  - Short distances, good accessibility
  - Same population or racial balance

On-site services

- „Service comes to the customer."
- Examples: Sales territories, districts for pickup/delivery operations
- Goals:
  - Little travel time
  - Same workload or earning opportunities

MULTI-PERIOD SERVICE TERRITORY DESIGN

Motivation

Companies with a field service workforce

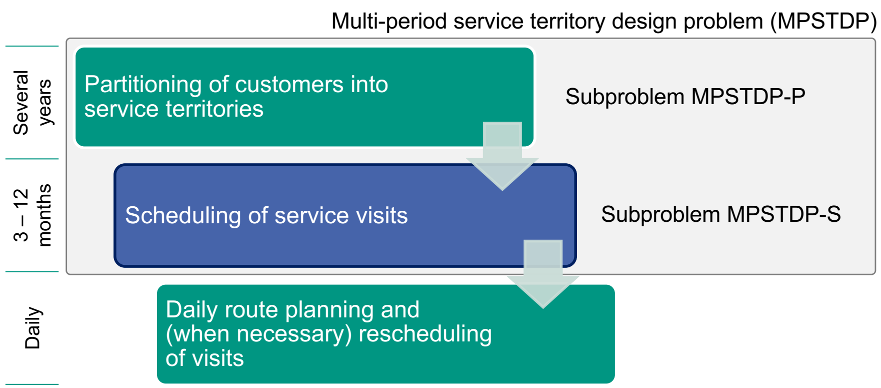
- Provide recurring services at customers' locations
- Examples:
  - Sales force of manufacturers and wholesalers of consumer goods (Fleischmann and Paraschis, 1988; Polacek et al., 2007)
  - Field service technicians of engineering companies (Blakeley et al., 2003)

Importance of service consistency

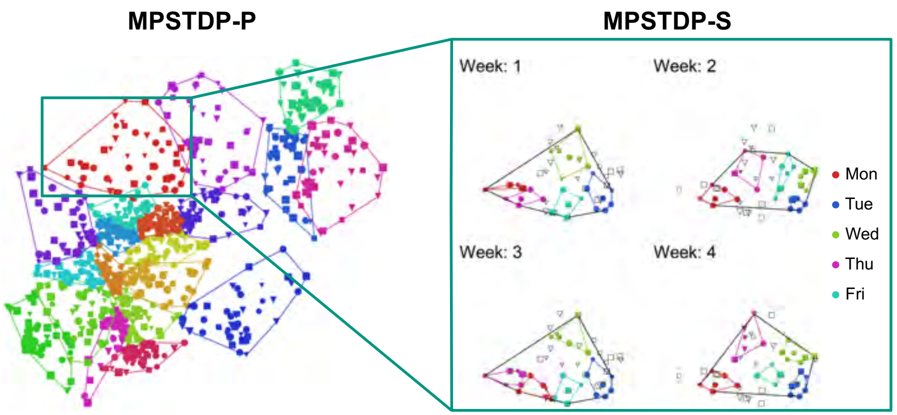
- Personal consistency: long-term personal relations with customers (Zoltners and Sinha, 2005)
- Temporal consistency: regularity of service visits (cf. Groër et al., 2009)

Blakeley, F., Argüello, B., Cao, B., Hall, W., and Knolmajer, J. (2003). Optimizing periodic maintenance operations for Schindler Elevator Corporation. *Interfaces*, 33(1):67–79.
Fleischmann, B. and Paraschis, J. N. (1988). Solving a large scale districting problem: A case report. *Computers & Operations Research*, 15(6):521–533.
Groër, C., Golden, B., and Wasil, E. (2009). The consistent vehicle routing problem. *Manufacturing & Service Operations Management*, 11(4):630–643.
Polacek, M., Doerner, K. F., Hartl, R. F., Kiechle, G., and Reimann, M. (2007). Scheduling periodic customer visits for a traveling salesperson. *EJOR*, 179:823–837.
Zoltners, A. A. and Sinha, P. (2005). Sales territory design: Thirty years of modeling and implementation. *Marketing Science*, 24(3):313–331.

