A decomposition Scheme for the Railway Rapid Transit Depot Location and Rolling Stock Circulation Problem

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OUTLINE

1

RRT Systems

Rolling Stock

Problem description & Math. Model

Solving approach

- Fleet size
- Train circulation (Fixed locations)
- Rest Facilities location

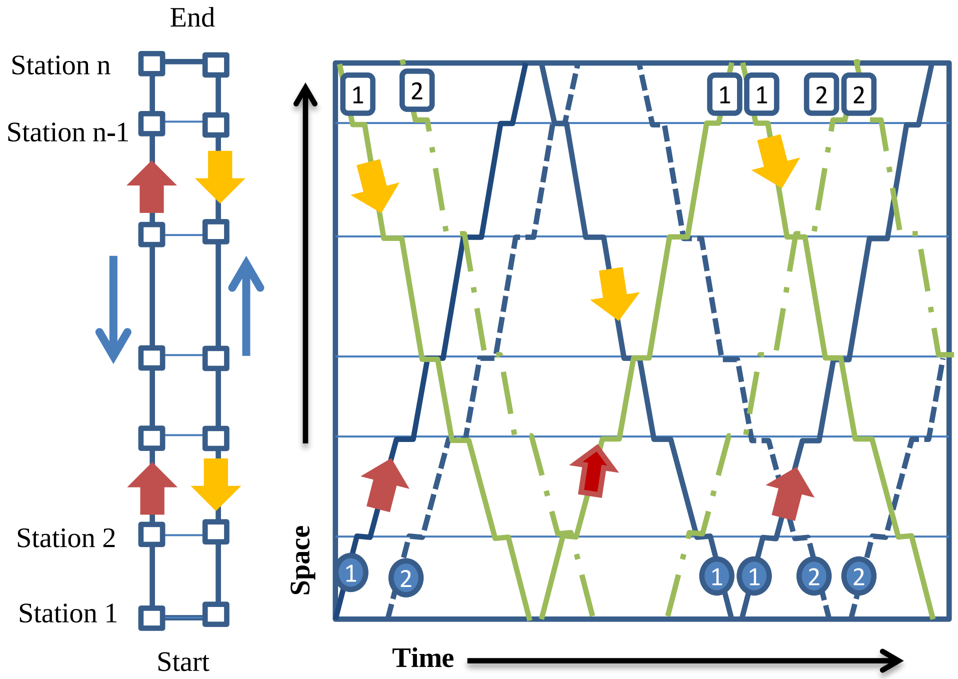
Illustration

Further research

RRT (Railway Rapid Transit) systems

Rapid transit systems are a type of high-capacity public transport generally found in urban and metropolitan areas. Unlike buses and trams, rapid transit systems **operate on an exclusive right-of-way** which is **usually grade separated** in tunnels or elevated railways (at least when conflict appears with other transportation modes).

Usually each line **has a track for each direction** and is operated in **a closed loop way** where train units start their journey at certain station and perform loops between the extreme stations of the line following certain schedule (not necessarily symmetric).



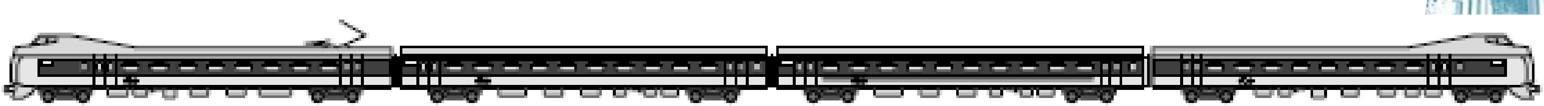
For each line, trains begin their service every morning at certain stations to ensure regularity in the timetable (they are firstly moved from a rest facility) and repeat the circulation until they finish their schedule. In this moment, trains are routed to a rest place until next day.

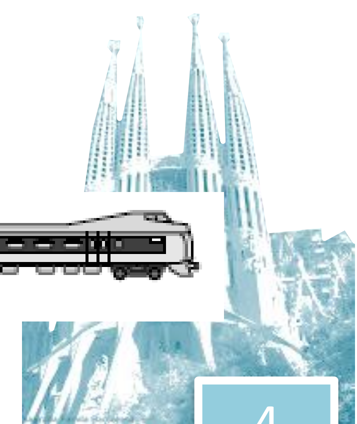
Rolling Stock & RRT systems

- **Trains are typically “train units”**, train in which all the carriages making it up are **shipped from the same origin to the same destination**, **without being split** up or stored on route.

- Carriages composing a unit are all of the same type with exception of the two extremes of the unit, where two **self-propelling carriages** are used. As a consequence, a train unit can move individually in both directions without a locomotive.

- When a train finishes the last service at one station without rest facilities, it must be routed over the network to a rest place without taken passengers. This kind of trips are called “**empty movements**” and should be minimized.





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• Rolling stock is one of the key operational issues for a railway transportation company. Typically, rolling stock and infrastructure maintenance account for **about 75%** of the total cost for a railway transportation network.

• The rolling stock circulation problem consists of **determining individual train paths over the network accomplishing a pre-defined timetable (set of services)**, fulfilling certain design criteria and minimizing facility and operations costs.

• The rolling stock circulation problem can be viewed as a special **multi-commodity capacitated minimum cost routing problem** where a set of different commodities (trains) **must be routed** every day through a network from certain **stations** (rest places) **in order to ensure a set of services and to guarantee minimum operation cost**.

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Rolling Stock - RRT systems assumptions

Trains' rotations can be of different length or even of different number of days.

In order to attend passenger demand for certain services, **several units can be coupled and decoupled**, **this issue is pre-defined accordingly with the timetable.**

The model can manage **different train unit types** (for instance, 4 wagons units, 6 wagons units) simultaneously.

