Strategic Oscillation for the Capacitated Single Assignment Hub Location Problem with Modular Link Capacities

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Introduction

Hub location problems

- Given a network $G = (V, E)$
- For each pair of nodes ij, there is a traffic t_{ij} to be transported
- It is assumed that direct transportation between nodes is not possible
- The traffic t_{ij} travels along a path $i \to k \to l \to j$, where i and j are assigned to hubs k and $l \in V$

HLP consist of:

- To identify an optimal subset of facilities $H \subseteq V$ (hubs)
- To find paths $i \to k \to l \to j$ to transport the traffics through the network in such a way a transportation cost function is minimized

In particular, the C-SA-HLP-wMLC

- The number of hubs that can be used is not fixed *a priori*.
- Each node i is assigned to only one hub $h_i \in H$.
- It consists of selecting a subset of nodes to be hubs and assigning the rest of nodes to them in such a way the transportation cost of all traffics is minimized while satisfying the capacity constraints.

It was formulated by Yaman and Carello in:

Solving the hub location problem with modular link capacities. *Computers & Operations Research*, 32 (12): 3227-3245, 2005.

It was proved that the problem is $\mathcal{NP}$-hard in:

H. Yaman. PhD thesis
Concentrator Location in Telecommunication Networks
Université Libre de Bruxelles, Brussels, Belgium. Dec 2002

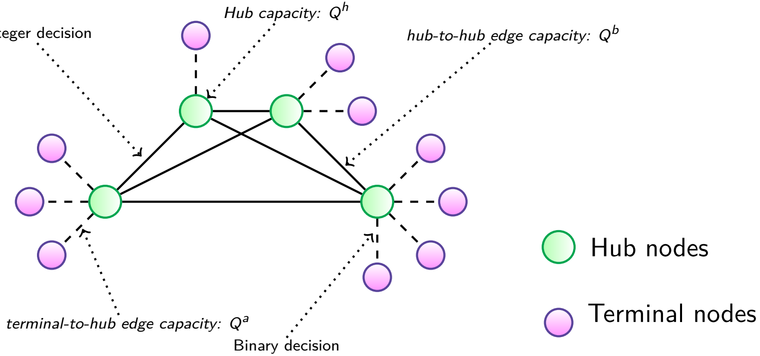
Problem characteristics

- Hubs can be located at any node $i \in V$, with an associated installation fixed cost C_{ii}. All hubs have the same maximum transit capacity Q^h.
- Edges connecting terminals with hubs have a fixed cost of assingment C_{ik}
- Edges between hubs have capacity Q^b in each direction. If the traffic between two hubs exceeds this amount, additional edges with Q^b capacity can be added. Each copy of this type of edge has a fixed cost of R_{kl}.

Decisions:

- Binary decisions of opening hubs and assigning nodes to hubs: x_{ik} and x_{kk}
- Integer decisions of how many copies of edges between hubs, w_{kl}, will be used

A graphical description



A mixed integer model with some non-linear constraints

Objective:

$$\min \quad \sum_{i \in V} \sum_{k \in V} C_{ik} x_{ik} + \sum_{\{k,l\} \in E} R_{kl} w_{kl} \tag{1}$$

Subject to:

$$\sum_{k \in V} x_{ik} = 1, \quad \forall i \in V \tag{2}$$

$$x_{ik} \leq x_{kk}, \quad \forall i \in V, \quad \forall k \in V \setminus \{i\} \tag{3}$$

$$\sum_{i \in V} \sum_{j \in V} (t_{ij} + t_{ji}) x_{ik} - \sum_{i \in V} \sum_{j \in V} t_{ij} x_{ik} x_{jk} \leq Q^h x_{kk}, \quad \forall k \in V \tag{4}$$

$$z_{kl} \geq \sum_{i \in V} \sum_{j \in V} t_{ij} x_{ik} x_{jl}, \quad \forall (k,l) \in A \tag{5}$$