Typical planning process



Visualization of the MPSTDP

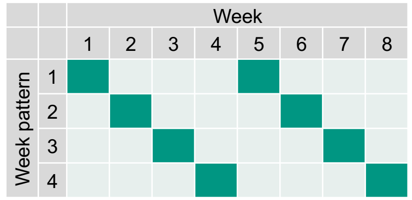


DESCRIPTION OF THE MPSTDP-S

Customer-specific visiting requirements

Week patterns

- Feasible combinations of visiting weeks
- Rigid week rhythm



Example: Week rhythm = 4

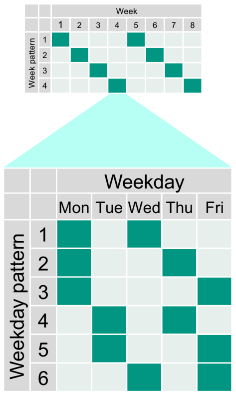
Customer-specific visiting requirements

Week patterns

- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns

- Feasible combinations of weekdays within visiting weeks



Example: Two service visits per week, but not on consecutive days

Customer-specific visiting requirements

Week patterns

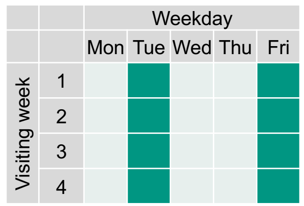
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns

- Feasible combinations of weekdays within visiting weeks

Weekday regularities

- Strict: same weekday pattern in each visiting week



Example: Visits always on Tuesday and Friday

Customer-specific visiting requirements

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed

		Weekday				
		Mon	Tue	Wed	Thu	Fri
Visiting week	1		■			■
	2		■			■
	3	■			■	
	4		■			■

Example: Visits always on Tuesday and Friday except for the third visiting week

Customer-specific visiting requirements

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed
- No regularity requirements: No restrictions

		Weekday				
		Mon	Tue	Wed	Thu	Fri
Visiting week	1		■			■
	2	■			■	
	3	■		■		
	4		■		■	

Example: Different weekday pattern in each visiting week

Customer-specific visiting requirements

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed
- No regularity requirements: No restrictions

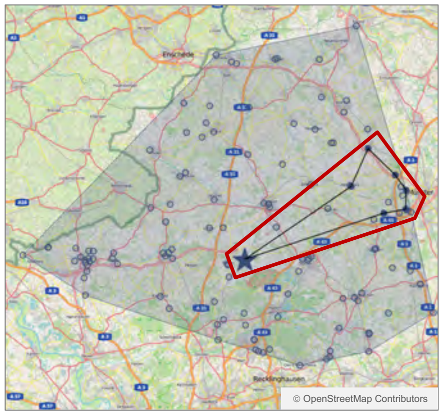
Service times
- Specific for each service visit

		Service time [min]
Visit no.	1	60
	2	25
	3	30
	4	45

Example: Four visits with different service times

Planning criteria

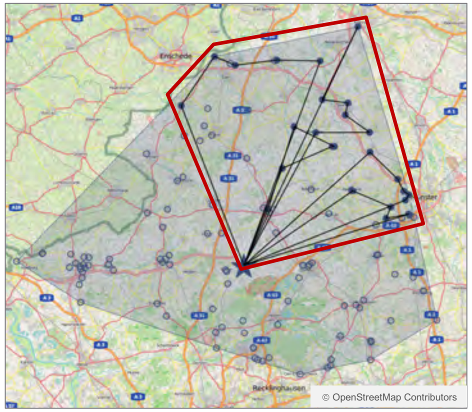
Geographic compactness
- Compact day clusters



Planning criteria

Geographic compactness

- Compact day clusters
- Compact week clusters



Planning criteria

Geographic compactness

- Compact day clusters
- Compact week clusters

Balance

- Service time evenly distributed across *days*
- Service time evenly distributed across *weeks*

Planning criteria

Geographic compactness

- Compact day clusters
- Compact week clusters

Balance

- Service time evenly distributed across *days*
- Service time evenly distributed across *weeks*

Feasibility

- Feasible schedule with respect to all customer-specific visiting requirements

Related Problems

	Application	Difference
Other multi-period districting problems	▪ Districting in a setting with a dynamically varying customer base ▪ Only two papers: Lei et al. 2015, 2016	No consideration of week or weekday patterns
Extensions of the vehicle routing problem	▪ Route planning across several time periods ▪ Examples: IRP (Irnich et al., 2014), PVRP (Coelho et al., 2014)	Optimization of routing cost instead of compactness
Multi-period scheduling problems	▪ Scheduling of tasks according to strict rhythms ▪ Examples: Machine maintenance (Wei and Liu, 1983), logistics (Campbell and Hardin, 2005)	No consideration of geographical aspects

