A MATHEURISTIC FOR THE RAPID TRANSIT NETWORK DESIGN PROBLEM WITH ELASTIC DEMAND

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LOGRO

UNIVERSIDAD DE CORDOBA

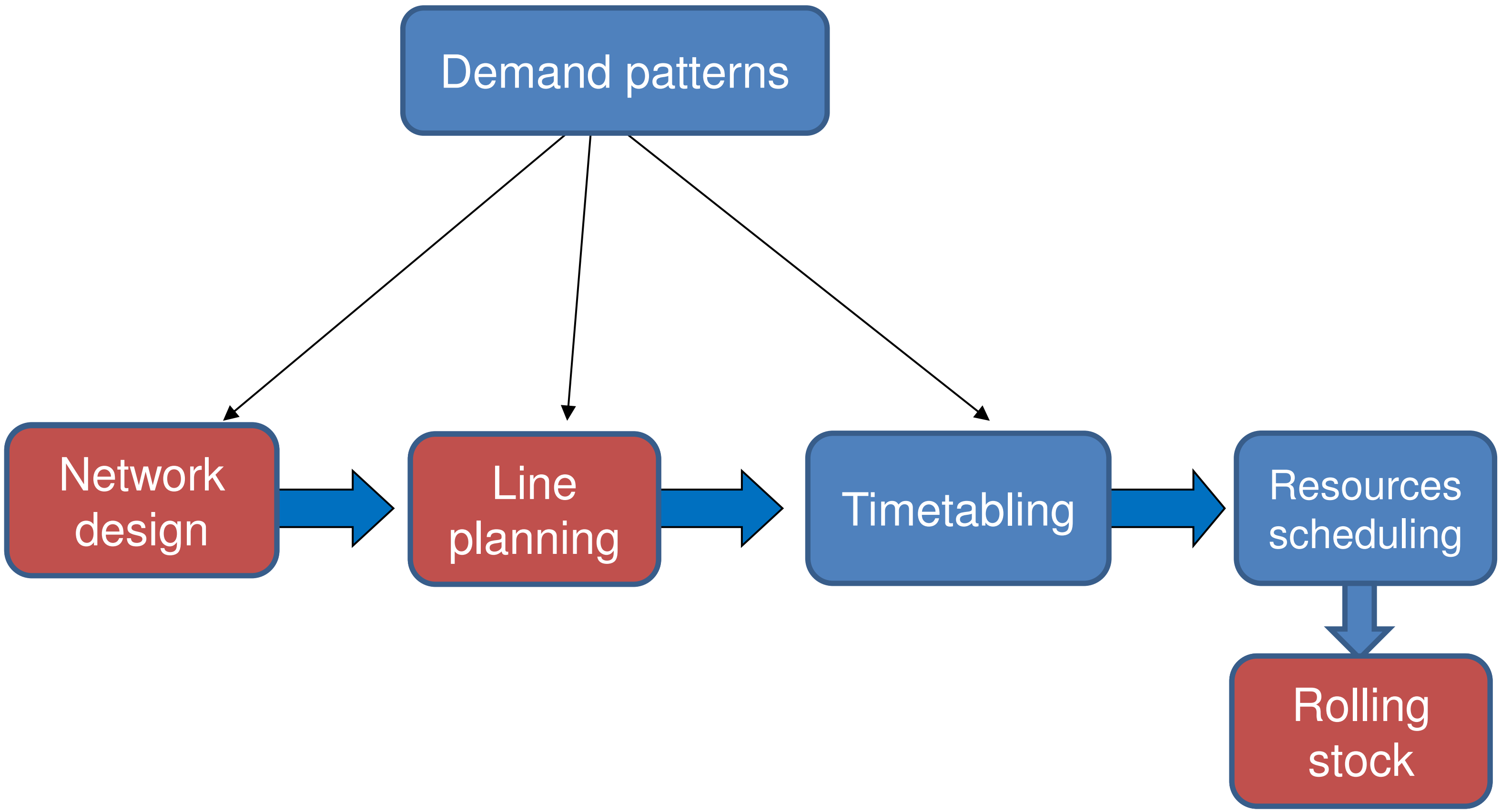
Rapid Transit System (RTS)

Metro of Seville

Wuppertal monorail
(Germany)

Light metro of Oporto

Transportation line planning process

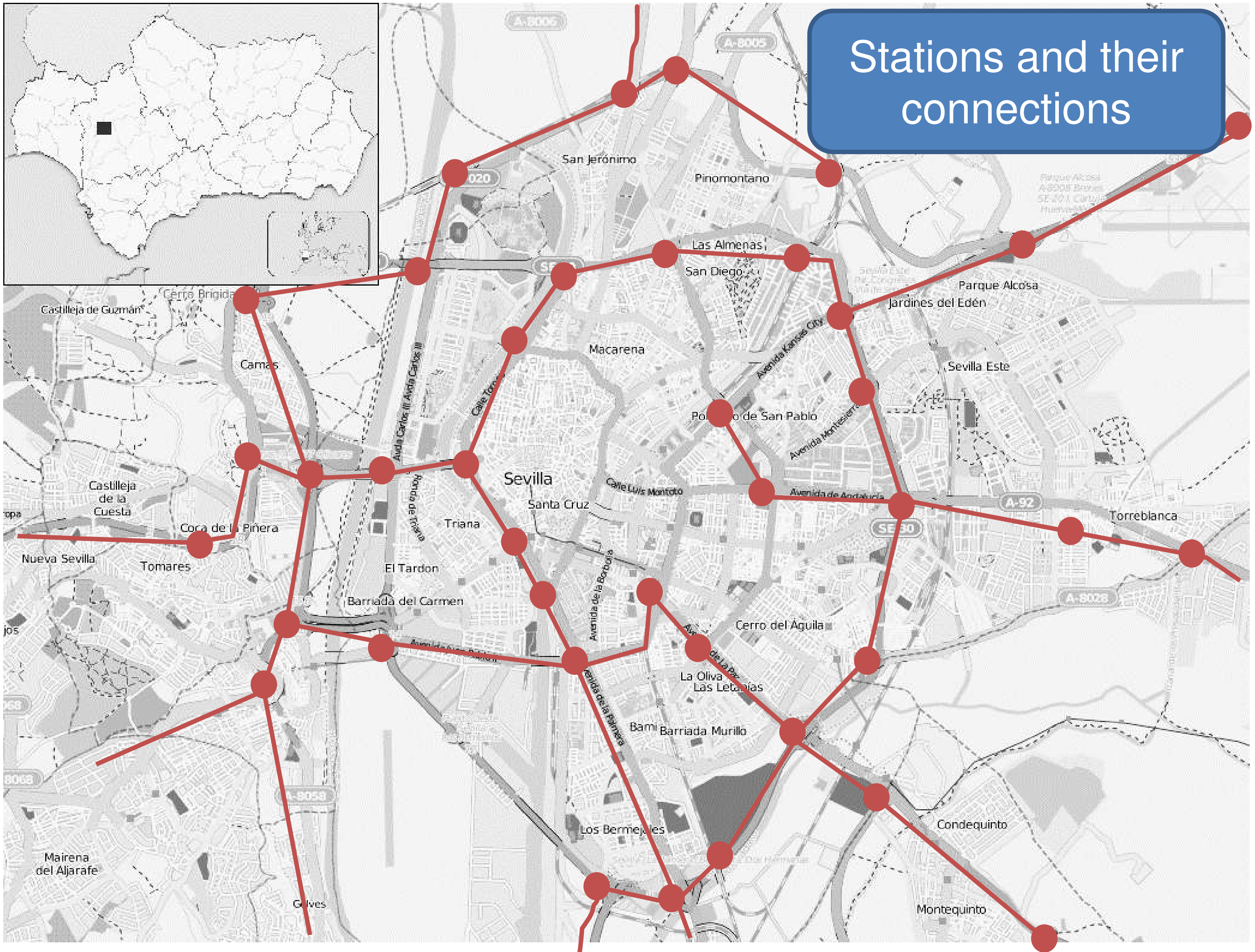


THE PROBLEM

The problem

San Jerónimo
Pinomontano
Las Almenas
Parque Alcosa
Jardines del Edén
Castilleja de Guzmán
Camas
Sevilla Este
Sevilla
Santa Cruz
Triana
Castilleja de la Cuesta
Coca de la Piñera
Torreblanca
Nueva Sevilla
Tomares
El Tardón
Barriada del Carmen
Cerro del Águila
La Oliva
Barriada Murillo
Los Bermejales
Condequinto
Mairena del Aljarafe
Gelves
Montequinto