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Further research

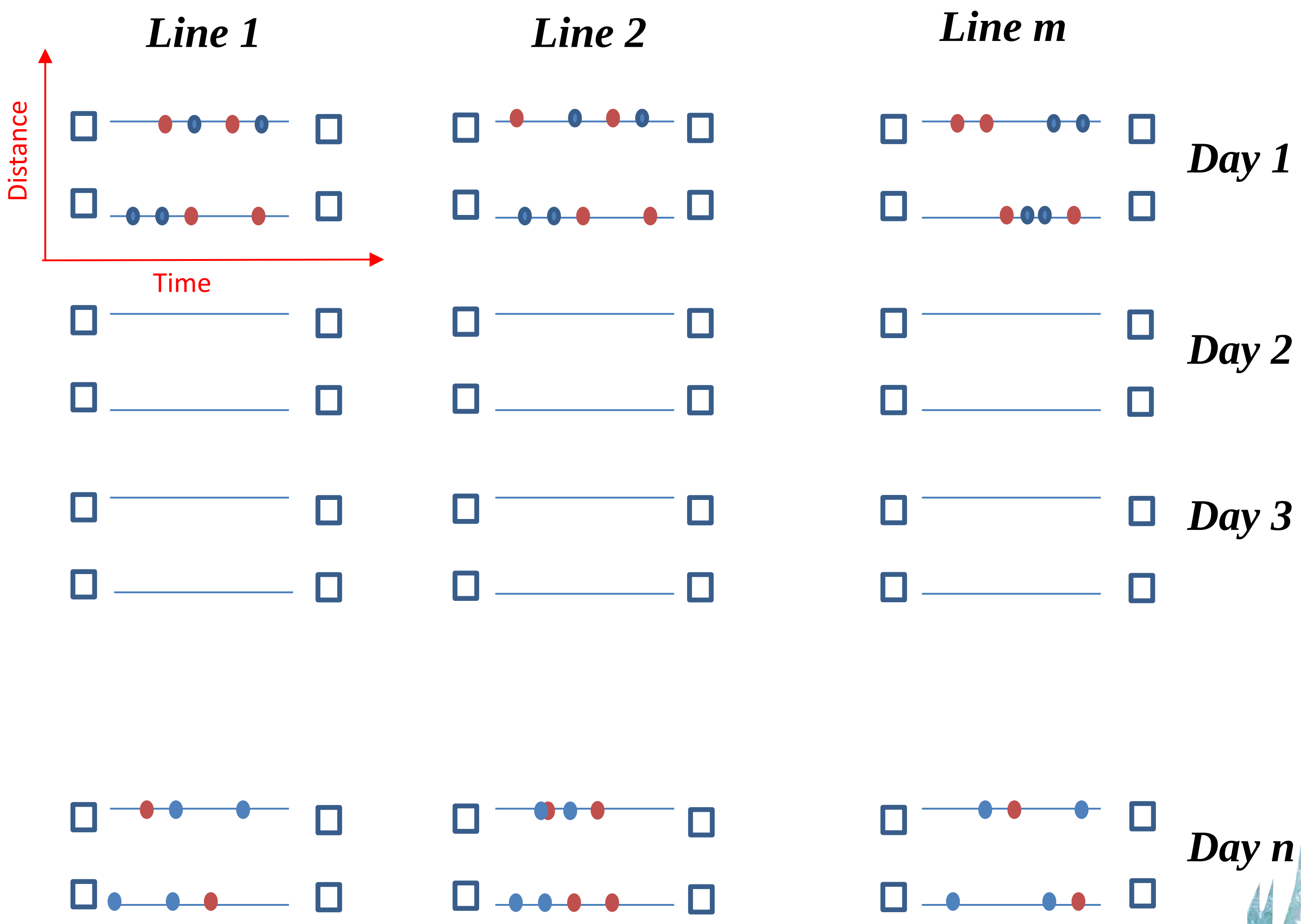
Problem description

Consider a multi-line RRT where:

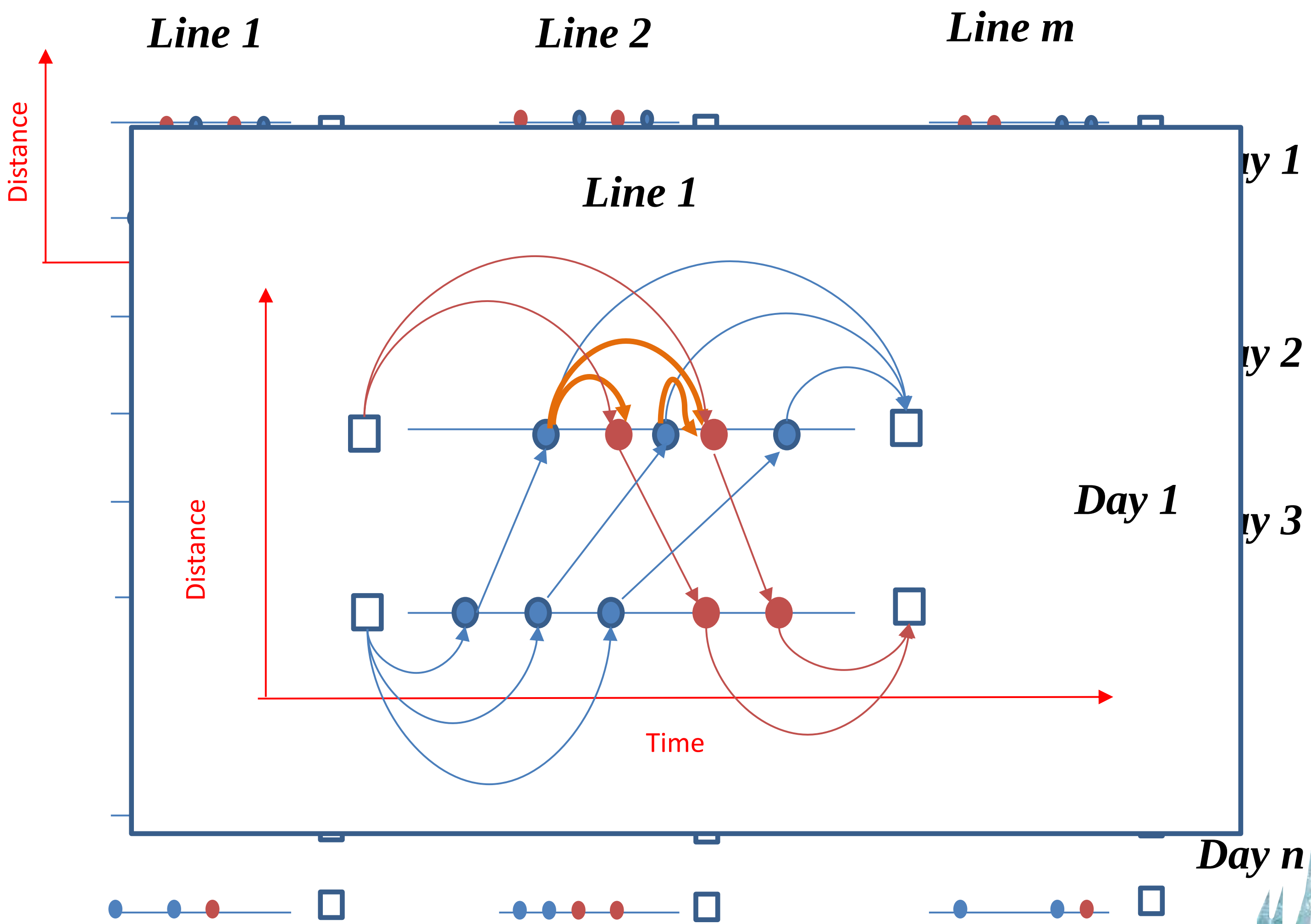
- The train timetable of each line is known in advance together with the specification of coupling/decoupling needs and unit types (as a function of expected passenger demand).

- RRT follow a **weekly pre-specified timetable** repeating the same schedule from Monday to Thursday and introducing some variants for the last three days of the week.