Campbell, A. M. and Hardin, J. R. (2005). Vehicle minimization for periodic deliveries. *European Journal of Operational Research*, 165(3):668–684.
Coelho, L. C., Cordeau, J.-F., and Laporte, G. (2014). Thirty years of inventory routing. *Transportation Science*, 48(1):1–19.
Irnich, S., Schneider, M., and Vigo, D. (2014). Four variants of the vehicle routing problem. In Toth, P. und Vigo, D. (Hrsg.), *Vehicle Routing: Problems, Methods, and Applications*, S. 241–271.
Lei, H., Laporte, G., Liu, Y., and Zhang, T. (2015). Dynamic design of sales territories. *COR*, 56:84–92.
Lei, H., Wang, R., and Laporte, G. (2016). Solving a multi-objective dynamic stochastic districting and routing problem with a co-evolutionary algorithm. *COR*, 67:12–24.
Wei, W. D. and Liu, C. L. (1983). On a periodic maintenance problem. *Operations Research Letters*, 2(2):90–93.

MATHEMATICAL FORMULATION OF THE MPSTDP-S

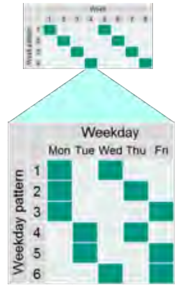
MIP Formulation of the MPSTDP-S

Decision variables

• Compactness • **Balance** • **Feasibility**

$$g_{bp} = \begin{cases} 1 & \text{if } \textbf{week pattern } p \in P(b) \\ & \text{is assigned to customer } b \in B \\ 0 & \text{otherwise} \end{cases}$$

$$h^w_{bq} = \begin{cases} 1 & \text{if } \textbf{weekday pattern } q \in Q(b) \\ & \text{is assigned to customer } b \in B \text{ in week } w \in W \\ 0 & \text{otherwise} \end{cases}$$



MIP Formulation of the MPSTDP-S

Auxiliary variables

• **Compactness** • Balance • Feasibility

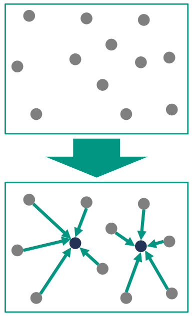
$$u^w_{ib} = \begin{cases} 1 & \text{if customer } b \in B \text{ is } \textbf{assigned to week center } i \in B \\ & \text{in week } w \in W \\ 0 & \text{otherwise} \end{cases}$$

$$v^d_{ib} = \begin{cases} 1 & \text{if customer } b \in B \text{ is } \textbf{assigned to day center } i \in B \\ & \text{on day } d \in D \\ 0 & \text{otherwise} \end{cases}$$

$$x^w_{b} = \begin{cases} 1 & \text{if customer } b \in B \text{ is selected as the } \textbf{week center} \\ & \text{in week } w \in W \\ 0 & \text{otherwise} \end{cases}$$

$$y^d_{b} = \begin{cases} 1 & \text{if customer } b \in B \text{ is selected as the } \textbf{day center} \\ & \text{on day } d \in D \\ 0 & \text{otherwise} \end{cases}$$

Compactness measurement



MIP Formulation of the MPSTDP-S

- Week | Day — Optimize compactness of week and day clusters
- Week — Week pattern selection
- Week — Assignment of customers to week centers
- Week — Week center selection
- Week — Weekly balance

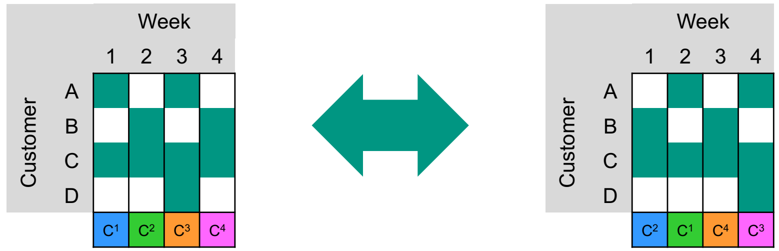
MIP Formulation of the MPSTDP-S

- Week | Day — Link week pattern and weekday pattern selection
- Day — Assignment of customers to day centers
- Day — Day center selection
- Day — Daily balance
- Week | Day — + Domain constraints

Symmetry

Symmetrical solutions

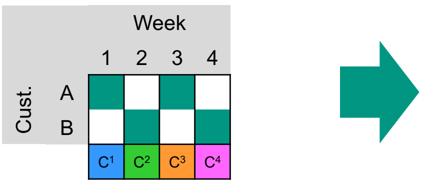
- Feasible permutations of clusters form symmetrical solutions
- Symmetry exists on the level of day and week clusters



Symmetry (cont.)