The problem


Stations and their connections
San Jerónimo
Pinomontano
Las Almenas
San Diego
Parque Alcosa
Jardines del Edén
Macarena
Sevilla Este
Castilleja de Guzmán
Camas
Sevilla
Santa Cruz
Triana
Castilleja de la Cuesta
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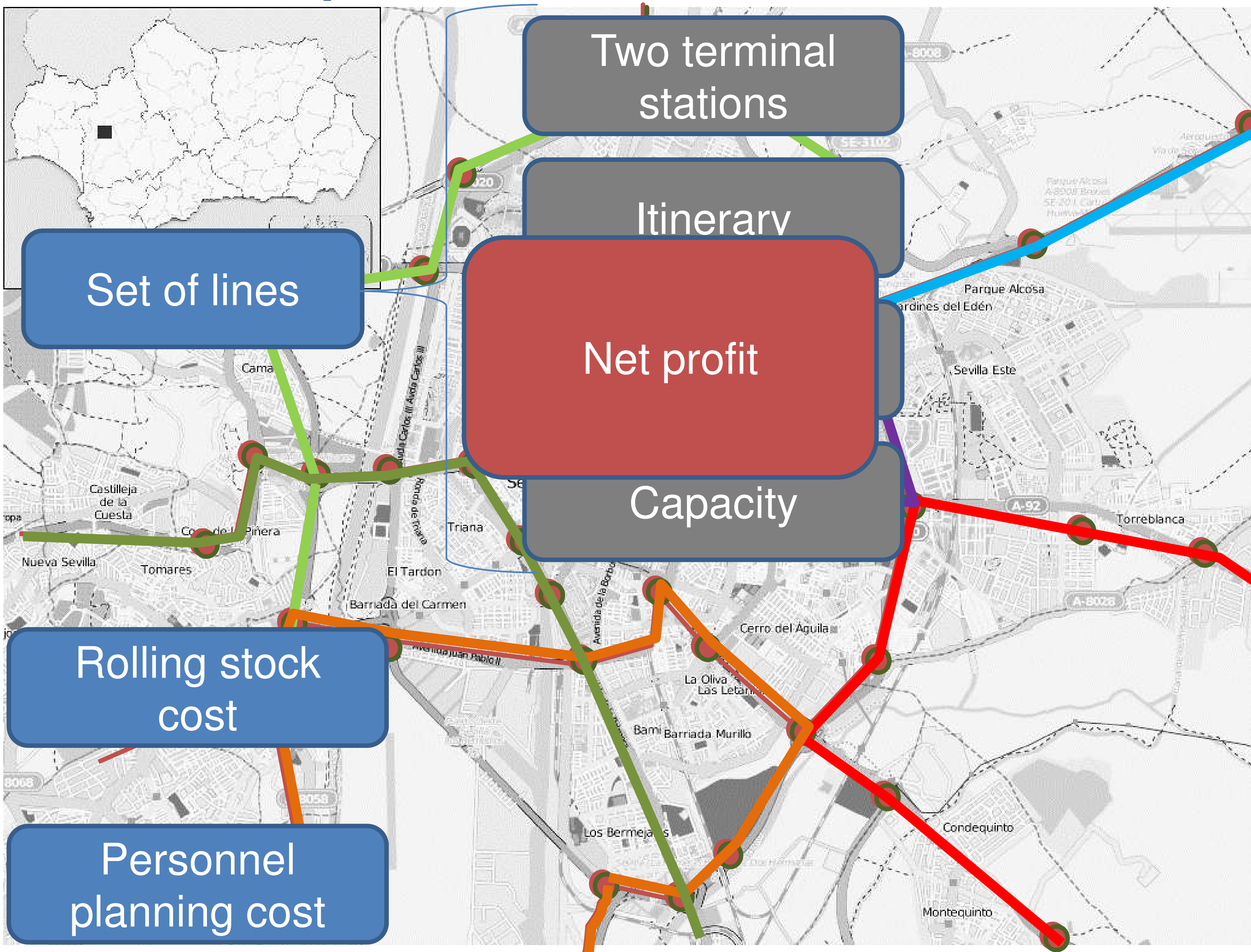
Mairena del Aljarafe
Gelves

