Locating Capacitated Unreliable Facilities

M. Albareda-Sambola, M. Landete,
J.F. Monge, and J.L. Sainz-Pardo

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and Related Problems
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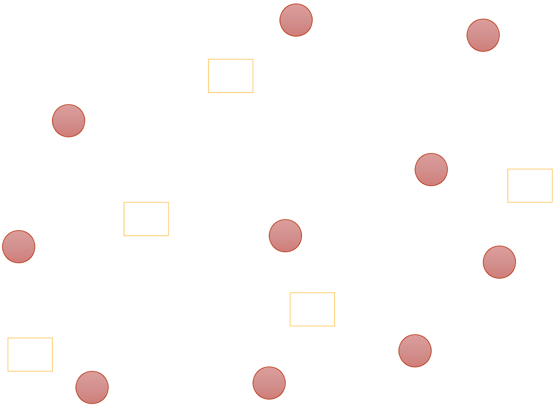
Outline

Unreliable facility location



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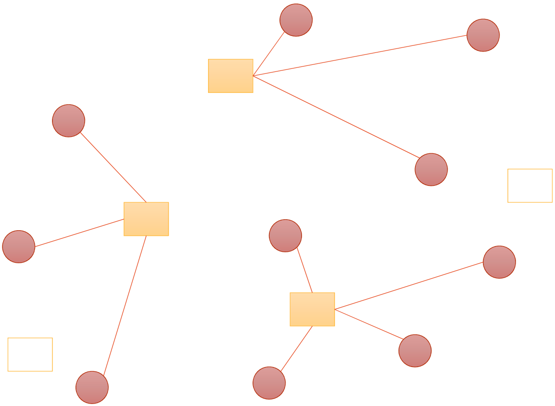

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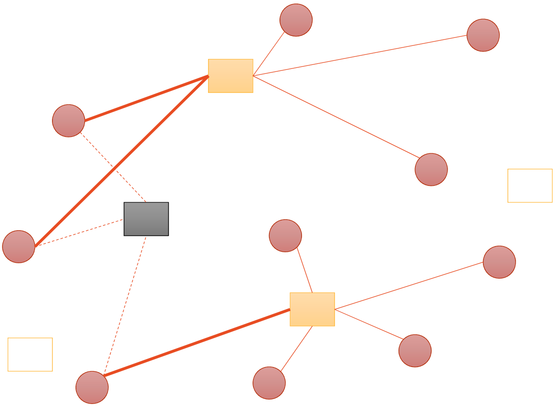

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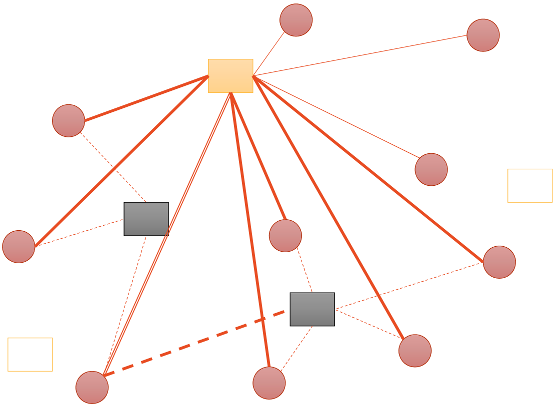

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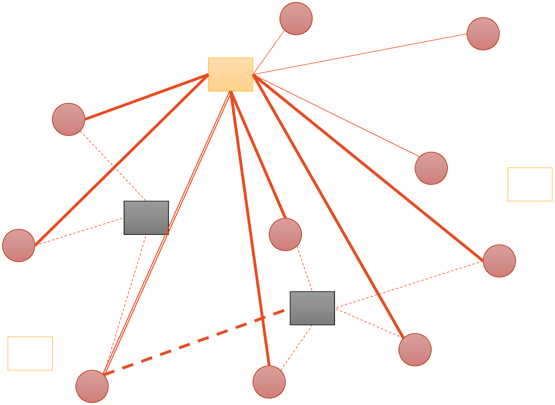

What if facilities are capacitated?