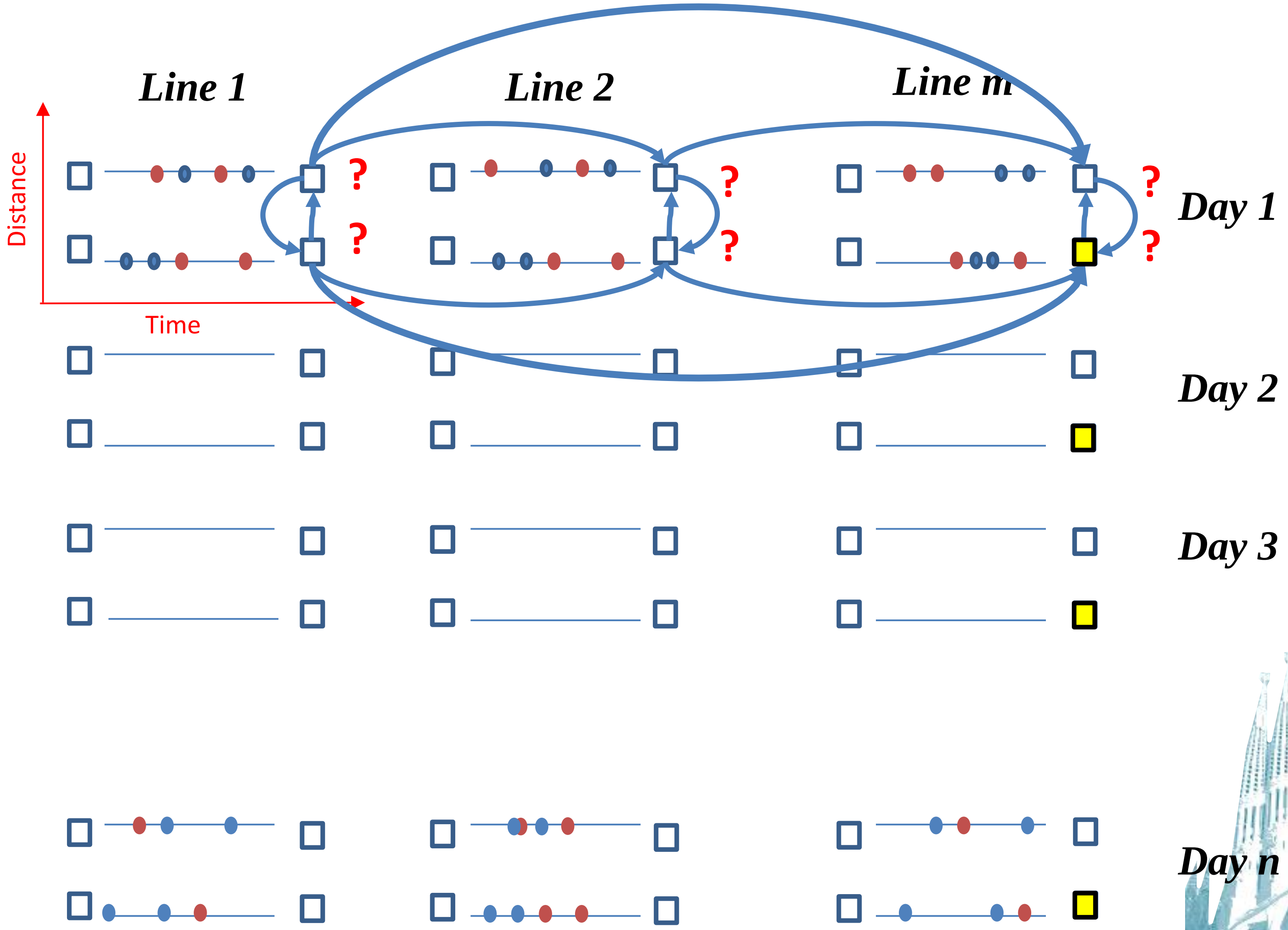
Event-Activity Graph



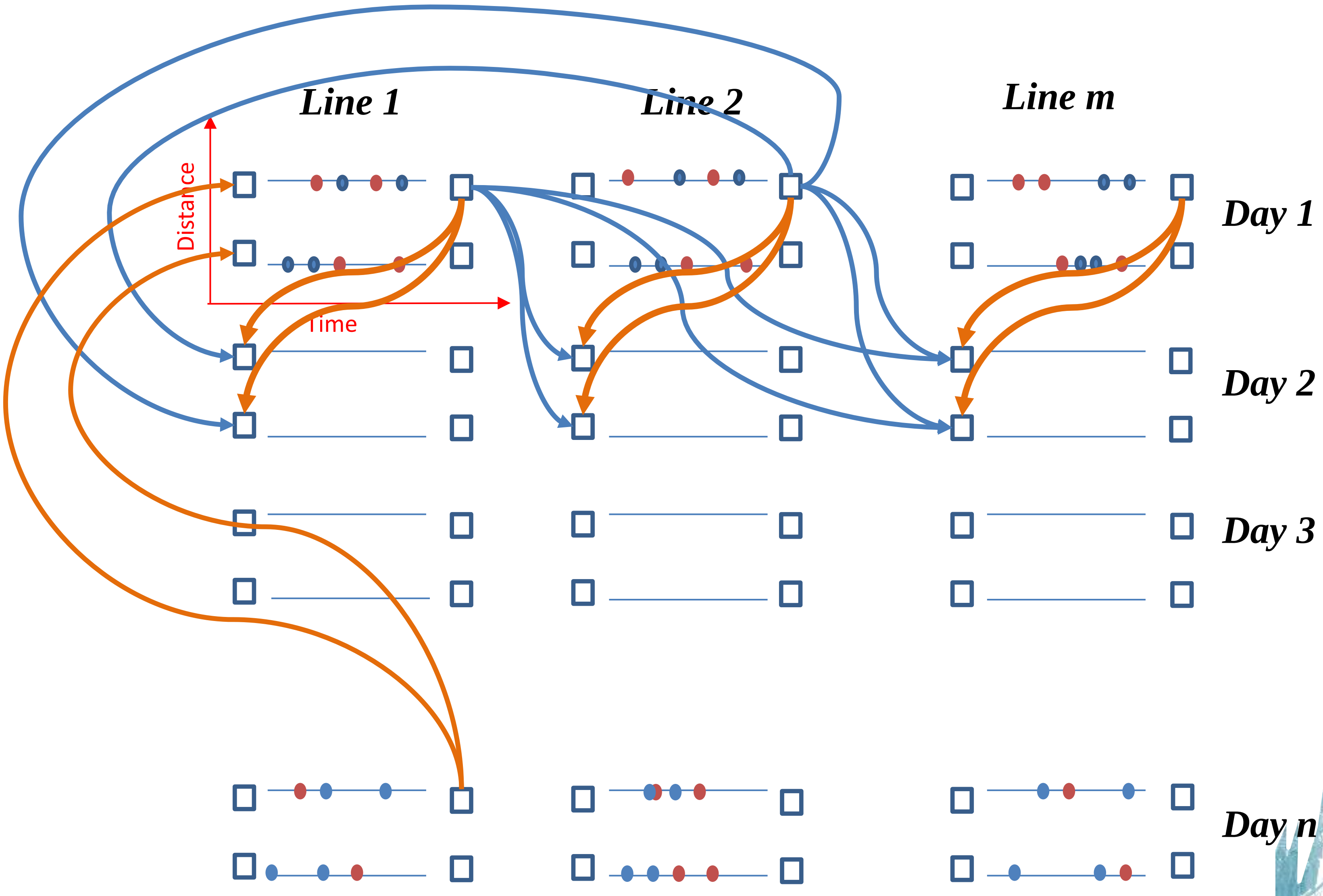
Event-Activity Graph



Event-Activity Graph