The problem



Two terminal stations

Itinerary

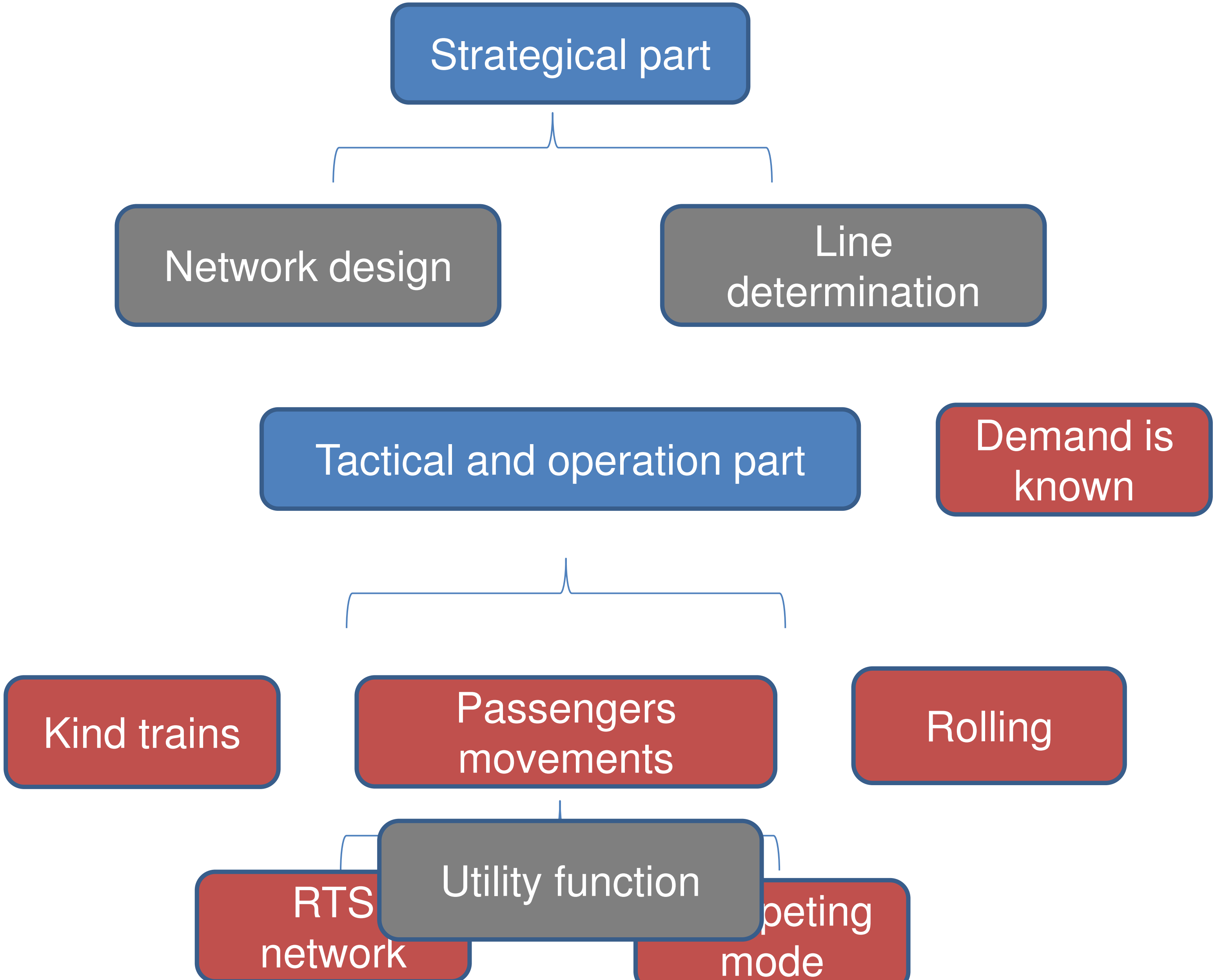
Net profit

Capacity

Set of lines

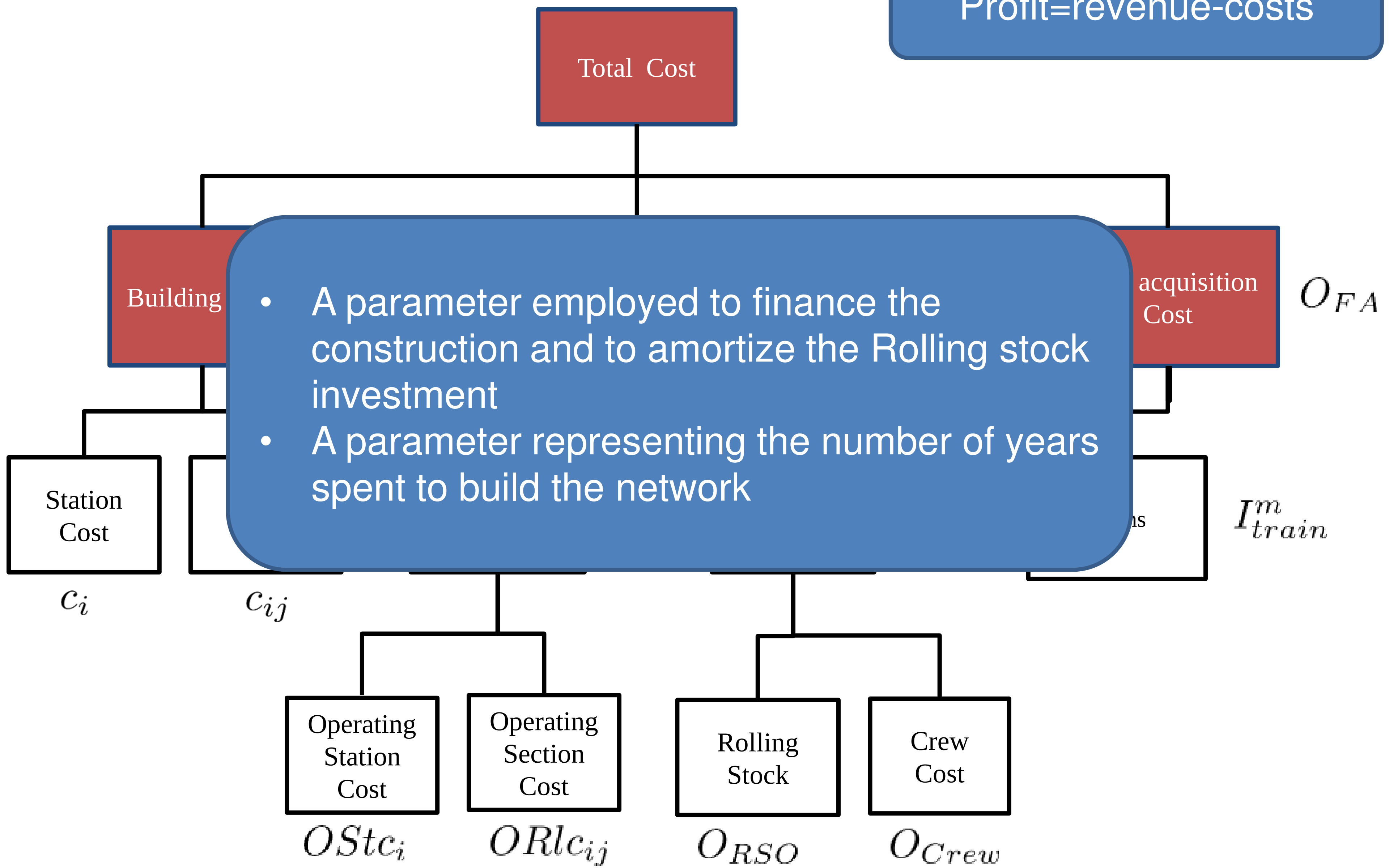
Rolling stock cost

Personnel planning cost


Strategical part
Network design
Line determination
Tactical and operation part
Demand is known
Kind trains
Passengers movements
Rolling
Utility function
RTS network
peting mode

IV. Costs

Profit=revenue-costs



- A parameter employed to finance the construction and to amortize the Rolling stock investment
- A parameter representing the number of years spent to build the network

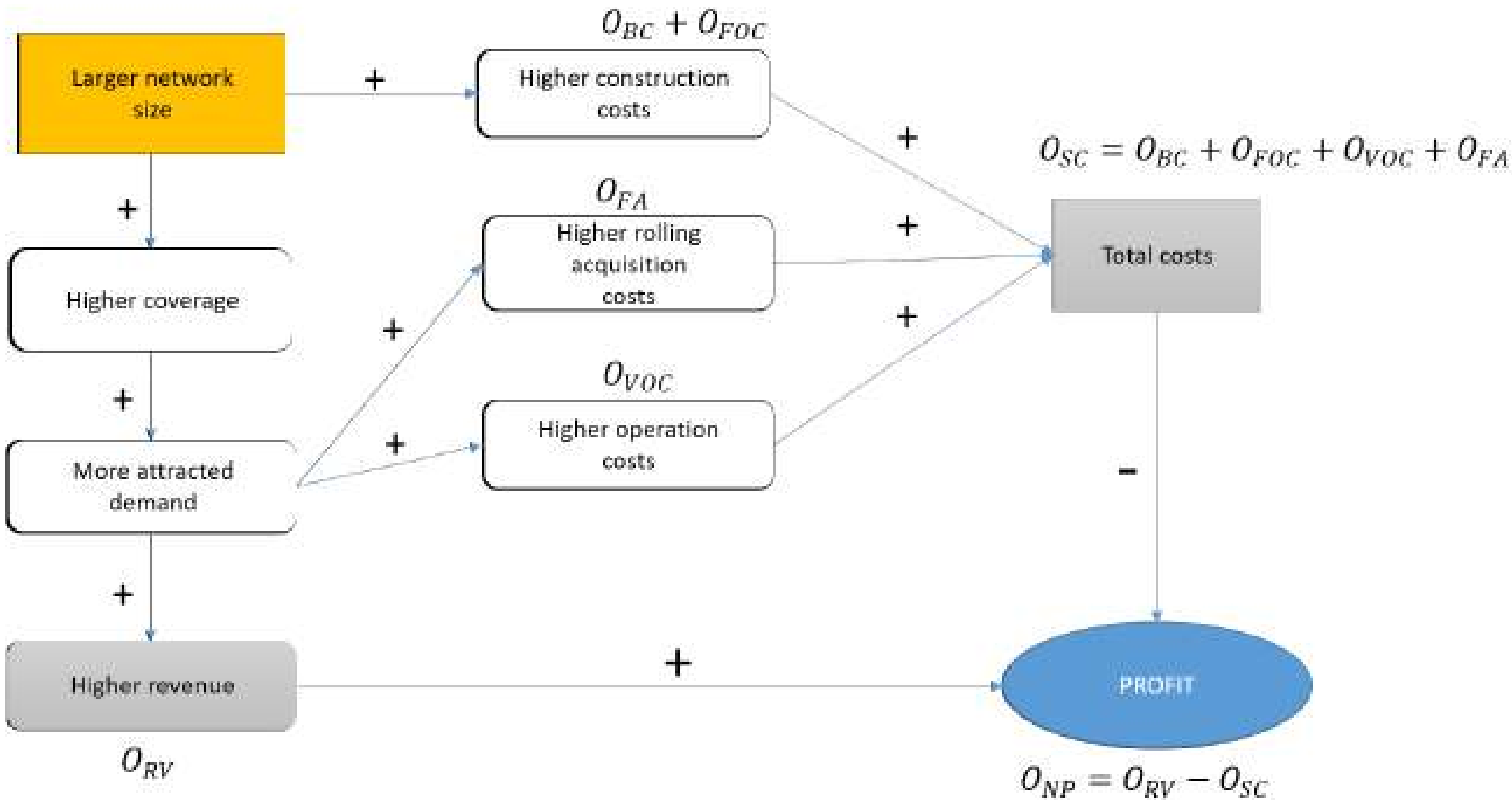
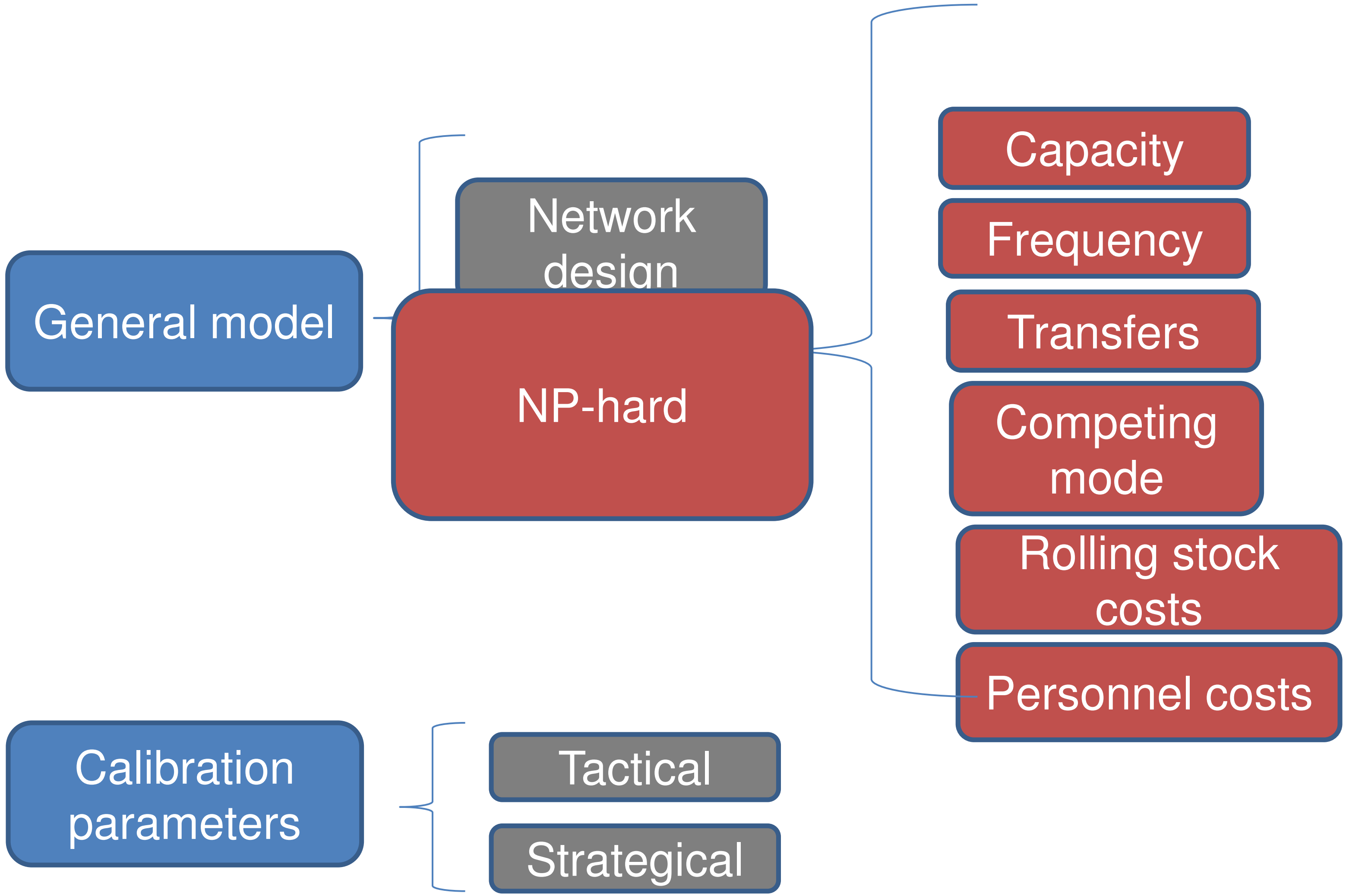


Figure 1: Integrated Network Design and Line Planning.

