Heuristics and models for the stochastic uncapacitated r-allocation p-hub median problem

Some managerial insights

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The uncapacitated r-allocation p-hub median problem

The uncapacitated r-allocation p-hub median problem

- Given a network $G = (V, E)$, for each pair of nodes i and $j \in V$, there is a traffic $t_{ij} \geq 0$ to be transported
- p, the number of hubs to be open, is an input
- Each node i is assigned to at most r of the p hubs (r is also an input)
- No capacity constraints
- Direct transportation between nodes is not common but possible. Traffic t_{ij} travels along a path $i \to k \to l \to j$ or uses a non-stop service.

The uncapacitated r-allocation p-hub median problem

It was originally formulated as a MILP by Hande Yaman:

Allocation strategies in hub networks.
European Journal of Operational Research, 211: 442–451, 2011.

We extend the model, using decision variables:

- Binary variables of opening hubs and assigning nodes to hubs:
  - $z_{kk} = 1$ if $k \in V$ is a hub, and 0 otherwise.
  - $z_{ik} = 1$ if node i is assigned to node k, and 0 otherwise.
  - $w_{kl} = 1$ to connect hubs k and l, and 0 otherwise.
- Continuous variables to route the traffics
  - x_{ijkl} the proportion of the traffic t_{ij} that travels along the path $i \rightarrow k \rightarrow l \rightarrow j$.
  - y_{ij} for the proportion of the traffic t_{ij} that travels using a non-stop service between i and j

We have extended the model

Objective:

$$\min \sum_{i,k \in V} a_{ik} z_{ik} + \sum_{k,\ell \in V,\, k<\ell} f_{k\ell} w_{k\ell} + \sum_{i,j,k,\ell \in V} c_{ijk\ell} t_{ij} x_{ijk\ell} + \sum_{i,j \in V} (d_{ij} t_{ij} + b_{ij}) y_{ij}, \tag{1}$$

Subject to:

$$\sum_{k \in V} z_{kk} = p, \tag{2}$$

$$\sum_{k \in V} z_{ik} \leq r, \qquad \forall i \in V, \tag{3}$$

$$z_{ik} \leq z_{kk}, \qquad \forall i, k \in V, \tag{4}$$

$$\sum_{\ell \in V} x_{ijk\ell} \leq z_{ik}, \qquad \forall i, j, k \in V, \tag{5}$$

$$\sum_{k \in V} x_{ijk\ell} \leq z_{j\ell}, \qquad \forall i, j, \ell \in V, \tag{6}$$

$$\sum_{k,\ell \in V} x_{ijk\ell} + y_{ij} = 1, \qquad \forall i, j \in V : t_{ij} > 0, \tag{7}$$

We have extended the model

$$w_{k\ell} \leq z_{kk}, \qquad \forall k, \ell \in V,\ k < \ell, \tag{8}$$

$$w_{k\ell} \leq z_{\ell\ell}, \qquad \forall k, \ell \in V,\ k < \ell, \tag{9}$$

$$z_{kk} + z_{\ell\ell} \leq w_{k\ell} + 1, \qquad \forall k, \ell \in V,\ k < \ell, \tag{10}$$

$$\sum_{j \in V} y_{ij} + \sum_{j \in V} y_{ji} \leq M\left(1 - z_{ii}\right), \qquad \forall i \in V, \tag{11}$$

$$x_{ijk\ell} \geq 0, \qquad \forall i, j, k, \ell \in V, \tag{12}$$

$$y_{ij} \geq 0, \qquad \forall i, j \in V, \tag{13}$$

$$z_{ik} \in \{0, 1\}, \qquad \forall i, k \in V, \tag{14}$$

$$w_{k\ell} \in \{0, 1\}, \qquad \forall k, \ell \in V, k < \ell. \tag{15}$$

We introduce uncertainty in the model

We will assume that demands and costs are **NOT KNOWN** in advance, but can be captured by a probability distribution.

We will use a two-stage stochastic programming model.

For every $i, j \in V$, we assume t_{ij}, c_{ij}, b_{ij}, and d_{ij} to be random. The random vector is
$\boldsymbol{\xi} = \left[[t_{ij}]_{i,j \in V}, [c_{ij}]_{(i,j) \in E}, [b_{ij}]_{(i,j) \in E}, [d_{ij}]_{(i,j) \in E} \right]$.