Event-Activity Graph



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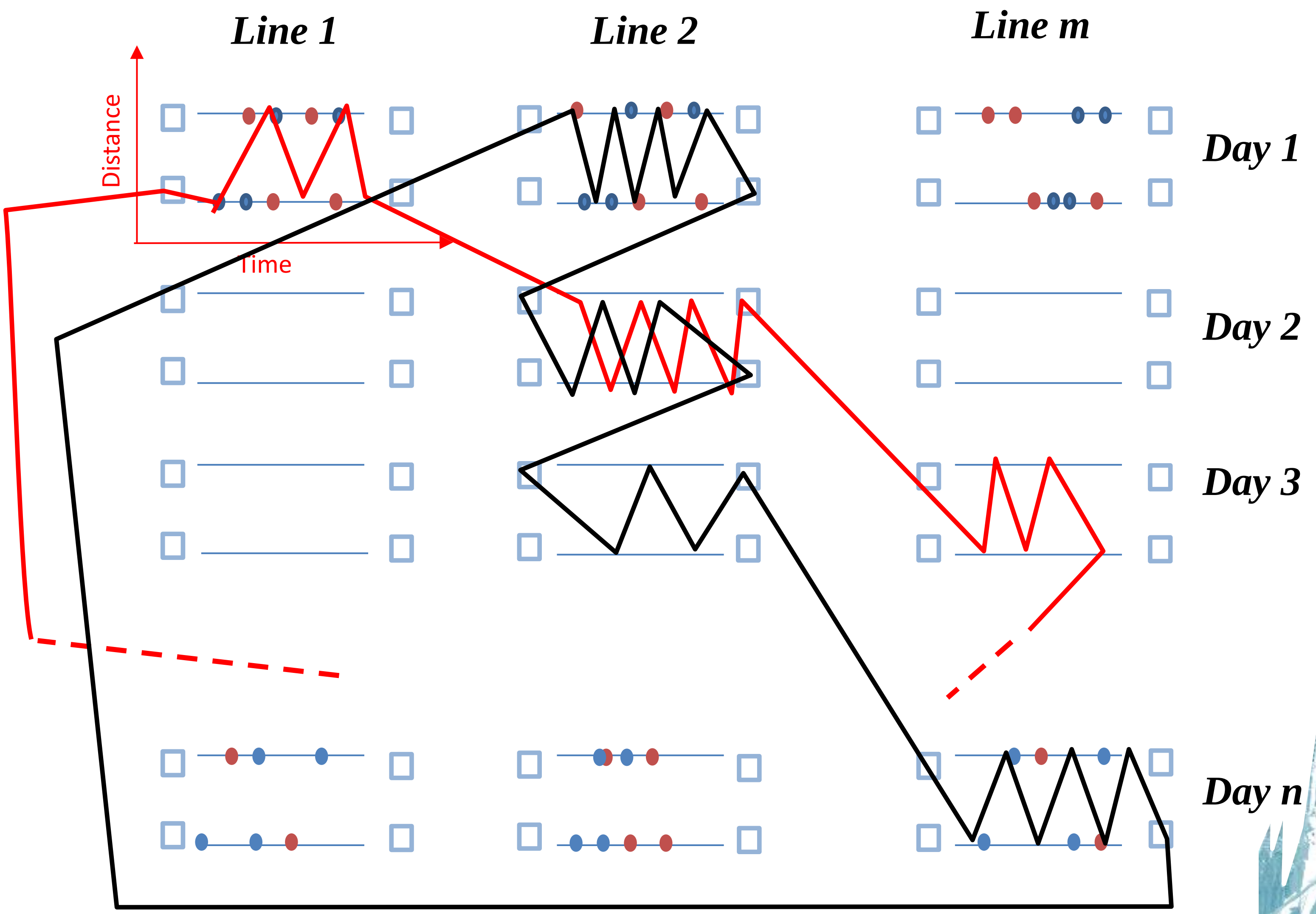
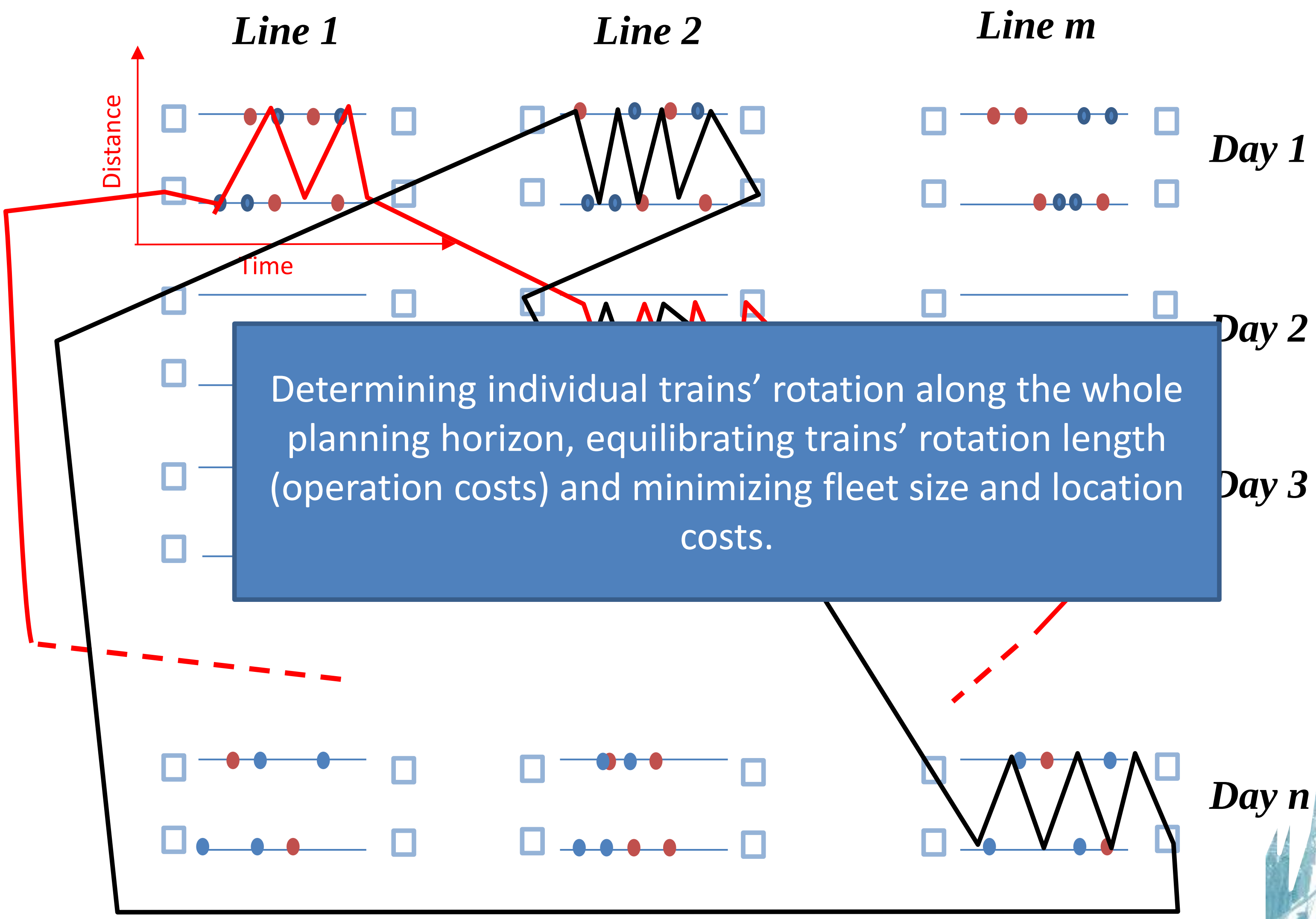