Literature review 1: uncapacitated

L. Snyder.
Facility location under uncertainty: a review.
IIE Transactions, 38(7):547–564, 2006.

J. R. O'Hanley, M. P. Scaparra, and S. García.
Probability chains: A general linearization technique for modeling reliability in facility location and related problems.
European Journal of Operational Research, 230:63–75, 2013.

O. Berman, D. Krass, and M. Menezes.
Location and reliability problems on a line: Impact of objectives and correlated failures on optimal location patterns.
Omega, 41:766–779, 2013.

O. Berman, D. Krass, and M. Menezes.
Locating facilities in the presence of disruptions and incomplete information.
Decision Sciences, 40(4):845– 868, 2009.

M. Albareda-Sambola, Y. Hinojosa, and J. Puerto.
The reliable p-median problem with at-facility service.
European Journal of Operational Research, 245:656–666, 2015.

Literature review 2: capacitated

D. Gade and E. Pohl. (2009)
Sample average approximation applied to the capacitated-facilities location problem with unreliable facilities. *J of Risk and Reliability*: 259–269.

N. Aydin and A. Murat.(2013)
A swarm intelligence based sample average approximation algorithm for the capacitated reliable facility location problem. *Int J Prod Econ*, 145:173–183.

Y. An, B. Zeng, Y. Zhang, and L. Zhao.(2014)
Reliable p-median facility location problem: two stage robust models and algorithms. *Transport Res B-Meth*, 64:54–72.

I. Espejo, A. Marín, and A. M. Rodríguez-Chía.(2015)
Capacitated p-center problem with failre foresight. *EJOR*, 247:229–244.

N. Azad, H. Davoudpour, G. Saharidis, and M. Shiripour.(2014)
A new model for mitigating random disruption risks of facility and transportation in supply chain network design. *Int J Adv Manuf Tech*, 70:1757–1774.

K. Lim., A. Bassamboo, S. Chopra, and M. Daskin.(2013)
Facility location decisions with random disruptions and imperfect estimation. *M&SOM-Manuf Serv Op*, 15:239–249.

Basic assumptions and notation

- Each candidate facility location ($i \in I$):
  - has a fixed opening cost f_i,
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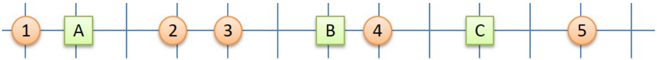
- A dummy facility models lost customers $\longrightarrow$ penalty.

Failure(s)! And now?



If one facility fails and full reassignments are allowed:

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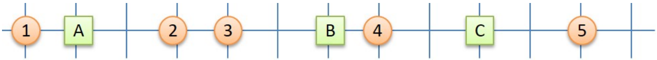

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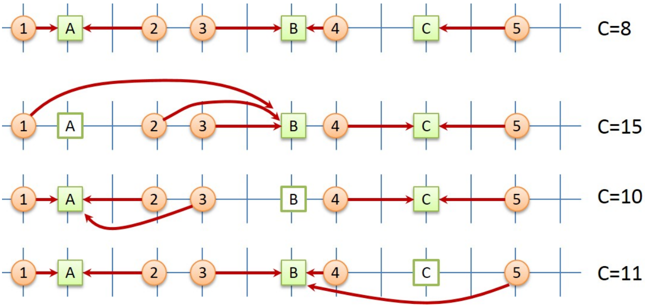

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- Less flexible $\longrightarrow$ maybe more expensive
- Capacity becomes more challenging.