Feasible permutations of week clusters

- The set of feasible permutations depends on the planning horizon and week rhythms
- A set of feasible permutations may be determined which is valid for all instances of a given horizon
- 4 weeks planning horizon: Symmetry only constrained by biweekly customers



C^1	C^2	C^3	C^4
C^1	C^4	C^3	C^2
C^2	C^3	C^4	C^1
C^2	C^1	C^4	C^3
C^3	C^4	C^1	C^2
C^3	C^2	C^1	C^4
C^4	C^1	C^2	C^3
C^4	C^3	C^2	C^1

SOLUTION APPROACHES FOR THE MPSTDP-S

Two new solution approaches

Location-allocation

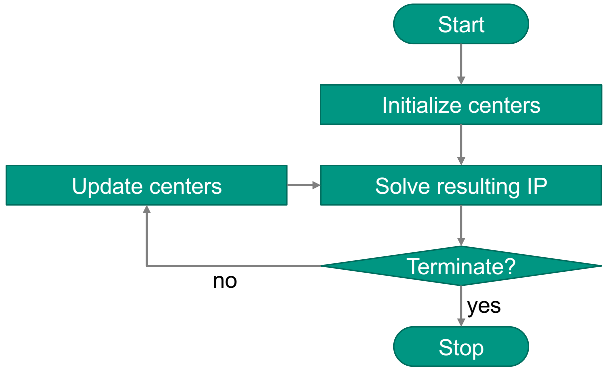
- Fast heuristic
- Covers all planning requirements of the MPSTDP-S
- Is based on model $SCHEDULE_{MIP}$ with variable fixations
- Extends the decomposition idea of Hess et al. (1965) to a multi-period setting

Branch-and-price

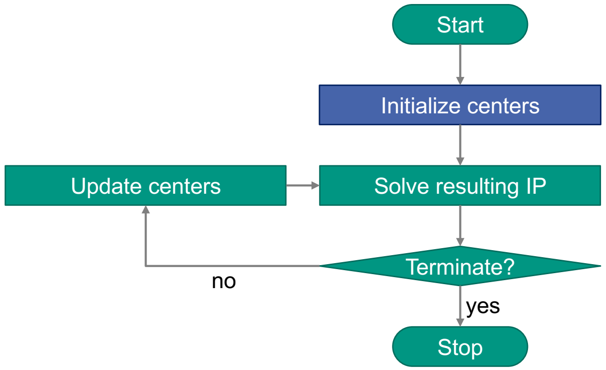
- Exact method
- For a special planning scenario of the MPSTDP-S
- Is based on a formulation with a huge number of variables
- Contains new, specially-tailored acceleration techniques

Hess, S.W., Weaver, J. B., Siegfeldt, H. J., Whelan, J. N., and Zitlau, P. A. (1965). Nonpartisan political redistricting by computer. *Operations Research*, 13(6):998–1006.

Location-allocation: Flowchart



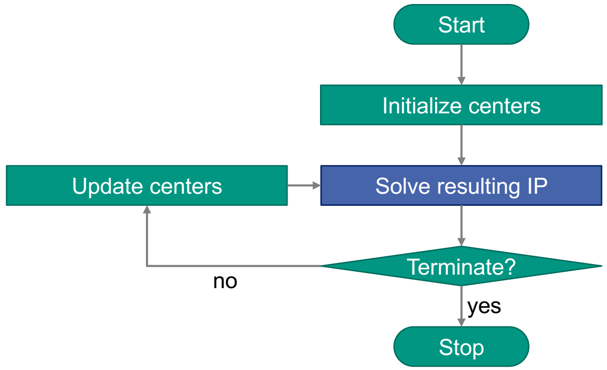
Location-allocation: Flowchart



Initialize centers

- Select suitable initial week and day centers

Location-allocation: Flowchart



Solve resulting IP

- Consider week and day centers in model $SCHEDULE_{MIP}$ as fixed
- Solve the resulting problem using the MIP solver Gurobi