Previous works

- D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. A General rapid network design, line planning and fleet investment integrated model. *Annals of Operational Research*
- D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. An adaptive neighborhood search metaheuristic for the integrated railway rapid transit network design and line planning problem. Computers & Operations Research. 78, pp. 1 - 14. 2017.
- D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. The railway network design, line planning and capacity problem: An adaptive large neighborhood search metaheuristic. Advances in Intelligent Systems and Computing. pp. 1 - 22. 2017.

D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. A General rapid network design, line planning and fleet investment integrated model. *Annals of Operational Research*



Variables

I. Design

$y_i^\ell,\, y_i \in \{0,1\}$ → Stations

$x_{ij}^\ell,\, x_{ij} \in \{0,1\}$ → Edges

$h_\ell \in \{0,1\}$ → Lines

II. Flow

$f_{ij}^{w\ell} \in [0,1]$ → Flow arc and line

$f_w^{ALT},\, f_w^{RTS} \in [0,1]$ → Proportion demand

$f_i^{w\ell\ell'} \in [0,1]$ → Transfers

Variables

III. Headway and frequency

$\varsigma_\ell \in H$ → Headway

$\varphi_\ell = 60/\varsigma_\ell$ → Frequency

IV. Capacity

$\delta_m^\ell \in \{0,1\}$ → Kind of train

V. Travel time

$U_w^{RRT} > 0$ → Travel time

VI. Fleet size

FS_ℓ → Required fleet line

D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. An adaptive neighborhood search metaheuristic for the integrated railway rapid transit network design and line planning problem. Computers & Operations Research. 78, pp. 1 - 14. 2017

- Binary assigment model (flow: binary variables)
- The passenger only can consider one path in the network
- We propose a metaheuristic ALNS for solving the problem.
- We compare the mathematical model againts the ALNS on small instances
- We solve the problem on a real life network using the ALNS.

D. Canca, A. De-Los-Santos, G. Laporte and J.A. Mesa. The railway network design, line planning and capacity problem: An adaptive large neighborhood search metaheuristic. Advances in Intelligent Systems and Computing. pp. 1 - 22. 2017.

- We use a new utility function considering prices of time

$$U_w^{RRT} = \eta + \beta_{tt} \cdot u_w^{RRT,tt} + \beta_{tr} \cdot u_w^{RRT,tr} + \beta_{tw} \cdot u_w^{RRT,tw}, \; w \in W$$

PROBLEM DESCRIPTION

- Original problem in which flow variables are continuos
- Several paths, for a same OD-pairs it´s possible
- Utility function considering value of time

Input data

I. Topology

$N = \{1, \ldots, n\}$ → Nodes

$A \subseteq N \times N$ → Arcs

$E = \{\{i,j\} : (i,j) \in A, i < j\}$ → Edges

$N(i) = \{j \in N : \{i,j\} \in E\}$ → Incidents nodes

II. Metric and demand

$d_{ij}, \forall \{i,j\}$ → Distance

$\lambda,\ \eta,\ \mu$ → Speed, fare, subsidy

$W = \{w_1, \ldots, w_v\}$ $g_w,\ u_w^{ALT}$ → OD pairs, demand
Alternative time

Input data

III. Lines

$\mathcal{L}, \ell_{min}, \ell_{max}$ → Lines, minimum and maximum length

$\gamma, \mathcal{H}$ → Maximum number of lines per edge, set of possibles headways

$K^{\ell}_{train}, m \in M$ → Capacity for kind of trains

Variables

I. Design

$y_i^\ell,\ y_i \in \{0,1\}$ → Stations

$x_{ij}^\ell,\ x_{ij} \in \{0,1\}$ → Edges

$h_\ell \in \{0,1\}$ → Lines

II. Flow

$f_{ij}^{w\ell} \in [0,1]$ → Flow arc and line

$f_w^{RRT} \in [0,1]$ → Proportion demand

$f_i^{w\ell\ell'} \in [0,1]$ → Transfers

Variables

III. Headway and frequency

$\varsigma_\ell \in H$ → Headway

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V. Travel time

$U_w^{RRT} > 0$ → Travel time

VI. Fleet size

FS_ℓ → Required fleet line

Constraints

edges

stations

Budget constraint

$$\sum_{\{i,j\}\in E} c_{ij}x_{ij} + \sum_{i\in N} c_i y_i \leq C_{max}$$

Upper bound

Design constraints

$$x_{ij}^{\ell} \leq y_i^{\ell},\ \{i,j\}\in E,\ i<j,\ \ell\in\mathcal{L}$$

$$x_{ij}^{\ell} \leq y_j^{\ell},\ \{i,j\}\in E,\ i<j,\ \ell\in\mathcal{L}$$

$$x_{ij}^{\ell} = x_{ji}^{\ell},\ \{i,j\}\in E,\ i<j,\ \ell\in\mathcal{L}$$

$$x_{ij} \leq \sum_{\ell\in\mathcal{L}} x_{ij}^{\ell},\ \{i,j\}\in E,\ i<j$$

$$\sum_{\ell\in\mathcal{L}} x_{ij}^{\ell} \leq \gamma x_{ij},\ \{i,j\}\in E,\ i<j$$

$$\sum_{j\in N(i)} x_{ij}^{\ell} \leq 2,\ i\in N,\ \ell\in\mathcal{L}$$

Edges and stations

Lines

Design constraints

$$h_\ell + \sum_{ij \in E} x_{ij}^\ell = \sum_{i \in N} y_i^\ell,\ \ell \in \mathcal{L}$$

$$\ell_{\min} h_\ell \leq \sum_{\{i,j\} \in E} x_{ij}^\ell \leq \ell_{\max} h_\ell,\ \ell \in \mathcal{L}$$

$$\sum_{i \in B} \sum_{j \in B} x_{ij}^\ell \leq |B| - 1,\ B \subseteq N,\ |B| \geq 2,\ \ell \in \mathcal{L}$$

Line

Sub-tour elimination

Flow

Transfers

Flow conservation

$$\sum_{\ell \in \mathcal{L}} \sum_{j \in N(w_s)} f_{w_s j}^{w\ell} = f_w^{RRT},\ w = (w_s, w_t) \in W$$

Outgoing origin

$$\sum_{\ell \in \mathcal{L}} \sum_{i \in N(w_t)} f_{i w_t}^{w\ell} = f_w^{RRT},\ w = (w_s, w_t) \in W$$

Ingoing destination

$$\sum_{i \in N(k)} f_{ik}^{w\ell} - \sum_{\ell' \in \mathcal{L} \setminus \{\ell\}} f_k^{w\ell\ell'} + \sum_{\ell' \in \mathcal{L} \setminus \{\ell\}} f_k^{w\ell'\ell} - \sum_{j \in N(k)} f_{kj}^{w\ell} = 0$$

$$\ell \in \mathcal{L},\ w = (w_s, w_t) \in W,\ k \neq \{w_s, w_t\},\ k \in N$$

Balance

Location-allocation constraints

$$f_{ij}^{w\ell} \leq x_{ij}^{\ell},\ \ell \in \mathcal{L},\ w \in W, \{i,j\} \in E, i < j$$
$$f_i^{w\ell\ell'} \leq y_i^{\ell},\ \ell, \ell' \in \mathcal{L}$$
$$f_i^{w\ell\ell'} \leq y_i^{\ell'},\ \ell, \ell' \in \mathcal{L}$$

Demand and design

Splitting demand constraints

$$f_w^{RRT} \leq \frac{1}{1 + e^{(\beta(u_w^{ALT} - u_w^{RRT}))}},\ w \in W$$

Logit function

Capacity constraints

$$\sum_{w \in W} f_{ij}^{w\ell} g_w \leq \psi_\ell \sum_{m \in M} C_{train}^m \cdot \delta_\ell^m,\ \ell \in \mathcal{L}, \{i,j\}$$

Needed trains

Frequency

Kind of train

$$\sum_{m \in M} \delta_\ell^m = 1,\ \ell \in \mathcal{L}$$

Only one type of train

Travel time constraints

$$u_w^{RRT,tt} = (60/f_w^{RRT}\lambda)\sum_{\ell\in L}\sum_{(i,j)\in A(\ell)} f_{ij}^{w\ell} d_{ij},\ w\in W$$

$$u_w^{RRT,tr} = 1/f_w^{RRT}\sum_{\ell\in L}\sum_{\ell'\neq\ell}\sum_{i\in\ell\cup\ell'} f_i^{w\ell\ell'}(\varsigma_\ell' + uc_i),\ w\in W$$

$$u_w^{RRT,wt} = 1/f_w^{RRT}\sum_{\ell\in L}\sum_{(w^s,j)\in A(\ell)} \varsigma_\ell f_{w^s j}^{w\ell}/2,\ w\in W$$

Riding time

Transfer time

Waiting time

$$U_w^{RRT} = \eta + \beta_{tt}\cdot u_w^{RRT,tt} + \beta_{tr}\cdot u_w^{RRT,tr} + \beta_{tw}\cdot u_w^{RRT,tw},\ w\in W$$

$$FS_\ell = \lceil 120/(\varsigma_\ell \lambda) \sum_{\{i,j\} \in E} d_{ij} x_{ij}^\ell \rceil, \ \ell \in L$$