Event-Activity Graph



Determining individual trains' rotation along the whole planning horizon, equilibrating trains' rotation length (operation costs) and minimizing fleet size and location costs.

DATA

N	Set of nodes in the Event-Activity Network
A	Set of directed links in the Event-Activity Network
$Aserv$	Sub-set of linls in A corresponding to train services $Aserv(t), Aserv(l,t)$
$Nserv(t)$	Sub-set of nodes in N corresponding to end of service nodes of day t
Dep	Sub-set of nodes corresponding to "posible " depot locations, $Dep\ (t, t+1)$
L	Set of lines
K	Set of trains
T	Set of days
d_{ij}, c_{ij}	Lenght of link (i,j) / Operating cost of link (i,j)
c_l, c_l'	Cost of locating a depot at the start (, end) node of line l

VARIABLES

x_{ij}^k	**Binary variables**. x_{ij}^k=1 if train k traverses link (i,j), 0 otherwise
R_k	Auxiliary variable representing the lenght of the rotation of train k
$\emptyset_l, \emptyset_l'$	**Binary variables** used to select location of depots (star,end) of lines
z	Auxiliary variable used to equlibrate trains rotation
$\hat{C}_l, \hat{C}_l'$	**Integer variables**. Capacity of depots located at the start (, end) of line l
$f(\hat{C}_l), f(\hat{C}_l')$	Non convex functions used to model the cost of location when depot capacities are considered (start, end) respectively.

Minimizing the total weekly number of Km (all the fleet)

Equilibrating length of individual (train unit) rotations.

Minimizing the cost of locating a depot at the start or end of lines.

MA

$$Min \sum_{(i,j)\in A} \sum_{k\in K} \boldsymbol{c_{ij}} d_{ij} x_{ij}^k + Mz + \sum_{\boldsymbol{l}} \boldsymbol{c_l} \emptyset_{\boldsymbol{l}} + \boldsymbol{c'_l} \emptyset'_{\boldsymbol{l}}$$

st:

$$\sum_{j\in D(i)} x_{ij}^k - \sum_{r\in A(i)} x_{ri}^k = 0 \qquad \forall\, (i,\mathrm{j}) \in \mathrm{A}, k \in K,$$

$$\sum_{k\in K} x_{ij}^k = 1 \qquad \forall\, (i,j) \in AServ \subset A,$$

$$x_{ij}^k \leq |k| \emptyset_l \qquad l \in \mathrm{L}, k \in K,$$

$$x_{i,j}^k \leq |K| \emptyset'_l \qquad l \in L, k \in K,$$

$$R_k = \sum_{(i,j)\in A} d_{ij} x_{ij}^k \qquad \forall k \in K,$$

$$R_k \leq z \qquad \forall k \in K,$$

$$x_{ij}^k, \emptyset_l, \emptyset_l^{\grave{}\prime} = \{0,1\} \qquad \forall k \in K, \forall\, i,j \in A, \forall l \in L$$

Forcing unitary flows on service links for each train (The cardinality of the train set should be defined a priori)

Counting weekly mileage of train *k*

Ingoing flows on depots are allowed only if depot locations are selected

Upper bound of train mileage for every train *k*

Minimizing the total weekly number of Km (all the fleet)

Equilibrating length of individual (train unit) rotations.

Minimizing the cost of locating a depot at the start or end of lines. Non convex function cost. Economies of scale.

MB

$$Min \sum_{(i,j)\in A} \sum_{k\in K} \boldsymbol{c_{ij}} d_{ij} x_{ij}^k + Mz + \sum_{\boldsymbol{l}} \boldsymbol{f}(\widehat{\boldsymbol{C}}_{\boldsymbol{l}}) + \boldsymbol{f}(\widehat{\boldsymbol{C}}'_{\boldsymbol{l}})$$

st:

$$\sum\nolimits_{j\in D(i)} x_{ij}^k - \sum\nolimits_{r\in A(i)} x_{ri}^k = 0 \qquad \forall\, (i,\mathrm{j}) \in \mathrm{A}, k \in K,$$

$$\sum_{k\in K} x_{ij}^k = 1 \qquad \forall\, (i,j) \in AServ \subset A,$$

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$$x_{i,j}^k \le |K| \emptyset'_l \qquad l \in L, k \in K,$$

$$R_k = \sum\nolimits_{(i,j)\in A} d_{ij} x_{ij}^k \qquad \forall k \in K,$$

Upper bound of train mileage for every train *k*

$$R_k \le z \qquad \forall k \in K,$$

$$\hat{C}_l \le \emptyset_l |K|$$

$$\hat{C}'_l \le \emptyset'_l |K|$$

Ensuring the total capacity is enough to manage all trains

$$\sum_{\boldsymbol{l}} \widehat{\boldsymbol{C}}_{\boldsymbol{l}} + \widehat{\boldsymbol{C}}'_{\boldsymbol{l}} \ge \max_{\boldsymbol{t\in T}} \{ \sum_{\substack{\boldsymbol{(i,j):} \\ \boldsymbol{i\in Nserv(t), j\in Dep}}} \sum_{\boldsymbol{k\in K}} \boldsymbol{x_{ij}^k} \}$$

Ingoing flows on depots are allowed only if depot locations are selected

$$x_{ij}^k, \emptyset_l, \emptyset_l^{\prime} = \{0,1\} \qquad \forall k \in K, \forall\, i,j \in A, \forall l \in L$$

OUTLINE

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4 Solving approach

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Illustration

Further research

Decomposition Modelling Approach

? Rest facilities location

Model M1 (Global and continuous)

Determining the minimum number of trains required to perform the timetable

Model M2 (One per line)
Easily solvable heuristically

Determining best local solution for each line
Number of trains required at the start/end stations

Model M3 (Global)
Reduced binary model

Determining depot location and train circulations

Train timetable and rest facilities location

No matter about depots location

Minimum number of trains needed to perform the timetable. Train number is measured as the outgoing flow for each origin node of each line.