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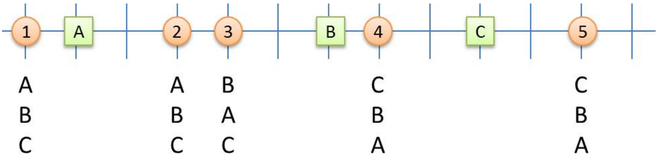

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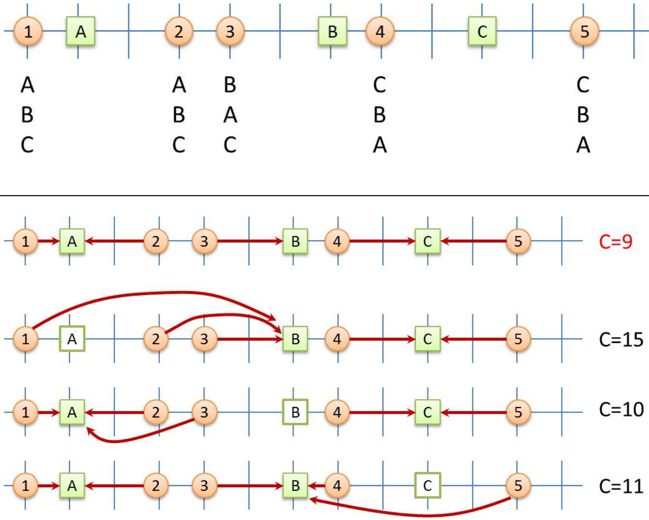

Capacity constraints satisfaction

- Should be granted in regular conditions:

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US Small overloads might be assumed in emergency situations.

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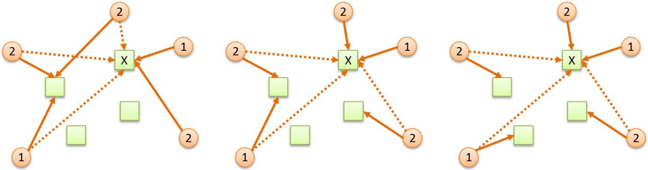
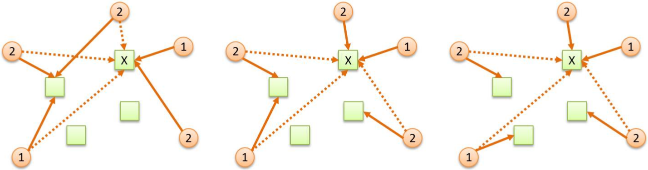
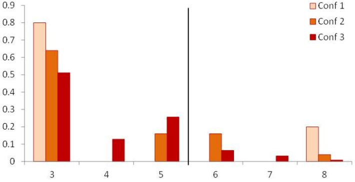


Conf 1 Conf 2 Conf 3

Capacity constraints satisfaction

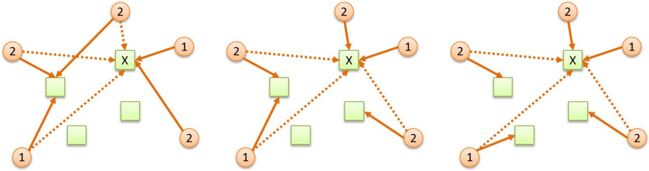


Demand distribution at X for $q = 0.2$



	conf. 1	conf. 2	conf. 3
$\mathbb{E}$(dem)	4	4	4
$\mathbb{P}$(overload)	0.2	0.2	0.104
$\mathbb{E}$(overload)	0.6	0.28	0.152

Capacity constraints satisfaction



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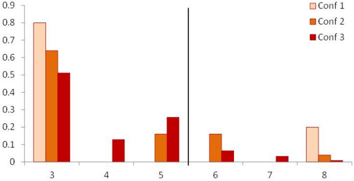

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Expected values miss relevant information!

Limits on expected loads - LEL(V, γ)

Expected demands can exceed the capacities in at most γ facilities by, at most, V

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$$
\begin{aligned}
&\sum_{j\in J} h_j \sum_{r\in R} q^r Y_{ijr} \leqslant Q_i y_i + \nu_i && i \in I \\
&\nu_i \leqslant V u_i && j \in J \\
&\sum_{i\in I} u_i \leqslant \gamma && \\
&\nu_i \geqslant 0 && i \in I \\
&u_i \in \{0,1\} && i \in I
\end{aligned}
$$

Expected overloads - E(X)

$$\sum_{i \in O(X)} \mathbb{E}\left[\left(\xi_i \cdot \underbrace{\sum_{j \in J} h_j \left(\sum_{r \in R} x_{ijr} \cdot \prod_{s<r} \left(\sum_{j' \in O(X)} x_{i'js}(1-\xi_{i'})\right)\right)}_{\text{demand at } i \text{ according to } \xi} - Q_i\right)^+\right]$$

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- Far from linear!