Fleet size constraints

Objective function

$$O_{NP} = O_{RV} - O_C = O_{RV} - O_{FOC} - O_{VOC} - O_{FA} - O_{BC}$$ Net profit

$$O_{RV} = \sum_{k=t_i}^{t_f-1} \frac{1}{e^{rk}} \left[(\xi + \eta) h_{year} \sum_{w \in W} g_w \cdot f_w^{RRTN} \right]$$ Revenue

$$O_{FOC} = \sum_{k=t_i}^{t_f-1} \frac{1}{e^{rk}} \left[\sum_{\{i,j\} \in E} ORlc_{ij} x_{ij} + \sum_{i \in N} OStc_i y_i \right]$$ Fixed cost

$$O_{VOC} = \sum_{k=t_i}^{t_f-1} \frac{1}{e^{rk}} \left[(h_{year} \cdot \lambda) \sum_{\ell \in \mathcal{L}} FS_\ell (\sum_{m \in M} c_{train}^m \delta_\ell^m) + c_{crew} \sum_{\ell \in \mathcal{L}} FS_\ell \right]$$ Variable cost

$$O_{FA} = \sum_{k=t_i}^{t_f-1} \frac{1}{e^{rk}} \left[\frac{\chi}{(t_f - t_i)} \sum_{\ell \in \mathcal{L}} FS_\ell (\sum_{m \in M} I_{train}^m \delta_\ell^m) \right]$$ Acquisition cost

$$O_{BC} = \frac{1}{t_f} \sum_{k=0}^{t_f-1} \frac{1}{e^{rk}} \left[\sum_{\{i,j\} \in E} c_{ij} x_{ij} + \sum_{i \in N} c_i y_i \right]$$ Building cost

Binary constraints

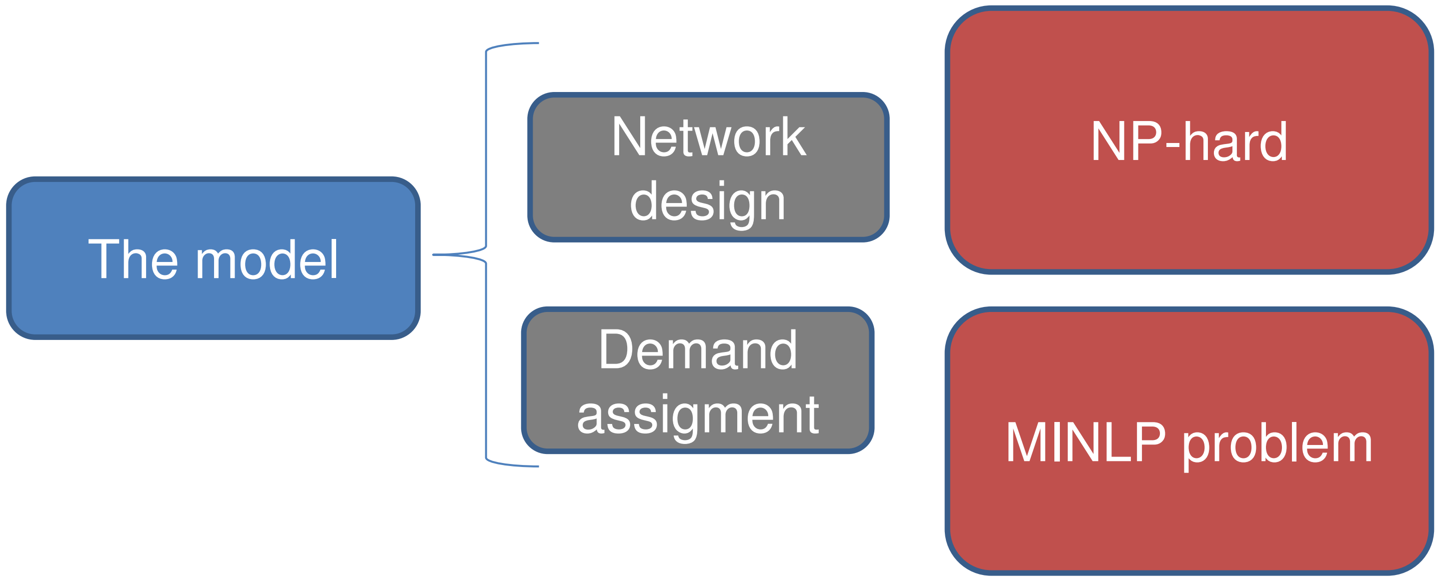
$$x_{ij}, y_i, x_{ij}^{\ell}, y_i^{\ell}, h_\ell, \delta_\ell^m \in \{0,1\}$$
$$i \in N, \{i,j\} \in E, \ell \in \mathcal{L}, w \in W$$

Integer constraints

$$\varsigma_\ell \in H,\ \ell \in \mathcal{L}$$
$$FS_\ell,\ \ell \in \mathcal{L}$$
$$\psi_\ell \in \{\psi^{min}, \dots, \psi^{max}\},\ \ell \in \mathcal{L}$$

Bounding constraints

$$f_{ij}^{w\ell}, f_i^{w\ell\ell'}, f_w^{RTS}, f_w^{ALT} \in [0,1]$$
$$i \in N, \{i,j\} \in E, \ell, \ell' \in \mathcal{L}, w \in W$$
$$u_w^{RRT,tt}, u_w^{RRT,tr}, u_w^{RRT,tw}, U_w^{RRT} \geq 0$$


The model
Network design
Demand assigment
NP-hard
MINLP problem

THE MATHEURISTIC

- Full linearized model
- Define the mechanism in the matheuristic

Model linearization

Capacity constrains

$$\sum_{w\in W} f_{ij}^{w\ell} g_w \leq \psi_\ell \sum_{m\in M} C_{train}^{m} \cdot \delta_\ell^m,\ \ell \in \mathcal{L}, \{i,j\} \in E$$

Fleet size constraints

$$FS_\ell = \lceil 120/(\varsigma_\ell \lambda) \sum_{\{i,j\}\in E} d_{ij} x_{ij}^{\ell} \rceil,\ \ell \in L$$

Utility constraints

$$u_w^{RRT,tt} = (60/f_w^{RRT}\lambda) \sum_{\ell\in L} \sum_{(i,j)\in A(\ell)} f_{ij}^{w\ell} d_{ij},\ w \in W$$

$$u_w^{RRT,tr} = 1/f_w^{RRT} \sum_{\ell\in L} \sum_{\ell'\neq\ell} \sum_{i\in\ell\cup\ell'} f_i^{w\ell\ell'} (\varsigma_\ell' + uc_i),\ w \in W$$

$$u_w^{RRT,wt} = 1/f_w^{RRT} \sum_{\ell\in L} \sum_{(w^s,j)\in A(\ell)} \varsigma_\ell f_{w^s j}^{w\ell}/2,\ w \in W$$

Modal split constraints

$$f_w^{RRT} \leq \frac{1}{1+e^{(\beta(u_w^{ALT}-u_w^{RRT}))}},\ w \in W$$

Objective function

Model linearization

Capacity constrains

$$\sum_{w \in W} f_{ij}^{w\ell} g_w \leq \psi_\ell \sum_{m \in M} C_{train}^m \cdot \delta_\ell^m,\ \ell \in \mathcal{L}, \{i,j\} \in E$$

Fleet size constraints

$$FS_\ell = \lceil 120/(\varsigma_\ell \lambda) \sum_{\{i,j\} \in E} d_{ij} x_{ij}^\ell \rceil,\ \ell \in L$$

Utility constraints

$$u_w^{RRT,tt} = (60/f_w^{RRT} \lambda) \sum_{\ell \in L} \sum_{(i,j) \in A(\ell)} f_{ij}^{w\ell} d_{ij},\ w \in W$$

$$u_w^{RRT,tr} = 1/f_w^{RRT} \sum_{\ell \in L} \sum_{\ell' \neq \ell} \sum_{i \in \ell \cup \ell'} f_i^{w\ell\ell'} (\varsigma_\ell' + uc_i),\ w \in W$$

$$u_w^{RRT,wt} = 1/f_w^{RRT} \sum_{\ell \in L} \sum_{(w^s,j) \in A(\ell)} \varsigma_\ell f_{w^s j}^{w\ell}/2,\ w \in W$$

Modal split constraints

$$f_w^{RRT} \leq \frac{1}{1 + e^{(\beta(u_w^{ALT} - u_w^{RRT}))}},\ w \in W$$

Objective function

Capacity Constraints linearization

$$\sum_{w \in W} f_{ij}^{w\ell} g_w \leq \psi_\ell \sum_{m \in M} C_{train}^m \cdot \delta_\ell^m, \ \ell \in \mathcal{L}, \{i,j\} \in E$$

Product of integer and binary variables

- Introduce new binary variables: γ_ℓ^p
- $fv = \{fv_p : p = 1, \ldots, |P|\}$ represents the set of feasibles frequencies

$$\psi_\ell = \sum_{p \in P} fv_p \cdot \gamma_\ell^p, \ \ell \in \mathcal{L}$$

$$\varsigma_\ell = \sum_{p \in P} 60/fv_p \cdot \gamma_\ell^p, \ \ell \in \mathcal{L}$$

$$\sum_{p \in P} \gamma_\ell^p = 1, \ \ell \in \mathcal{L}$$

$$\psi_\ell \cdot \delta_\ell^m = \sum_{p \in P} f v_p \cdot \gamma_\ell^p \cdot \delta_\ell^m, \ \ell \in \mathcal{L}$$