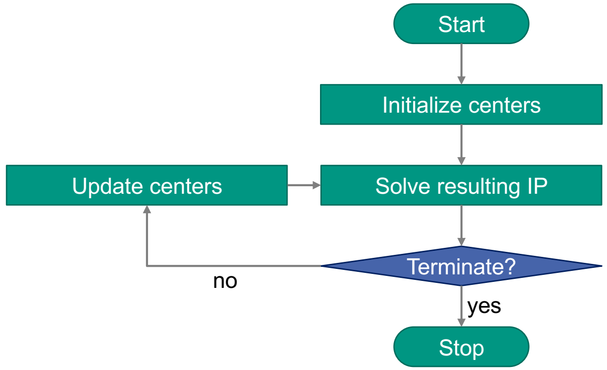
Integer Program with Fixed Centers

Attach distances to pattern variables

$$\lambda \sum_{b\in B} \sum_{p\in P_b} \bar{c}_{bp} g_{bp} + (1-\lambda) \sum_{b\in B} \sum_{q\in Q_b} \sum_{w\in W} \bar{c}^w_{bq} h^w_{bq} \to \min \quad (1)$$

- Week — Week pattern selection
- Week — Weekly balance
- Week, Day — Link week pattern and weekday pattern selection
- Day — Daily balance
- Week, Day — Week and weekday pattern variables

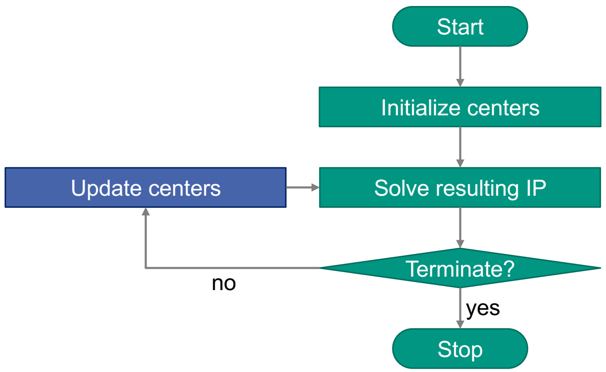
Location-allocation: Flowchart



Check termination
- Terminate if
  - no improvement

    or
  - iteration limit reached

Location-allocation: Flowchart



Update centers
- Select new week and day centers based on current solution

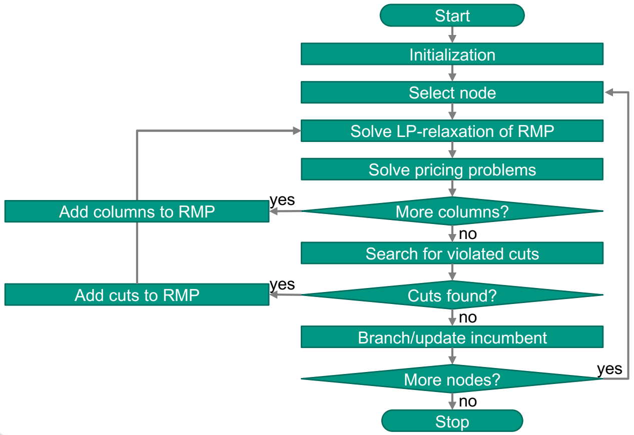
Branch-and-price: Introductory remarks

Considered planning scenario
- At most one service visit per customer and week
- No customer-specific restrictions of the feasible visiting days
- Identical service times for each service visit of the same customer

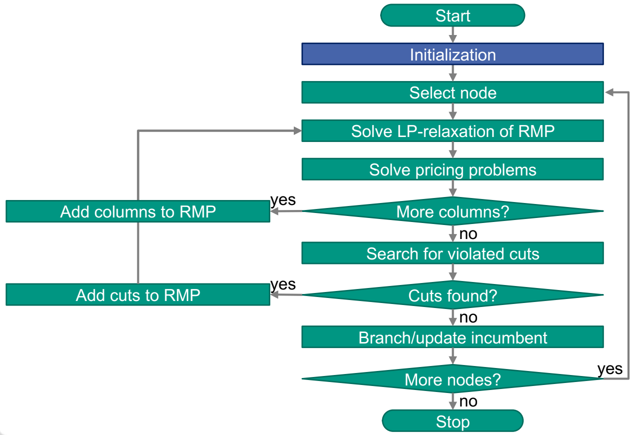
→ **Highly relevant scenario in practice**

Basic idea of the approach
- Do not consider individual customers, but feasible week and day clusters as variables/columns in the model.
- Select optimal combination of week and day clusters.
- Work with restricted master problem (RMP), which contains only a subset of all clusters, and generate new clusters only when needed.