Flow conservation at every node

Forcing unitary flows on service links

x_{ij} $\quad$ *flow on link* $(i,j) \in A$

(M1)

$$\boldsymbol{K}^* = Min \sum_{\substack{i \in NServ: \\ j \in Dep}} x_{ij}$$

st:

$$\sum_{j \in D(i)} x_{ij} - \sum_{k \in A(i)} x_{ki} = 0 \qquad \forall i \in N,$$

$$x_{i,j} = 1 \qquad \forall (i,j) \in Aserv,$$

$$x_{ij} \geq 0.$$

Up direction

Down direction

$$\boldsymbol{K}_{\boldsymbol{u}}^*(l) = \max_{t \in T} \sum_{\substack{i \in NServ(l,t): \\ j \in Dep_u}} x_{ij}^* \qquad \boldsymbol{K}_{\boldsymbol{d}}^*(l) = \max_{t \in T} \sum_{\substack{i \in NServ'(l,t): \\ j \in Dep_d}} x_{ij}^*$$

Train requirements for all lines, day by day

(M2) $$\boxed{\forall l \in L, t \in T, \quad K^*(l) = K_u^*(l) + K_d^*(l).}$$

$$Min \quad Mz$$

st:

$$\sum_{j \in D(i)} x_{ij}^k - \sum_{r \in A(i)} x_{ri}^k = 0 \qquad \forall i \in N, k = 1..K^*(l),$$

$$\sum_{k=1..1..K^*(l)} x_{ij}^k = 1 \qquad \forall (i,j) \in Aserv,$$

$$R_k = \sum_{(i,j) \in A} d_{ij} x_{ij}^k \qquad k = 1..K^*(l),$$

$$R_k \leq z \qquad k = 1..K^*(l),$$

$$x_{ij}^k = \{0,1\} \qquad \forall (i,j) \in \ Aserv, \ k = 1..K^*(l),$$

$$x_{ij}^k \geq 0 \qquad \forall (i,j) \in N \backslash Aserv, \ k = 1..K^*(l)$$

Equilibrating length of individual (train unit) circulation.

Counting **daily** mileage of train *k*

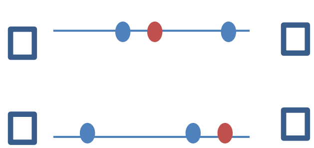
Forcing unitary flows on service links for each train

Upper bound of train **daily** mileage for every train *k*

Only flows at service links must be binary variables (due to the multiplicity on k – train number)

x_{ij}^{k*} ***Internal linkage of departure-arrivals with similar daily mileage and train requirements every day at the start and end of lines***

$\forall l \in L, t \in T$

$N_o^e(l,t)$  $N_f^e(l,t)$

$N_o^s(l,t)$ $N_f^s(l,t)$

$$N_f^e(l,t) = \sum_{\substack{i \in NServ(l,t): \\ j \in Dep_e}} x_{ij}^{k*}$$

$$A' = A \backslash \text{Aserv} \ \cup \{Iserv\}, N' = N \backslash \text{Nserv}$$

$\boldsymbol{N_o^e}(l,t)$ $\boldsymbol{N_f^e}(l,t)$

$\boldsymbol{N_o^s}(l,t)$ $\boldsymbol{N_f^s}(l,t)$

Iserv links

(M3)

$$Min \sum_{(i,j)\in \boldsymbol{A'}} \sum_{k\in K} \mathbf{c_{ij}} d_{ij} x_{ij}^k + Mz + \sum_{l} \boldsymbol{c_l \emptyset_l} + \boldsymbol{c_l' \emptyset_l'}$$

Equilibrating length of individual (train unit) circulation.

Minimizing the cost of locating a depot at the start or end of lines.

Minimizing the total weekly number of Km (all the fleet)

$st:$

$$\sum_{j\in D(i)} x_{ij}^k - \sum_{r\in A(i)} x_{ri}^k = \begin{cases} \boldsymbol{N_f^e}(l,t) & i \in Dep_f^s(l,t)) \\ \boldsymbol{N_f^s}(l,t) & i \in Dep_f^s(l,t) \\ -\boldsymbol{N_o^e}(l,t) & i \in Dep_o^s(l,t) \\ -\boldsymbol{N_o^s}(l,t) & i \in De_o^s p(l,t) \end{cases}$$

$$\sum_{j\in Dep(t+1)} x_{ij}^k \le \boldsymbol{N_f^e}(l,t)\boldsymbol{\emptyset_l'} \qquad \forall k \in K^*, i \in Dep^e(t)$$

$$\sum_{j\in Dep(t+1)} x_{ij}^k \le \boldsymbol{N_f^s}(l,t)\boldsymbol{\emptyset_l} \qquad \forall k \in K^*, i \in Dep^s(t)$$

Linking consecutive days depending on the existence of depots at depots locations

Counting weekly mileage of train *k*

$$R_k = \sum_{(i,j)\in \boldsymbol{A'}} d_{ij} x_{ij}^k \qquad \forall k \in K^*,$$

$$R_k \le z \qquad \forall k \in K^*,$$

Upper bound of train mileage for every train *k*

$$x_{ij}^k = \{0,1\} \qquad \forall (i,j) \in A', k = 1..K^*$$

Train requirements for all lines and every day

$$A' = A \backslash \mathrm{Aserv} \cup \{Iserv\}, N' = N \backslash \mathrm{Nserv}$$

$\boldsymbol{N_o^e}(l,t)$ → $\boldsymbol{N_f^e}(l,t)$

$\boldsymbol{N_o^s}(l,t)$ → $\boldsymbol{N_f^s}(l,t)$

Iserv links

Equilibrating length of individual (train unit) circulation.

Minimizing the cost of locating a depot at the start or end of lines.

(M3)

$$Min \sum_{(i,j)\in A'} \sum_{k\in K} \mathbf{c_{ij}} d_{ij} x_{ij}^k + Mz + \sum_{l} \boldsymbol{c_l \emptyset_l} + \boldsymbol{c_l' \emptyset_l'}$$

Minimizing t
weekly num
Km (all the f

Option 1. Metaheuristic (In progress)

Option 2. Branch and Price.