Two binary variables

$$\xi_p^{m\ell} = \psi_\ell \cdot \delta_\ell^m, \ \ell \in \mathcal{L}, m \in M, p \in P$$

$$\delta_\ell^m + \gamma_p^\ell \leq 1 + \xi_p^{m\ell}, \ \ell \in \mathcal{L}, m \in M, p \in P$$

$$2\xi_p^{m\ell} \leq \delta_\ell^m + \gamma_\ell^p, \ \ell \in \mathcal{L}, m \in M, p \in P$$

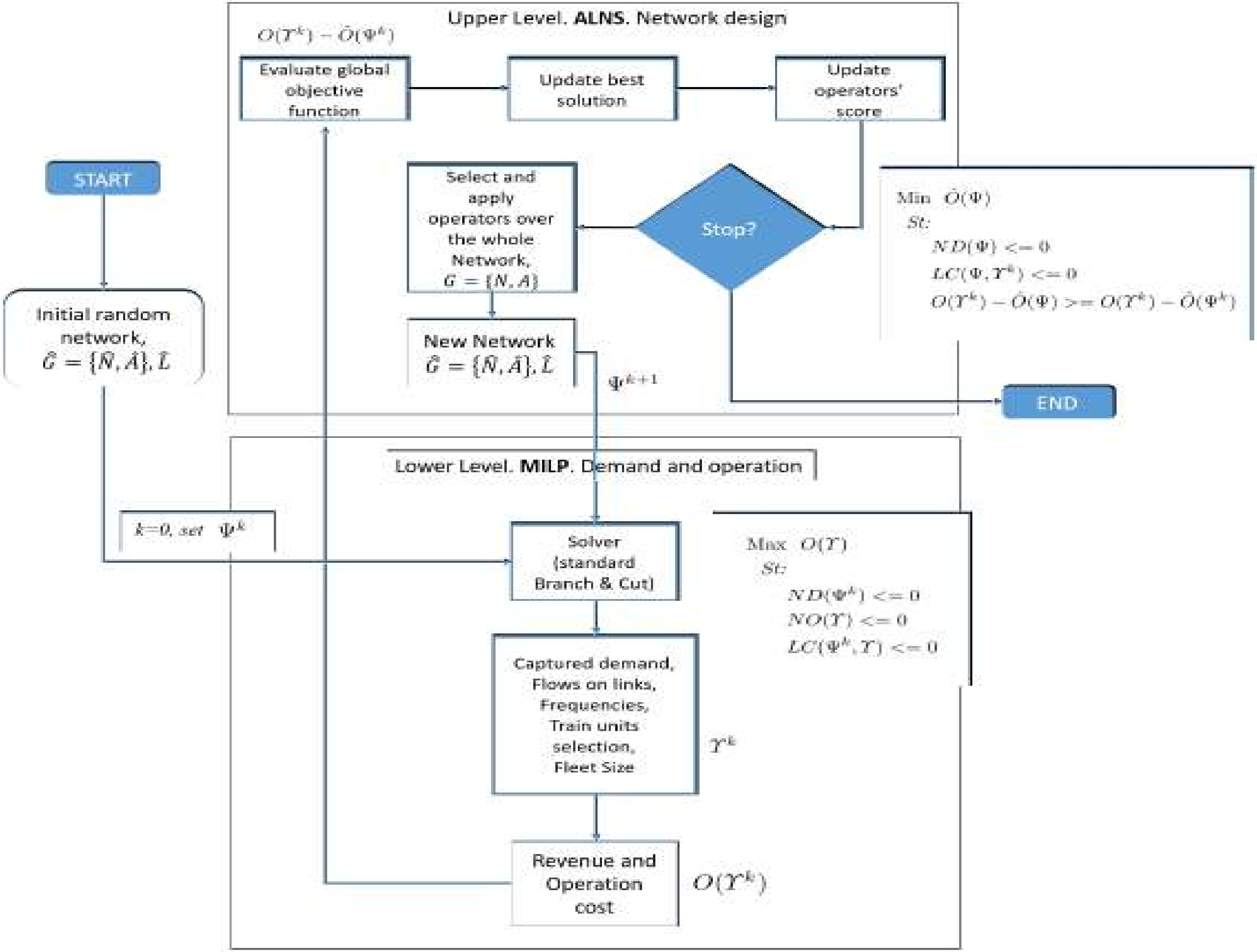
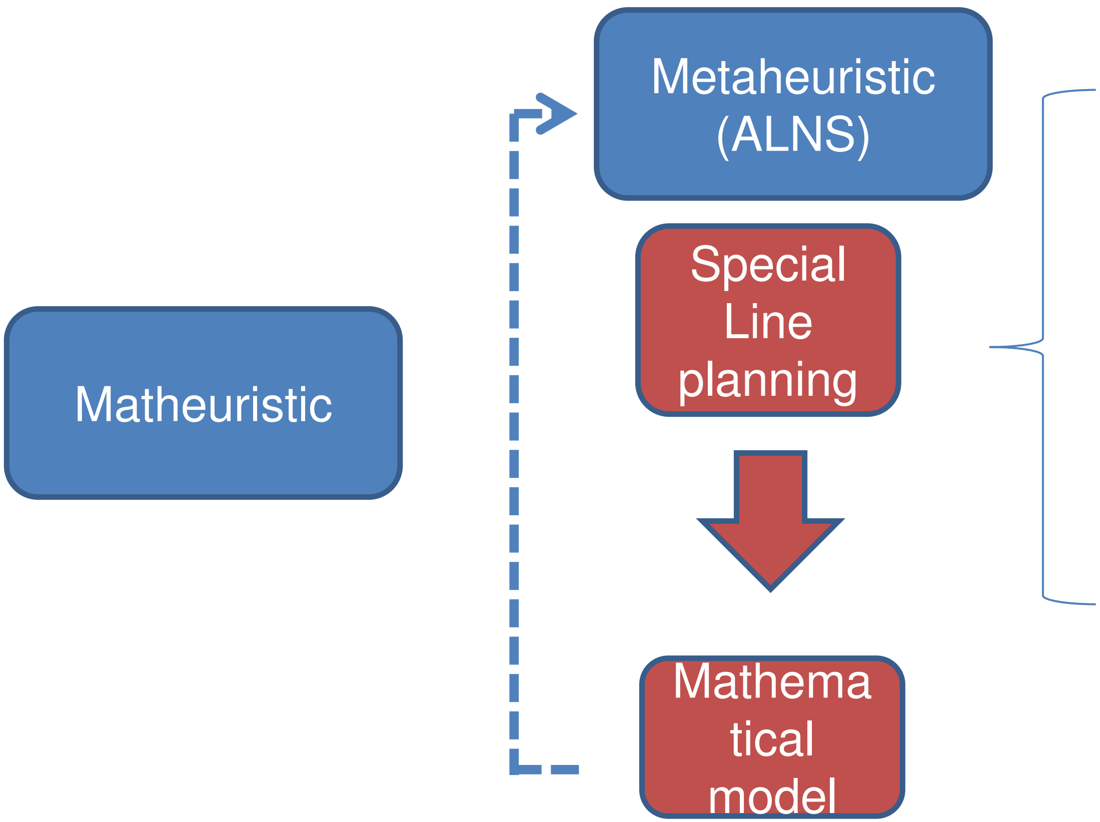


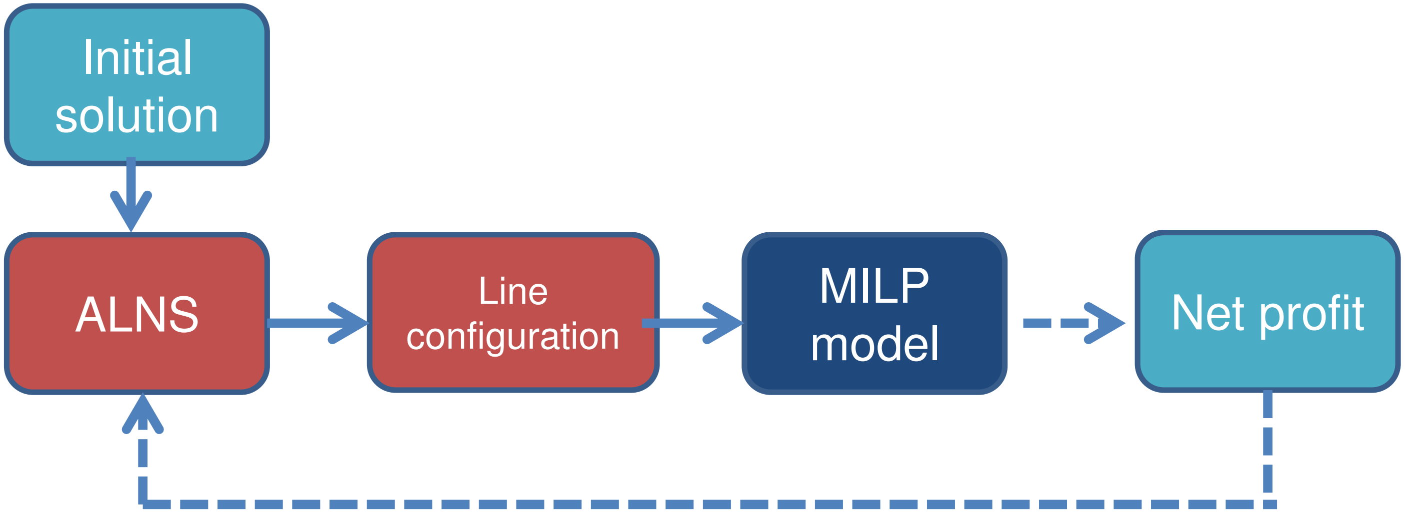
Figure 2: Integrated Network Design and Line Planning solving scheme.