Each realization of $\boldsymbol{\xi}$ is a scenario.

A two-stage stochastic version of the UrApHMP-NSS

$$\min \quad \sum_{i,k\in V} a_{ik} z_{ik} + \sum_{k,\ell\in V,\, k<\ell} f_{k\ell} w_{k\ell} + \mathcal{Q}(\mathbf{z},\mathbf{w}), \tag{16}$$

$$\text{s. t.} \quad (2)-(4), (8)-(10), (14), (15).$$

where $\mathcal{Q}(\mathbf{z},\mathbf{w}) = E_{\boldsymbol{\xi}}[Q(\mathbf{z},\mathbf{w},\boldsymbol{\xi})]$ is the mathematical expectation with respect to $\boldsymbol{\xi}$, and

each $Q(\mathbf{z},\boldsymbol{\xi}) =$

$$Q(\mathbf{z},\mathbf{w},\boldsymbol{\xi}) = \min \sum_{i,j,k,\ell\in V} c_{ijk\ell} t_{ij} x_{ijk\ell} + \sum_{i,j\in V} (d_{ij} t_{ij} + b_{ij}) y_{ij}, \tag{17}$$

$$\text{s. t.} \sum_{\ell\in V} x_{ijk\ell} \le z_{ik}, \qquad \forall i,j,k\in V, \tag{18}$$

$$\sum_{k\in V} x_{ijk\ell} \le z_{j\ell}, \qquad \forall i,j,\ell\in V, \tag{19}$$

$$\sum_{k,\ell\in V} x_{ijk\ell} + y_{ij} = 1, \qquad \forall i,j\in V : t_{ij} > 0, \tag{20}$$

$$\sum_{j\in V} y_{ij} + \sum_{j\in V} y_{ji} \le M\,(1 - z_{ii}), \quad \forall i\in V, \tag{21}$$

$$x_{ijk\ell} \ge 0, \qquad \forall i,j,k,\ell\in V, \tag{22}$$

$$y_{ij} \in \{0,1\}, \qquad \forall i,j\in V. \tag{23}$$

The deterministic equivalent model if the support of $\boldsymbol{\xi}$ is finite

$$\min \sum_{i,k\in V} a_{ik} z_{ik} + \sum_{k,\ell\in V,\, k<\ell} f_{k\ell} w_{k\ell} + \sum_{s\in S} \pi_s \left[\sum_{i,j,k,\ell\in V} c_{ijk\ell s} t_{ijs} x_{ijk\ell s} + \sum_{i,j\in V} (d_{ijs} t_{ijs} + b_{ijs}) y_{ijs} \right], \tag{24}$$

s. t. $(2)-(4), (8)-(10), (14), (15),$

$$\sum_{\ell\in V} x_{ijk\ell s} \leq z_{ik}, \qquad \forall i,j,k\in V,\ s\in S, \tag{25}$$

$$\sum_{k\in V} x_{ijk\ell s} \leq z_{j\ell}, \qquad \forall i,j,\ell\in V,\ s\in S, \tag{26}$$

$$\sum_{k,\ell\in V} x_{ijk\ell s} + y_{ijs} = 1, \qquad \forall i,j\in V,\ s\in S, \tag{27}$$

$$\sum_{j\in V} y_{ijs} + \sum_{j\in V} y_{jis} \leq M\left(1 - z_{ii}\right), \qquad \forall i\in V,\ s\in S, \tag{28}$$

$$x_{ijk\ell s} \geq 0, \qquad \forall i,j,k,\ell\in V,\ s\in S, \tag{29}$$

$$y_{ijs} \in \{0,1\}, \qquad \forall i,j\in V,\ s\in S. \tag{30}$$

Our proposal: Heuristic optimization

Our assumption:

Good solutions for the single-scenario problems $\mathcal{P}_s$, $s \in S$, may contain information about good attributes of a solution to $\mathcal{P}$

- Build solutions to $\mathcal{P}$ using the solutions of $\mathcal{P}_s$, $s \in S$.
- Note that solutions for problems $\mathcal{P}_s$, $s \in S$, may render different network designs:
  - distinct hubs selection
  - distinct allocations of terminals
  - the number of hubs to which a terminal is assigned to may be different for one scenario to another