The location variables (realization) are maintained in the pricing problem and used in the Branching Procedure . The column generation proposes new circuits linking depot nodes for successive days. We follow the same scheme as in "Berger et al. (2007)" *A Branch-and-Price Algorithm for Combined Location and Routing*.

Counting we
mileage of t

Linking consecutive days depending on the existence of depots at depots locations

Upper bound of train mileage for every train *k*

$R_k \le z \qquad \forall k \in K^*,$

$x_{ij}^k = \{0,1\} \qquad \forall (i,j) \in A', k = 1..K^*$

Rotating maintenance schedule

- **The solution of M3 yields** a directed graph containing the circulation of each train during a complete week, that is K^* **circuits,** incorporating solutions at local (line) levels.

- This information is used to build an “a posteriori” rotating maintenance plan and determine the **Reserve Fleet Size**.

- This is possible thanks to the equilibrium in weekly mileages.

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5 Illustration

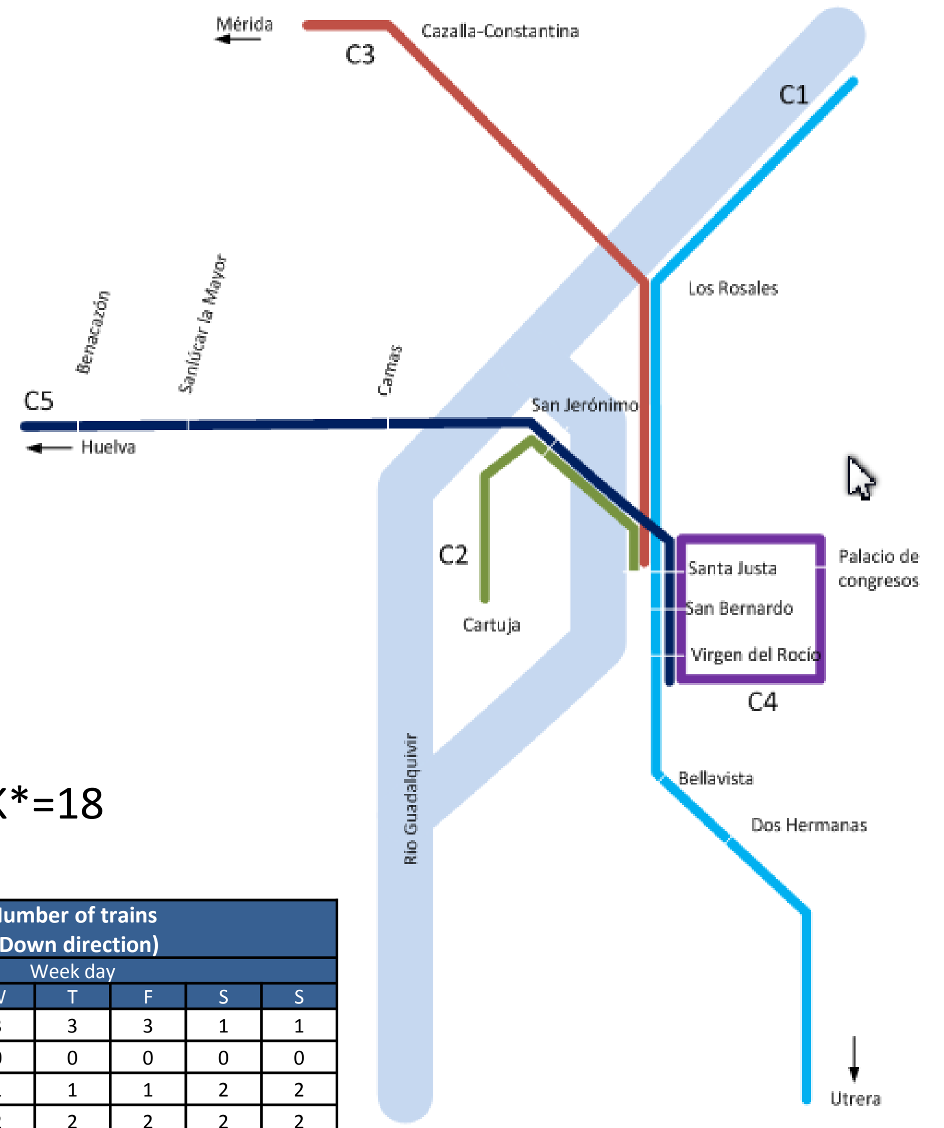
Further research

Illustration

Scenario Data

Line	Number of services Monday to Friday	Number of services Weekends
C1	27	27
C2	15	7
C3	4	2
C5	21	12
C1’	39	19