Solving a rapid transit network design problem



- Input: line network
- Main variables:
  Frequency
  Capacity
- Competing mode
- Objective: profit

ALNS: Adaptive Large Neighborhood Search



Computational experiments

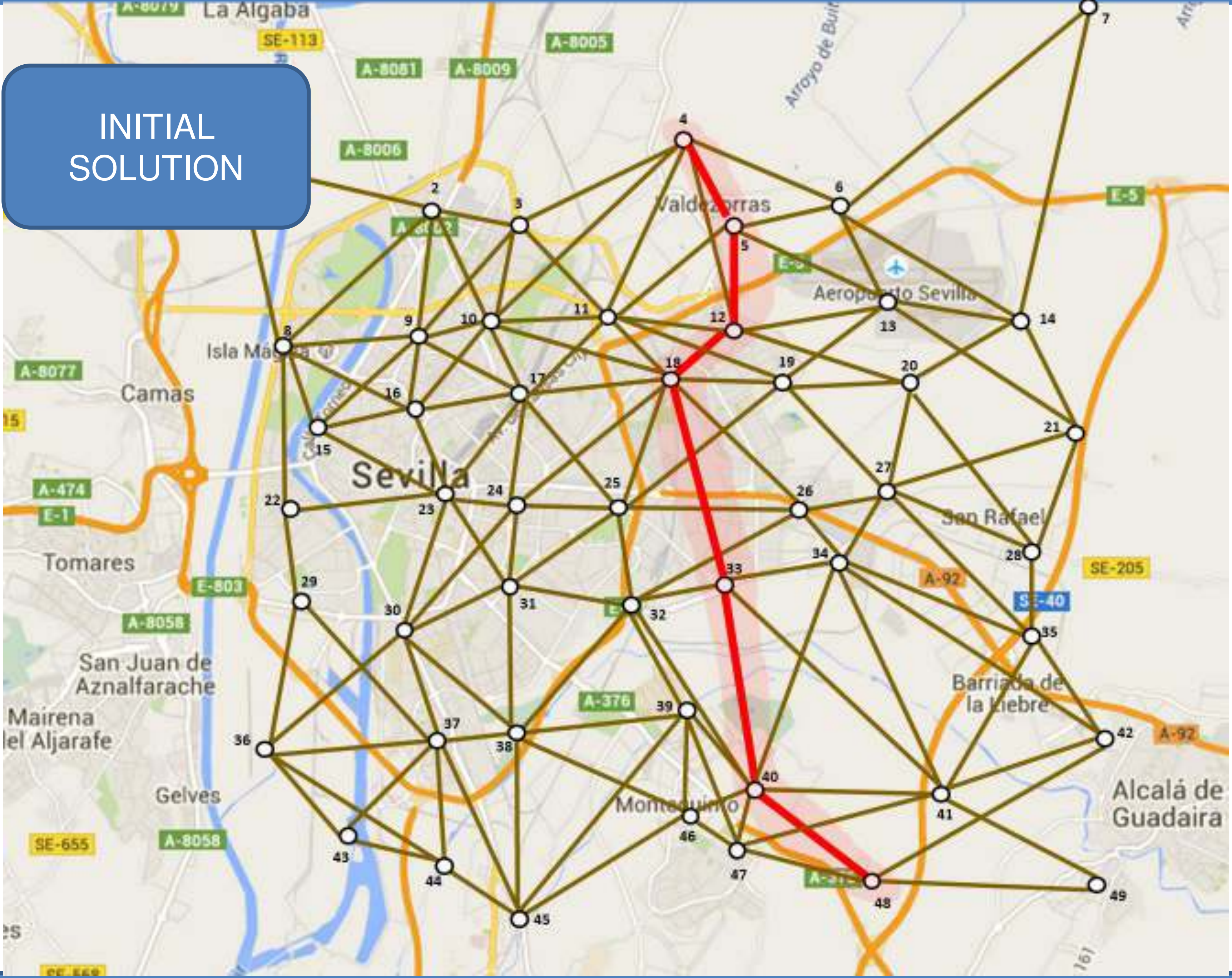
- We carried out several experiments using Java code and Gurobi 7.5.1
- We tested the matheuristic over small instances.
- We are testing the algorithm on a real life instance: Seville.
- We solve several scenarios for input parameters related to fares and values time.

Computational experiments

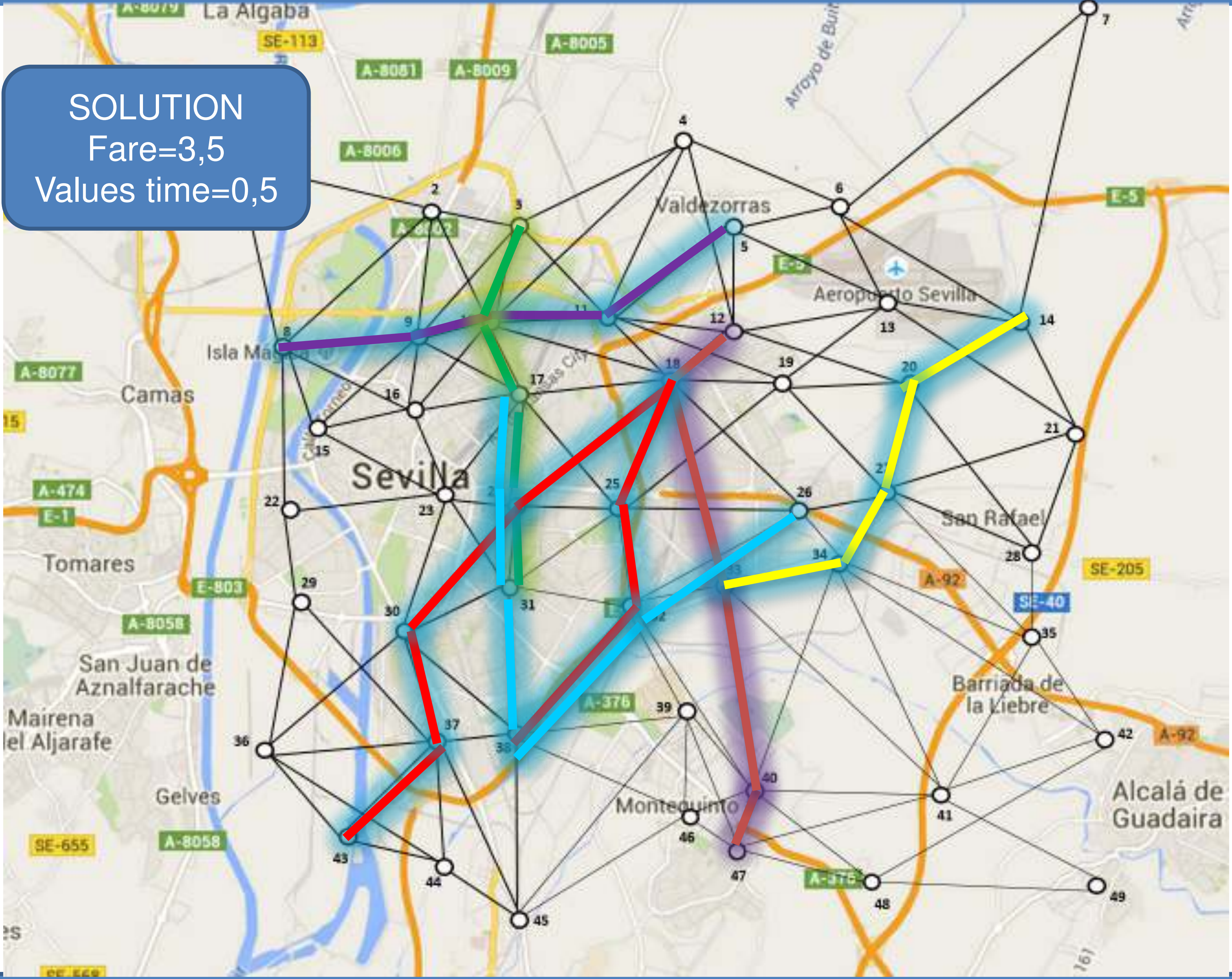
Parameters		
Name	**Description**	**Value**
$\hat{\rho}$	years to recover the purchase	20
h_{year}	number of years spent to build the network	10
ρ	number of operative hours per year	6935
m	model of train	463, 464, 465
ORC_{ij}	operating rail cost measured in € per year	$6 \cdot 10^4$
OSC_i	operating station cost expressed in € per year	$6 \cdot 10^4$
c_i	building cost of station at node i [€]	10^6
c_{ij}	building cost of link (i,j) [€]	$20^6 \cdot d_{ij}$
c^m_{train}	operating cost of a train per kilometer [€/km]	$3, 3.1, 3.2$
c_{crew}	per crew and year for each train m [€/ year]	$75 \cdot 10^3$
I_{train}	purchase cost of one train Civia in €	$4.4, 5.2, 5.9 \cdot 10^6$
K^m_{train}	capacity of each train (number of passengers)	$607, 832, 997 \cdot 10^2$
N_{min}	lower bound on the number of nodes of each line	3
N_{max}	upper bound on the number of nodes of each line	9
$\mathcal{H}$	Possible headways [min]	{3,4,5,6,10,12,15,20}
β^{tr}	Perceived value of time spent transferring in [€/ min]	0.25
β^{tw}	Perceived value of time spent for waiting at the origin station in [€/ min]	0.25
β^{tt}	Perceived value of time spent for riding in train in [€/ min]	0.083
$\mu + \eta$	fare plus subsidy	3.5