Branch-and-Price: Flowchart



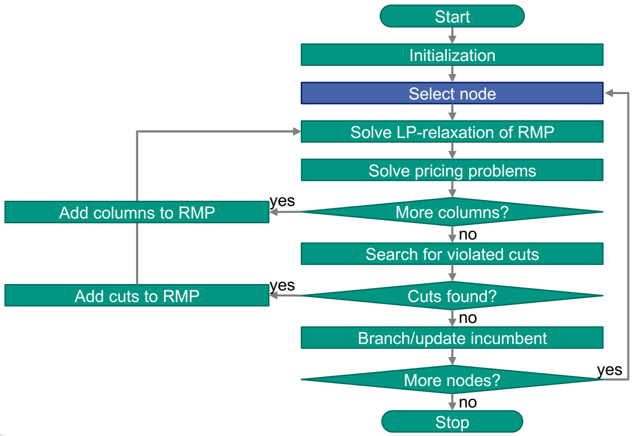
Branch-and-Price: Flowchart



Initialization

- Generate initial set of columns using the location-allocation heuristic
- Add root node to the set of active nodes

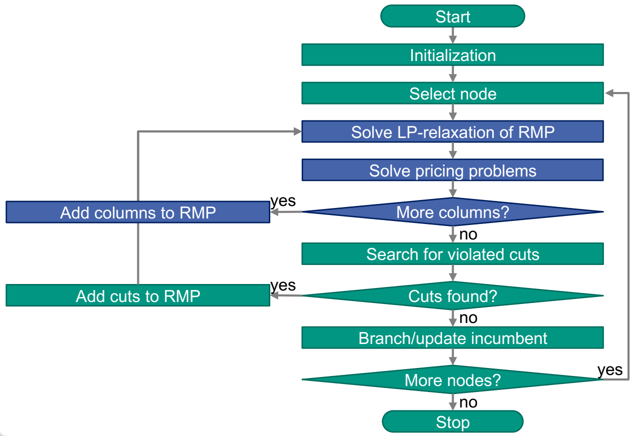
Branch-and-Price: Flowchart



Node selection

- Best-first strategy

Branch-and-Price: Flowchart



Column generation

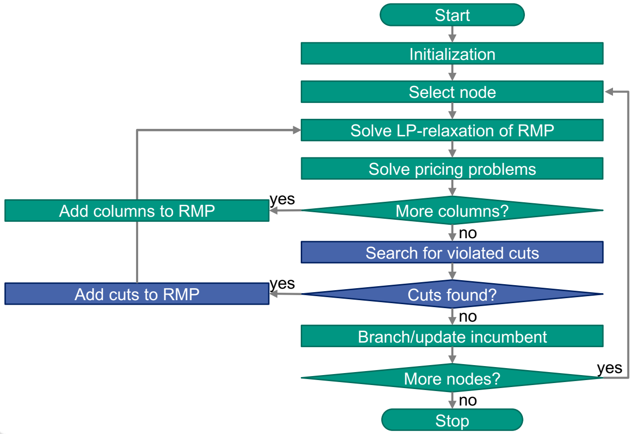
LP-relaxation of the RMP:

- Solve with MIP solver Gurobi
- Feasibility guaranteed through artificial columns

Pricing:

- Multiple independent problems
- Solve hierarchically: 1. heuristic, 2. exact

Branch-and-Price: Flowchart

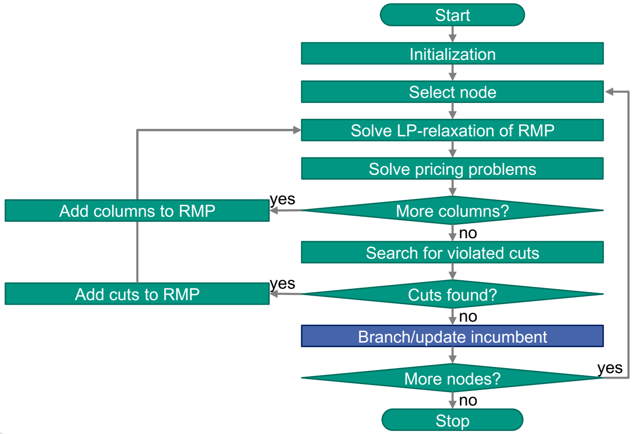


Cutting

- Subset-row inequalities (Jepsen et al., 2008)

Jepsen, M., Petersen, B., Spoorendonk, S., and Pisinger, D. (2008). Subset-row inequalities applied to the vehicle-routing problem with time windows. Operations Research, 56(2):497–511.