Name: Greedy Attributive Scenario Based Constructive Method

Computational experiments and some managerial insights

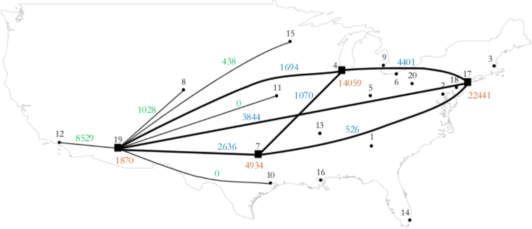
Competitive testing for CAB-based instances

					Heuristic					
			CPLEX		CPU (sec.)			Dev (%)		
n	p	r	#solved	CPU (sec.)	min	avg	max	min	avg	max
15	3	3	10	456	20	27	34	0.1	1.2	2.3
15	3	2	10	745	22	25	33	0.5	1.4	2.3
15	3	1	10	1118	20	28	34	0.4	2.0	4.9
20	3	3	10	3986	117	127	138	0.3	1.6	2.4
20	3	2	10	4150	119	128	138	0.6	1.8	3.0
20	3	1	3	6970	126	134	137	2.3	2.8	3.4
20	4	4	10	4924	297	331	365	0.9	1.9	2.7
20	4	3	10	6437	301	328	354	0.1	1.6	2.7
20	4	2	10	6414	297	338	368	0.1	2.2	4.1
20	4	1	2	7352	322	326	329	3.8	4.0	4.1

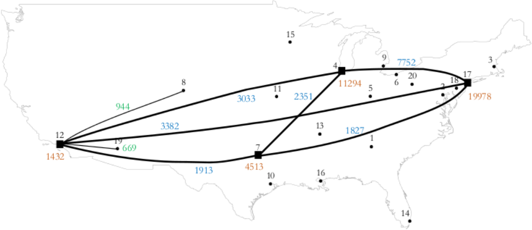
Competitive testing for AP-based instances

			CPLEX		Heuristic					
					CPU (sec.)			Dev (%)		
n	p	r	#solved	CPU (sec.)	min	avg	max	min	avg	max
15	3	3	3	575	31	32	33	0.2	2.3	3.5
15	3	2	3	794	23	25	28	0.1	0.4	0.9
15	3	1	3	1141	23	27	30	0.9	1.5	2.3
20	3	3	3	4878	120	122	124	1.8	2.1	2.5
20	3	2	3	5916	127	127	129	0.3	1.8	3.2
20	3	1	0	n/a	n/a	n/a	n/a	n/a	n/a	n/a
20	4	4	3	5603	268	278	288	1.1	1.6	2.1
20	4	3	3	6041	292	316	332	2.6	3.2	3.9
20	4	2	2	6711	250	277	305	1.7	2.7	3.8
20	4	1	1	7224	169	169	169	3.5	3.5	3.5

The optimal solution is not optimal for any scenario



(a) $\alpha = 1.0$.



(b) $\alpha = 0.6$.

The effect in here-and-now decisions when α changes

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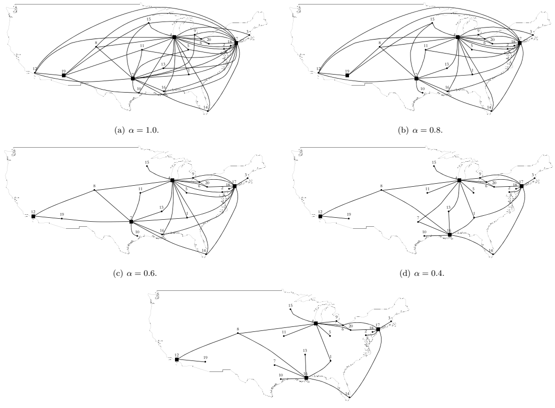

(a) $\alpha = 1.0$.

(b) $\alpha = 0.8$.

(c) $\alpha = 0.6$.

(d) $\alpha = 0.4$.

(e) $\alpha = 0.2$.

Figure 3: 20-node CAB instance with $p = 4$, $r = 4$: representation of the hubs and allocations.

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