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- Far from linear!
- Resort to bounds/approximations

Bounding-bound expected overload- B(V)

Total expected overloads are Bbounded above by V

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$$
\begin{array}{ll}
\sum_{s=1}^{r} \sum_{j \in J} h_j Y_{ijs} \leqslant Q_i + \nu_{ir} & \forall i \in I, r \in R \\
\lambda_{i1} = \nu_{i1} & \forall i \in I \\
\lambda_{ir} = \nu_{ir} - \nu_{ir-1} & \forall i \in I, r > 1 \\
\sum_{i \in F} \sum_{r>0} q^r (1-q) \lambda_{ir} + \sum_{i \in NF} \sum_{r>0} q^r \lambda_{ir} \leq V & \\
\lambda_{ir}, \nu_{ir} \geqslant 0 & \forall i \in I, r \in R
\end{array}
$$

Bounding-estimate expected overload- LR(V)

Estimated total expected overloads are bounded above by V

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Estimated total expected overloads are bounded above by V

$$\lambda_{\bullet r} = \sum_{i \in I} \lambda_{ir} \qquad r \in \{1, \dots, 4\}$$

$$0.722844 q \lambda_{\bullet 1} + 0.335816 q^2 \lambda_{\bullet 2} + 0.233097 q^3 \lambda_{\bullet 3} + 0.374673 q^4 \lambda_{\bullet 4} \leqslant V$$

Staggered capacities- S(β)

Capacities scaled by $\beta > 1$ for unlikely needs

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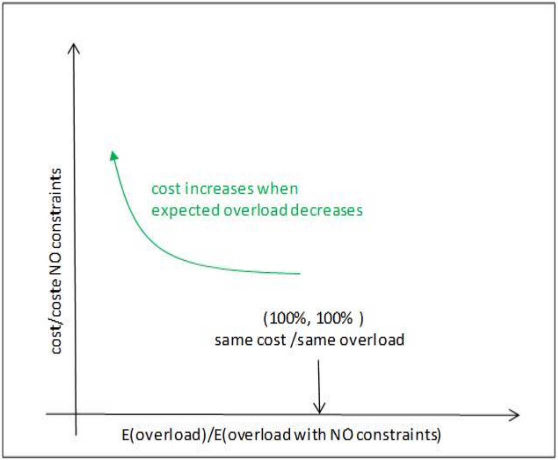
Capacities scaled by $\beta > 1$ for unlikely needs

$$\boxed{\sum_{s=0}^{r} \sum_{j \in J} h_i x_{ijs} \leqslant \beta^r Q_i y_i \qquad i \in I, r \geqslant 1}$$

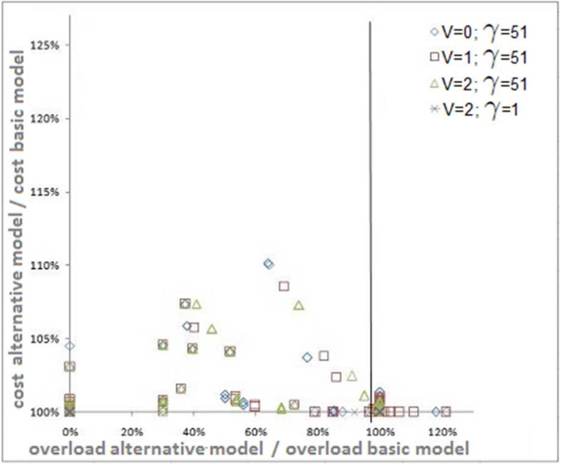
Computational experience

- All formulations implemented in Cplex 11.0
- Experiments run in a PC with a 2.33 GHz Intel Xeon dual core processor, 8.5 GB of RAM
- 140 instances generated from 10 ORLIB *p*-median instances:
  - $n \in \{20, 50\}$,
  - $q \in 0.05, 0.10, 0.20$,
  - $|NF| \in \{1, 16\}$ with two different relative costs
- Different formulation configurations (γ, V, β).
- Disregarded $r > 4$.