Branch-and-Price: Flowchart



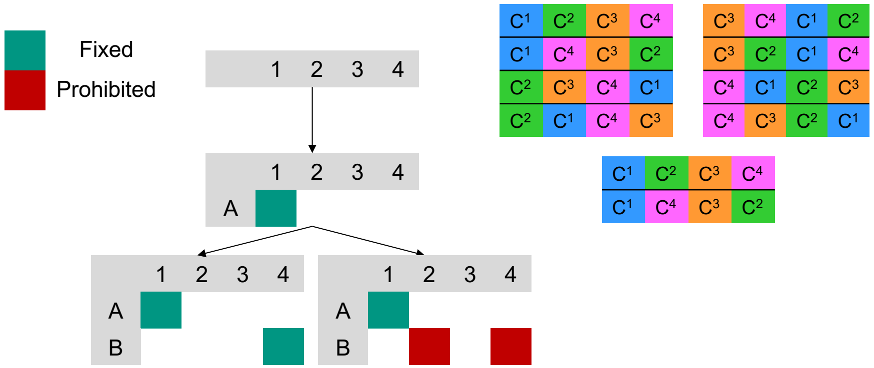
Branch

- Add further nodes to the set of active nodes
- Branch on week and day assignments of customers
- Special techniques for symmetry reduction

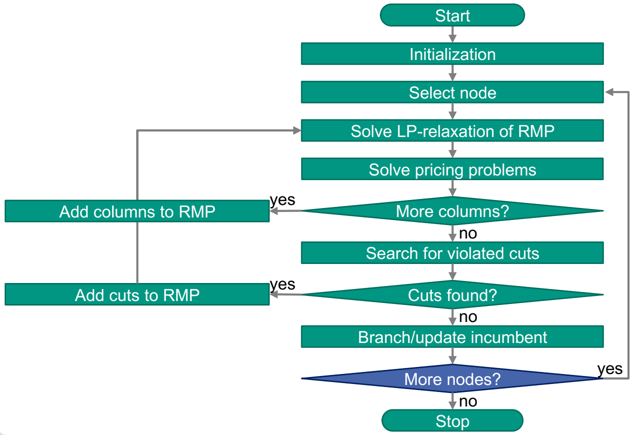
or

Update incumbent

Handling week symmetry



Branch-and-Price: Flowchart



Check termination

- Optimal solution found if set of active nodes empty

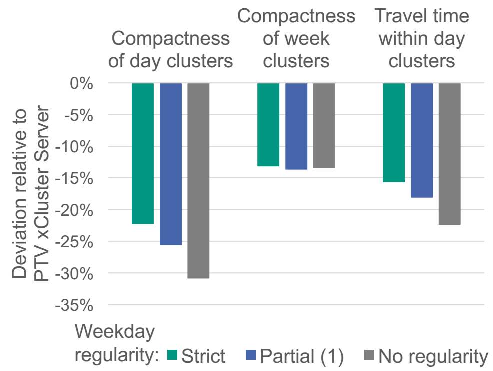
COMPUTATIONAL RESULTS

Evaluation of Location-Allocation Heuristic

Comparison with PTV xCluster Server version 1.18

- 480 real-world test instances and test instances derived from real-world data
  - On average 115 customers per test instance
  - Planning horizon consists of 16 or 48 weeks with 5 days per week
- Negative values correspond to improvements compared to xCluster.

→ **Significant improvement in all relevant measures**

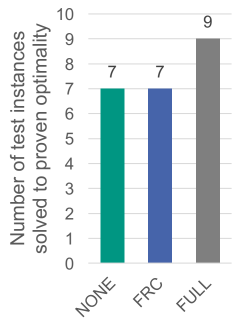


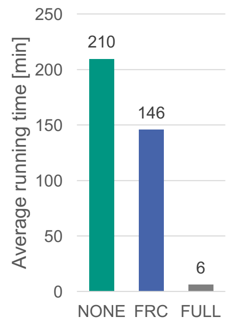
Evaluation of Branch&Price Algorithm

Symmetry breaking

- 9 real-world test instances
  - Between 25 and 35 customers per test instance
  - Planning horizon consists of 4 weeks with 5 days per week
- 3 tested settings
  - No symmetry reduction (NONE)
  - Fixing a reference customer (FRC)
  - Full symmetry reduction (FULL)
- Time limit of 10 h