Line length (km)	C1	C2	C3	C5	C1’	
	66	10	78	33	33	

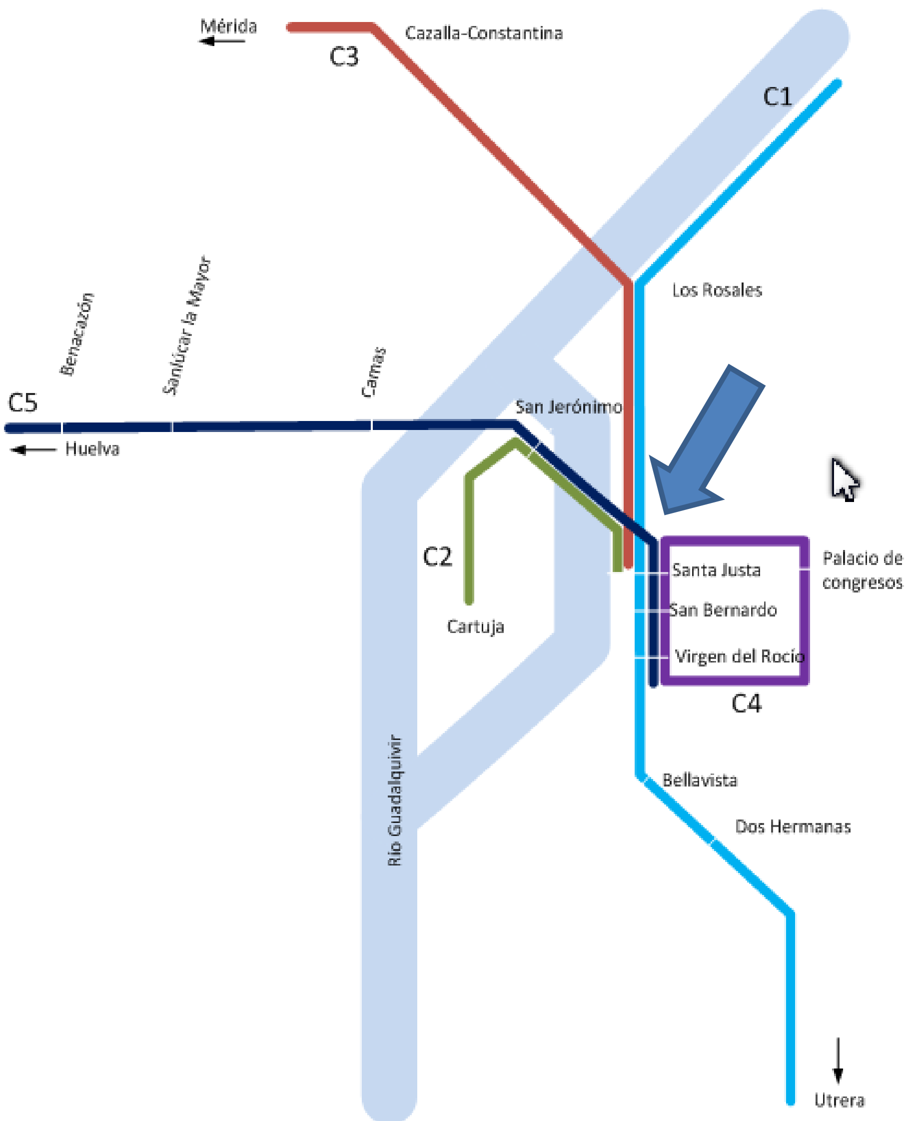


M1 Solution

Number of trains per day	Week day						
	M	T	W	T	F	S	S
	18	18	18	18	18	12	12

K*=18

	Number of trains (Up direction)								Number of trains (Down direction)						
	Week day								Week day						
	M	T	W	T	F	S	S		M	T	W	T	F	S	S
C1	3	3	3	3	3	2	2	C1	3	3	3	3	3	1	1
C2	1	1	1	1	1	1	1	C2	0	0	0	0	0	0	0
C3	1	1	1	1	1	0	0	C3	1	1	1	1	1	2	2
C5	1	1	1	1	1	1	1	C5	2	2	2	2	2	2	2
C1’	3	3	3	3	3	1	1	C1’	3	3	3	3	3	2	2

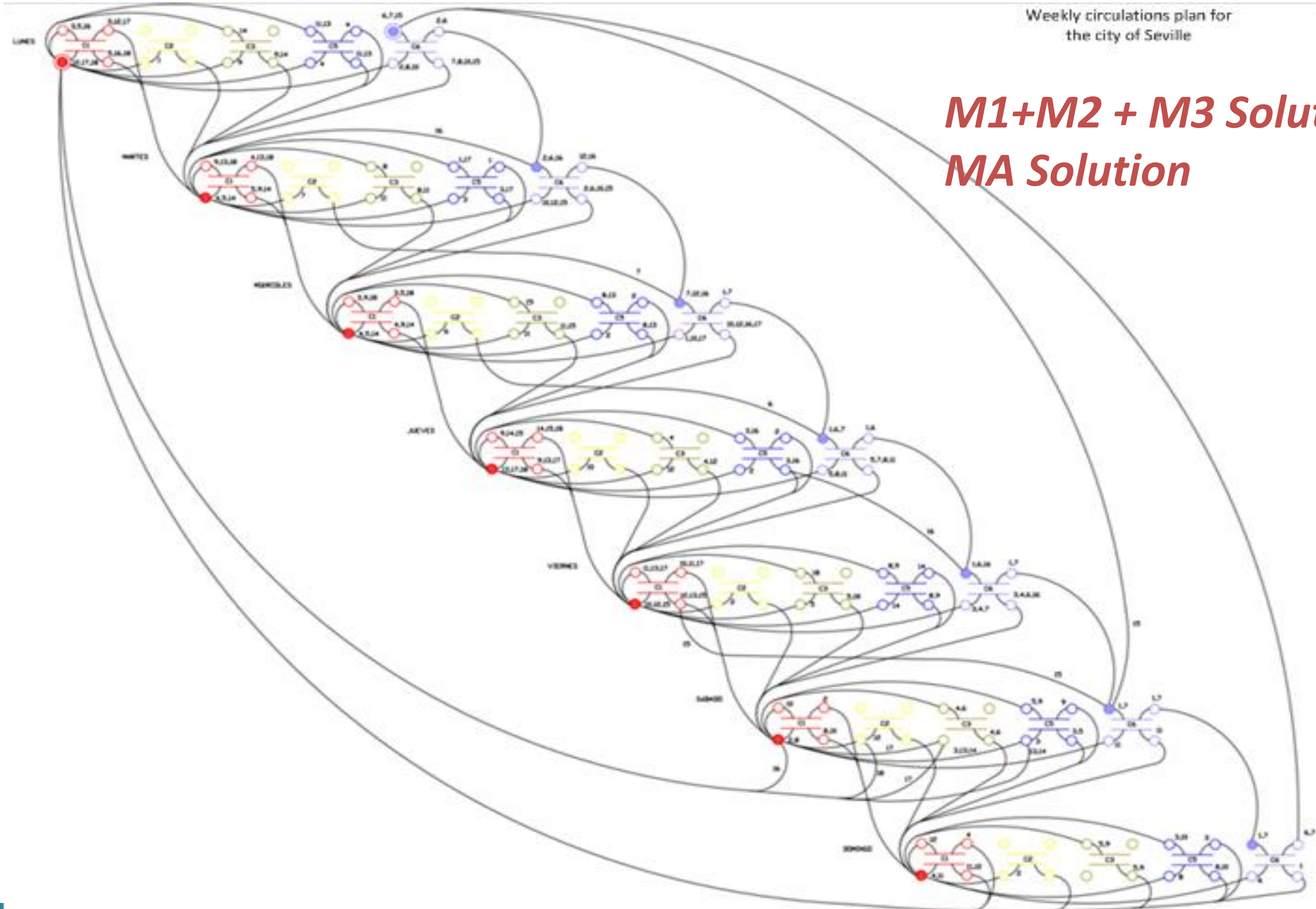


3000 Links, 2000 Nodes, 60000 binary variables

Model	MA	M1+M2 +M3	M3
Cplex Standard Branch and Bound	30 hours		
Cplex B&C		1 h 07 min	1 h 02 min
CG (Python+Gurobi API)		47 min	42 min

Intel i7 4 Core CPU 1,7 GHz, 8GB Ram

Illustration



M1+M2 + M3 Solution=
MA Solution

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6 Further research

FURTHER RESEARCH

Modelling Improvements

Working with detailed (within line circuits) and external (within days) ones, multiplexing the requirements of trains at depot nodes (Matching).

Now, we are working on the realistic cost function (Non-convex) for the **rest depot locations** and the influence on the obtained solutions (including the case of network expansion, a model to analyse the convenience of relocating/(adding) actual/(new) rest facilities).

Incorporate **some acceleration strategies** such as short turning in the train circulation model, i.e., other than extreme stations can be initial or final stations for some services and other special situations.

Solution improvements

Improve (fine tuning) the solution method for the circulation model (Column Generation).

Apply the methodology in bigger scenarios and use approximate methods (metaheuristics) to solve efficiently the problem (ALNS).

Thank you for your attention!.