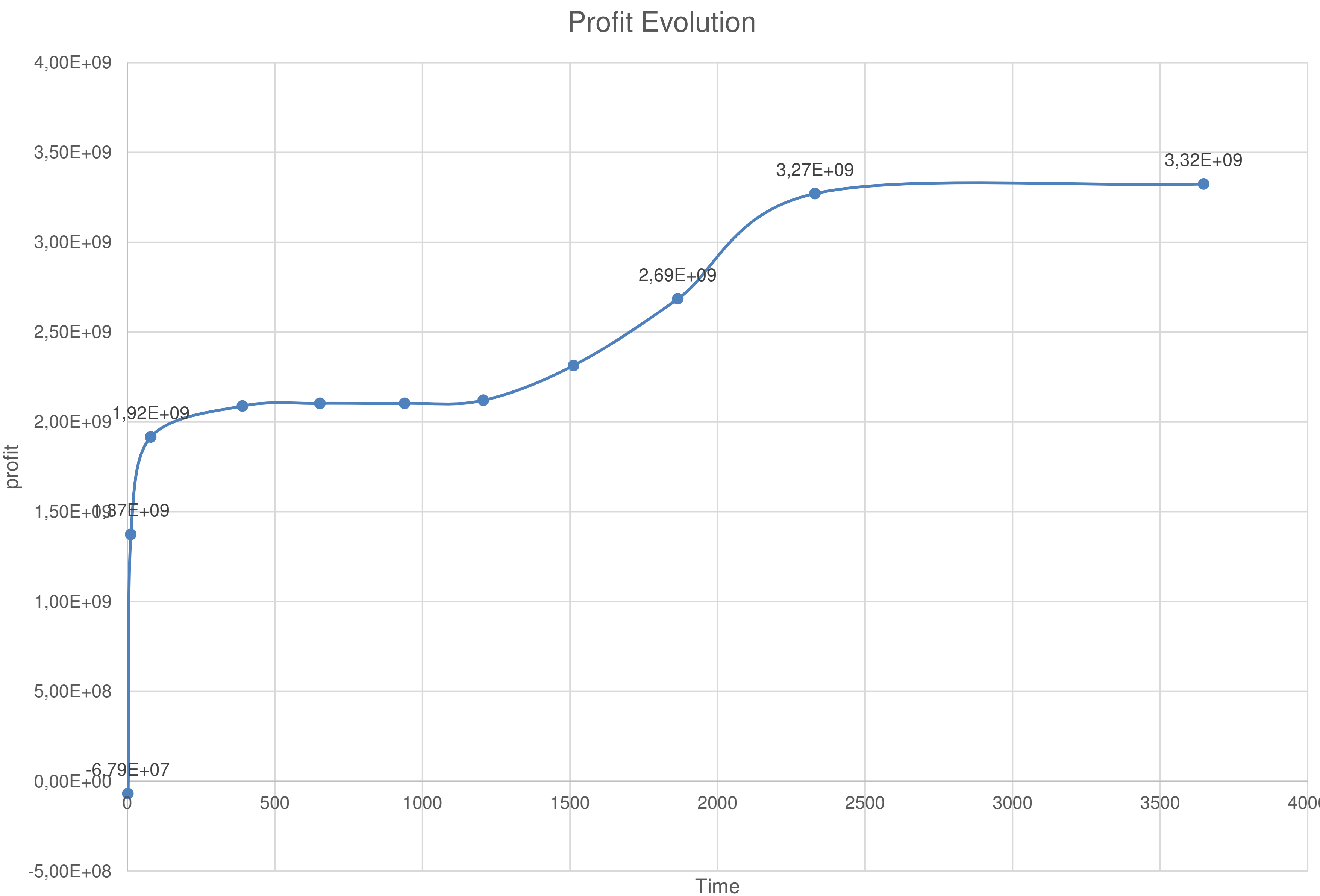
49 NODES
135 EDGES
2352 OD PAIRS
69,302 trips.

INITIAL SOLUTION

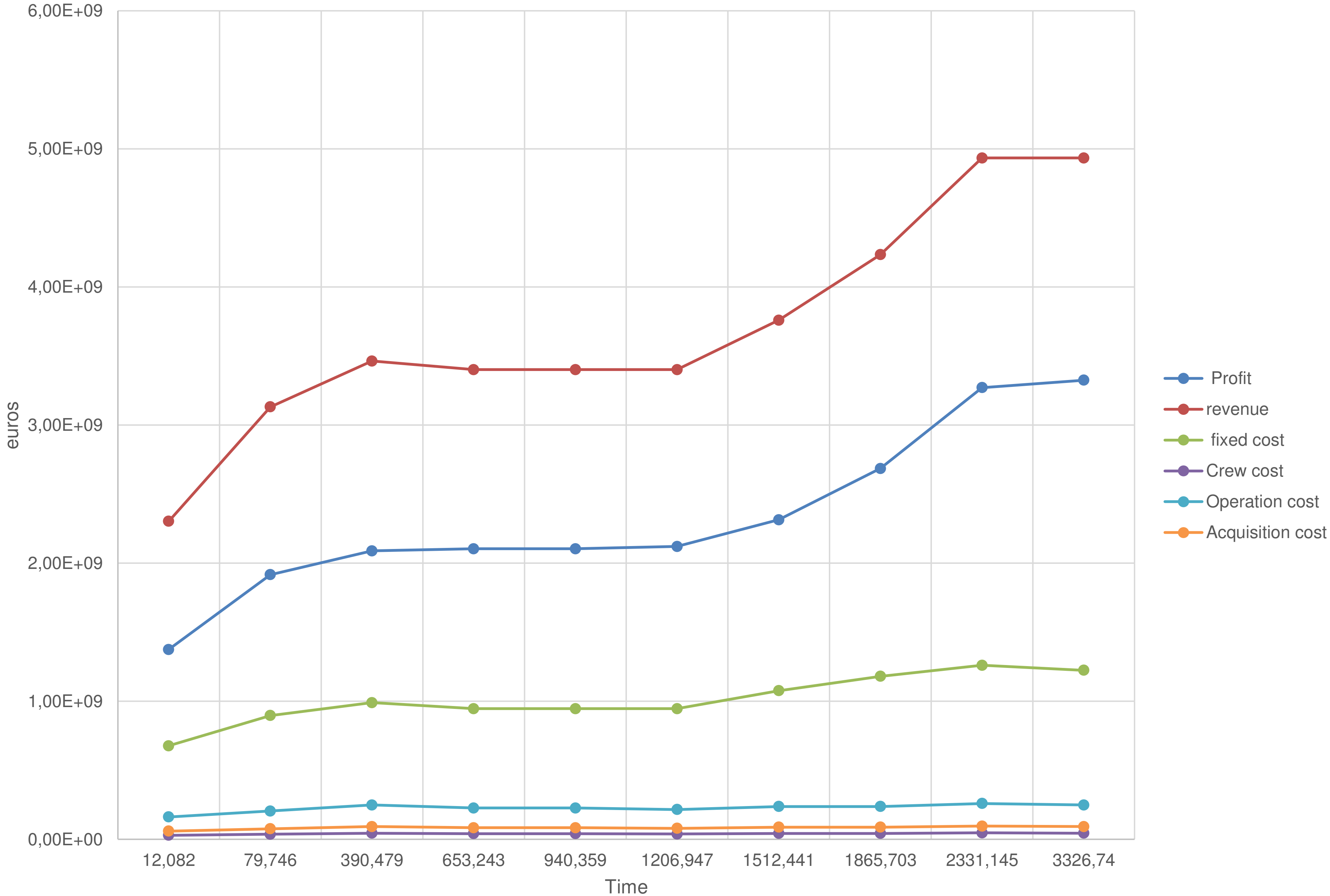


SOLUTION
Fare=3,5
Values time=0,5





Profit Evolution
4,00E+09
3,50E+09
3,00E+09
2,50E+09
2,00E+09
1,50E+09
1,00E+09
5,00E+08
0,00E+00
-5,00E+08
profit
0
500
1000
1500
2000
2500
3000
3500
4000
Time
-6,79E+07
1,37E+09
1,92E+09
2,69E+09
3,27E+09
3,32E+09



6,00E+09
5,00E+09
4,00E+09
3,00E+09
2,00E+09
1,00E+09
0,00E+00
euros
12,082
79,746
390,479
653,243
940,359
1206,947
1512,441
1865,703
2331,145
3326,74
Time
Profit
revenue
fixed cost
Crew cost
Operation cost
Acquisition cost

6. CONCLUSIONS AND FURTHER WORKS

- We have presented a mathematical programming program for the rapid transit network design.
- We have included a utility function considering values of time.
- We have proposed a new procedure which consists of a matheuristic considering a lineal mathematical model and the ALNS metaheuristic.
- We are going to test the matheuristic over real networks.

THANK YOU!!