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The generalized discrete (r|p)-centroid problem

Dolores R. Santos-Peñate[1], Clara M. Campos-Rodríguez[2] and José A. Moreno-Pérez[2]

[1]Universidad de Las Palmas de Gran Canaria, drsantos@dmc.ulpgc.es
[2]Universidad de La Laguna, {ccampos, jamoreno}@ull.es

Introduction

We consider a discrete competitive location model which is called (*r*|*p*)-centroid problem, leader-follower problem or Stackelberg problem in locations. The model represents a situation where two players, the leader and the follower make decisions sequentially in order to reach certain objectives. Leader and follower want to determine the locations for p and r facilities respectively. The objective of the follower, who makes the decision once the leader has selected its locations, is to maximize the demand captured by its facilities. The objective of the leader is to minimize the maximum demand that the follower could capture, as demand is assumed to be essential, this objective is equivalent to maximize the demand captured by its facilities. We study a generalized model in which the customer's choice rule is defined using a non-increasing capture function, the demand captured by the players is given by the value of this function for the difference between the distance from the demand point to the follower and the distance from the demand point to the leader. Given the set of *p* locations for the leader, *Xp*, the solution for the follower is an (*r*|*Xp*)-medianod. The solution for the leader is an (*r*|*p*)-centroid.

We present a linear programming formulation for the generalized (*r*|*p*)-centroid problem and an exact solution procedure. The model is illustrated with an example.

Statement of the problem

Let V be a set of points and C, L be subsets of V with cardinality |C|=n and |L|=m. C is the set of demand points or clients, and L is the set of potential locations for facilities. Let d(c,x) be the distance between point c є C and point x є L. For c є C and X subset of L, d(c,X) denotes the distance between c and X. Every point c є C has a weight w(c) which represents the demand at point c. Denote C={c_k: k є [1..n]} and L={l_i: i є [1..m]}, w_k=w(c_k), d_{ki} = d(c_k,l_i), and, for X subset of L, d_k(X) = d(c_k,X). Consider a market of essential goods, which means that the sum of demands served by the firms operating in the market is equal to the total existing demand.

Assume that two competing firms, A and B, operate in the market with p and r facilities located at X_p and Y_r, respectively. The demand at point c_k captured by the firms depends on the difference d_k=d_k(Y_r)-d_k(X_p). The market share for firms A and B are given by

$$W_A = W_A(X_p, Y_r) = \sum_{k=1}^{n} w_k\left(1 - f_k(d_k)\right)$$

$$W_B = W_B(X_p, Y_r) = \sum_{k=1}^{n} w_k f_k(d_k)$$

where f_k(d) is a non-negative and non-increasing function such that 0≤ f_k(d) ≤1 for d ≥ 0. The total demand is W_T=W_A+W_B.

Initially, no firm is operating in the market, firm A, the leader, wants to enter the market with p facilities taking into account that firm B, the follower, will enter later the market installing r facilities in the locations where the market share of B is maximum. Firm A wants to determine the p locations that minimize the demand captured by the competing firm B. As goods are essential, to minimize the demand captured by the competitor is equivalent to maximize the own market share.

If the leader has p facilities opened at X_p, the problem of the follower is to determine the set Y_r of r locations that maximizes its market share W_B(X_p,Y_r). An optimal solution to this problem, Y_r*(X_p), is an (r|X_p)-medianoid. The problem of the leader is to determine the set X_p that minimizes W_B(X_p,Y_r*(X_p)), that is, the set X_p which minimizes the maximum market share that the follower could achieve. An optimal solution to the problem of the leader is an (r|p)-centroid. Formally, the (r|p)-centroid problem or leader's problem is the following minimax problem

$$\min_{X\subset L,|X|=p}\ \max_{Y\subset L,|Y|=r} W_B(X,Y).$$

That is,

$$\min_{X\subset L,|X|=p} S(X)$$

where

$$S(X) = \max_{Y\subset L,|Y|=r} W_B(X,Y)$$

This is a bi-level problem where the lower level problem is the (r|X_p)-medianoid problem and the upper level problem is the (r|p)-centroid problem.

Linear formulations

The (r|X_p)-medianoid problem:

$$\max \sum_{i=1}^{m}\sum_{k=1}^{n} h_{ki} z_{ki}$$

$$\sum_{i=1}^{m} y_i = r$$

$$\sum_{i=1}^{m} z_{ki} \le 1, \qquad k \in [1..n]$$

$$z_{ki} \le y_i, \qquad i \in [1..m], k \in [1..n]$$

$$z_{ki}, y_i \in \{0,1\}, \qquad i \in [1..m], k \in [1..n]$$

where y_i=1 if the follower opens a facility at point l_i and y_i=0 otherwise, and z_{ki}=1 if client c_k visits a facility of the follower at point l_i and z_{ki} =0 otherwise. The coefficient h_{ki} in the objective function is

$$h_{ki} = w_k\left(1 - f_k(d_k)\right).$$

The (r|p)-centroid problem:

$$\min W$$

$$\sum_{i=1}^{m} x_i = p$$

$$\sum_{i=1}^{m}\sum_{k=1}^{n} h_{kij} u_{ki} \le W, \qquad j \in \left[1..\binom{m}{r}\right]$$

$$\sum_{i=1}^{m} u_{ki} = 1, \qquad k \in [1..n]$$

$$u_{ki} \le x_i, \qquad i \in [1..m], k \in [1..n]$$

$$u_{ki}, x_i \in \{0,1\}, \qquad i \in [1..m], k \in [1..n]$$

where x_i=1 if the leader opens a facility at point l_i and x_i=0 otherwise, u_{ki} =1 if client c_k visits a facility of the leader at point l_i and u_{ki} =0 otherwise, and