Solution quality (0)

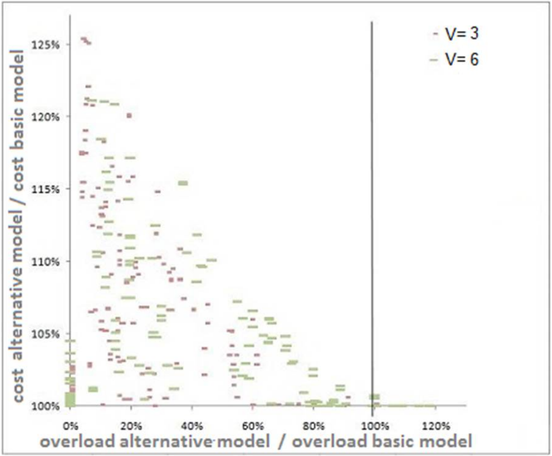


Solution quality- LEL(V,γ)



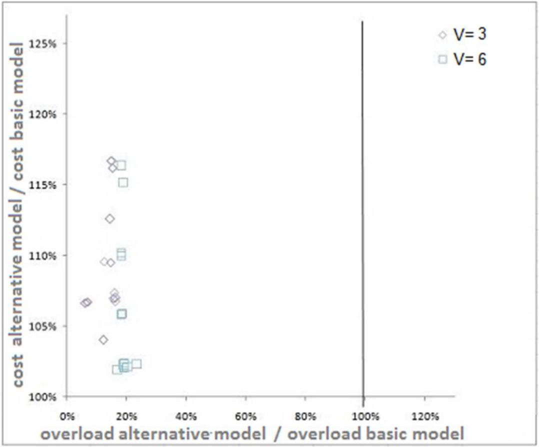
CRFLP - LEL

Solution quality-B(V)



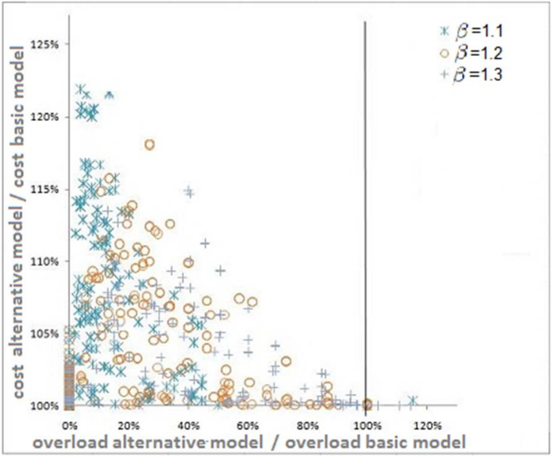
CRFLP - B1

Solution quality-LR(V)



CRFLP - LR

Solution quality-S(β)



CRFLP - S

CPU times

n	q	LEL		B1		LR		S		
		$V:1$ $\gamma:\lvert I\rvert$	$V:\infty$ $\gamma:2$	V:3	V:6	V:3	V:6	$\beta:1.1$	$\beta:1.2$	$\beta:1.3$
20	.05	11.9	7.4	30.2	32.3	31.7	17.8	59.4	42.1	39.5
	.10	109.4	7.2	80.4	64.2	244.1	55.5	241.7	89.7	62.7
	.20	71.0	7.4	236.1	954.2	744.6	944.1	1252.7	344.4	49.8
50	.05	751.1	703.0	187.0	215.0	1152.6	1188.7	2514.9	2554.8	3607.0

Gràcies!

maria.albareda@upc.edu