→ **Running time reduction of over 97% by full symmetry reduction**



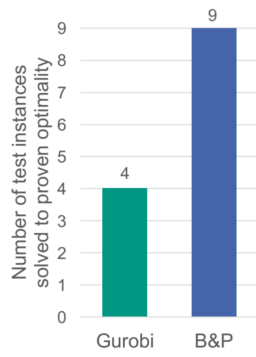


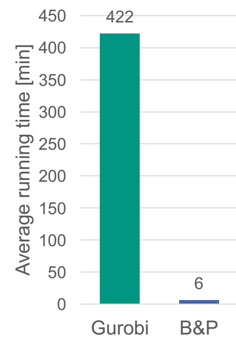
Evaluation of Branch&Price Algorithm

Comparison to MIP solver Gurobi

- 9 real-world test instances
  - Between 25 and 35 customers per test instance
  - Planning horizon consists of 4 weeks with 5 days per week
- Compact formulation
  - Includes symmetry-breaking constraints
  - Solved with MIP solver Gurobi
  - Time limit of 10 hours

→ **Running time reduction of over 98%**

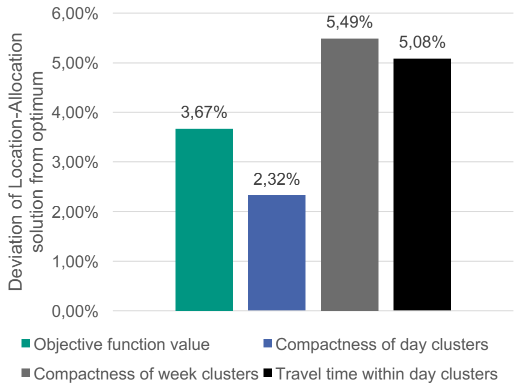




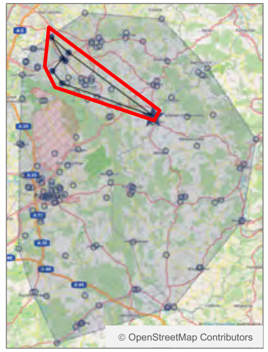
Comparison Location-Allocation and Branch&Price

Solution quality

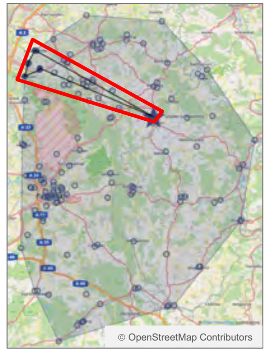
- 16 real-world test instances
  - Contain 25 – 55 customers
  - Planning horizon consists of 4 weeks with 5 days per week
- Percentage values correspond to the deviation of the location-allocation solution from the optimal solution



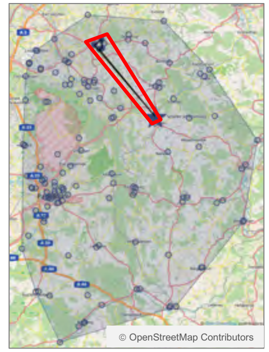
Exemplary day clusters: Mon – Wed



Monday

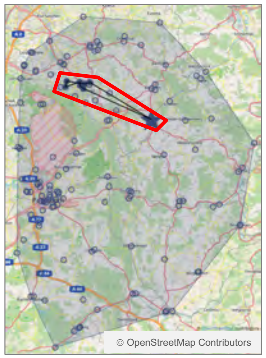


Tuesday

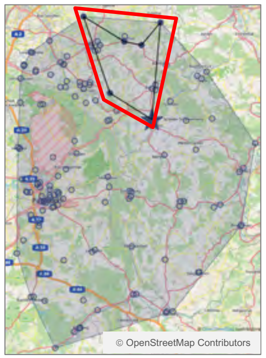


Wednesday

Exemplary day clusters: Thu – Fri

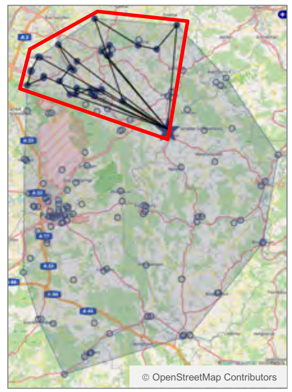


Thursday



Friday

Exemplary week cluster



CONCLUSION

Summary and Outlook

Summary

- We have introduced a highly relevant new problem.
- We have proposed two solution approaches and evaluated their performance:
  - The location-allocation heuristic clearly beats the software product PTV xCluster.
  - The branch-and-price algorithm outperforms the MIP solver Gurobi.
- With the release in December 2016, PTV Group has replaced the previous algorithm in their xCluster Server with an algorithm based on our location-allocation approach.

Outlook

- Integration of additional planning criteria, e.g.
  - Planning of overnight stays
  - Incorporation of travel time approximations

Literature

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