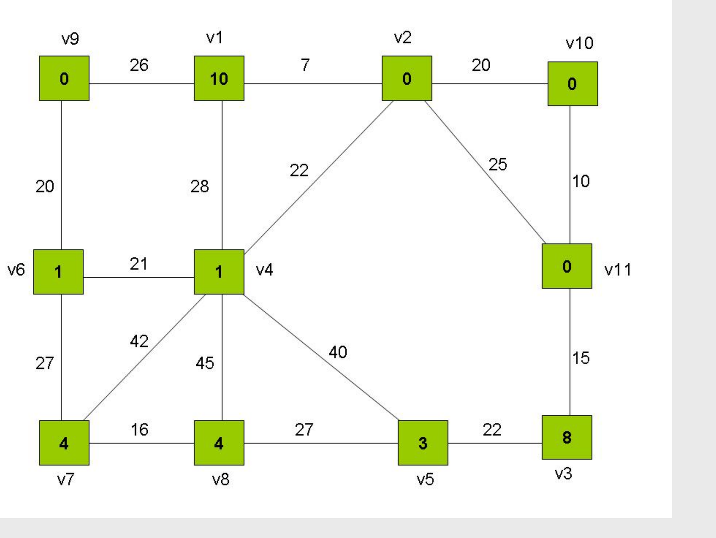
$$h_{kij} = w_k f_k\left(d_k(Y_j) - d_{ki}\right)$$

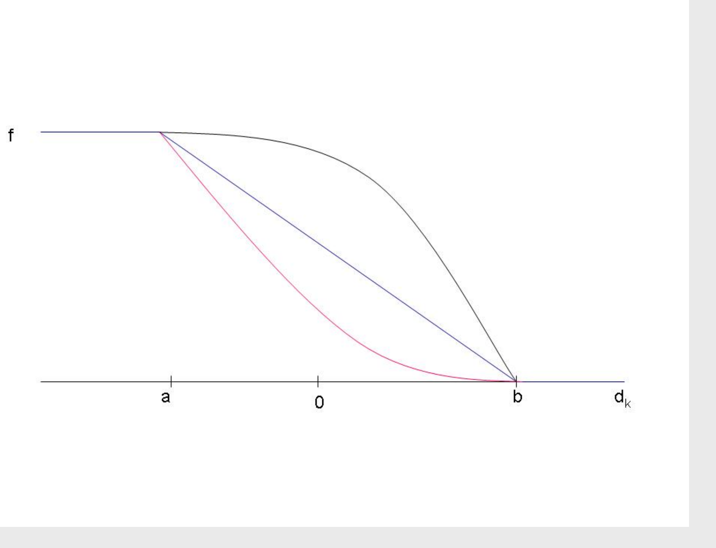
$$Y_j \in L_r = \{Y \subset L: |Y| = r\}, j \in \left[1..\binom{m}{r}\right].$$

L_r is the set of feasible solutions for the follower.

The value h_{kij} represents the demand at c_k captured by the follower if he/she opens facilities at Y_j and the closest leader's facility is located at l_i.

An example





Rule (p=r=2)	Coincidences allowed		Coincidences not allowed		X=Y
	X;Y	W=S(X)	X;Y	W=S(X)	W=S(X)
Binary (μ=0.5)	1,7; 1,3	16.50	1,3; 5,6	13	15.50
-a = b = 2.86					
Linear	1,7; 1,11	16.50	1,3; 5,6	13	15.50
Concave	1,3; 1,3	23.25	1,3; 5,6	13	23.25
Convex	1,3; 1,5	14	1,3; 5,6	13	7.75
a = - 2.86, b = 40					
Linear	1,3 ;1,3	28.93	1,5; 2,8	22.72	28.93
Concave	1,3 ;1,3	30.86	1,2; 5,9	26.23	30.86
Convex	1,3 ;1,3	27.00	1,5; 2,8	18.31	27.00
a = - 40, b = 2.86					
Linear	1,3; 5,8	9.88	1,3; 5,8	9.88	2.07
Concave	1,3; 5,6	11.16	1,3; 5,6	11.16	0.14
Convex	2,5; 3,7	7.30	2,5; 3,7	7.30	4.00

A solution procedure

Step 1.Initialization

1.1. Select s feasible leader's solutions X_i, i=1,...,s. Solve the follower's problem for X_i, i=1,...,s. An upper bound of the optimum W* is W_{UP} = $\min_i$ S(X_i). Let X*=X with S(X)=W_{UP}.
1.2. Let F be the selected family of good follower candidates, F= $\{Y_i\}_{1\le i\le s}$. Set W_{LO}=0.
1.3. MaxIte= maximum number of iterations. Let i=0.

Step 2. Iterations

Repeat, until W_{UP}=W_{LO} or until i=MaxIte, the steps:
2.1. Do i=i+1.
Solve the leader's problem constrained to F. Let X be the optimal solution obtained. If the optimal value obtained S_c(X) verifies S_c(X)>W_{LO} then do W_{LO}=S_c(X). If W_{LO}=W_{UP}, then W*= W_{LO}=W_{UP} is the optimal value and X*=X is the optimal location set for the leader.
2.2. Solve the follower's problem for X. If S(X)<W_{UP} then set W_{UP}=S(X) and X*=X. If W_{LO}=W_{UP}, then W*=W_{LO}=W_{UP} is the optimal value and X* is the optimal location set for the leader. Add Y(X) to F.

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