$$Q^b w_{kl} \geq z_{kl}, \quad \forall \{k,l\} \in E \tag{6}$$

$$Q^b w_{kl} \geq z_{lk}, \quad \forall \{k,l\} \in E \tag{7}$$

$$x_{ik} \in \{0,1\}, \quad \forall i,k \in V \tag{8}$$

$$w_{kl} \in \mathbb{Z}_+, \quad \forall \{k,l\} \in E \tag{9}$$

$$z_{kl} \geq 0, \quad \forall (k,l) \in A. \tag{10}$$

The state of the art

One publication:

- The problem is formulated as a quadratic mixed integer programming problem
- A Branch-and-Cut algorithm is proposed to optimally solve the problem
- A heuristic algorithm (a tabu search based method) is proposed to obtain a set of good initial solutions for the B&C

Our proposal:

A standalone heuristic algorithm based on Strategic Oscillation

The strategic oscillation methodology

- Memory structures can be implemented within a constructive process of a solution
  - To favor the inclusion of certain attractive elements
  - To avoid the inclusion of certain unattractive elements
- The neighborhood approach applies to constructive neighborhoods used in building solutions from scratch
- HOW? By orienting moves in relation to a critical level
- We consider a constructive/destructive type of strategic oscillation
  - Constructive: Add elements
  - Destructive: Drop elements

SO for the C-SA-HLP-wMLC pseudocode

While enough computing time, **do**:

1. Start from a complete initial solution
2. Destruction phase
3. Reconstruction phase
4. Local search phase
5. Acceptance criterion: Decide if the current solution becomes the new incumbent solution

End While

Destruction phase

We partially deconstruct the solution by removing some:

- **Hubs**
  1. A parameter of the search, δ, indicates the percentage of the total number of hubs that will be destructed
  2. The hubs are removed from the solution at random
- **Assignments** of the remaining hubs
  1. The same parameter, δ, indicates the percentage of terminals assigned to the still selected hubs that will be destructed
  2. The terminals with higher assignment cost to their associated non-removed hubs are selected and destructed.

Reconstruction phase

1. If the active hubs still have spare transit capacities
   - Assign as many orphan nodes as possible to them
2. Otherwise
   - Select a new node to be hub
   - Assign to this new hub as many terminals as its capacity permits

Improvement phase

Two neighborhoods, N_{pairs} and N_{alone}, to improve the assignments

- N_{pairs}
  - Implements an exchange movement between two terminals
  - Two terminals swap their corresponding hubs, if capacities permit
- N_{alone}
  - Implements an insertion movement
  - A terminal, previously assigned to a hub, is now assigned to another hub

Computational results

Test instances

Three families of instances:

- CAB, $n = \{10, 15, 25\}$
- AP, $n = \{10, \ldots, 200\}$
- USA423, $n = \{20, \ldots, 250\}$

A total of 170 instances have been used in the experiments

Comparison with optimal values

	CPLEX		SO	
Instance	**Value**	**CPU**	**Dev**	**CPU**
A1H	72710	24.02	0.0%	0.20
A2H	105477	254.30	0.0%	0.17
A3H	77516	23.69	0.0%	0.23
A4H	188200	139.00	0.0%	0.20
B1H	45636	75.80	0.0%	0.30
B2H	23818	4.66	0.0%	0.34
B3H	51387	31.08	0.0%	2.07
B4H	25410	4.71	0.0%	0.90
C1H	43526	4297.98	0.0%	0.50
C2H	43505	3304.48	0.0%	0.78
C3H	57905	33891.33	0.0%	1.08

Comparison with previous Tabu Search

		PrevTS			SO		
Type	**# Inst**	**Dev**	**# Best**	**CPU**	**Dev**	**# Best**	**CPU**
s	33	8.5%	7	1.0	0.0%	32	0.6
m	25	10.9%	3	88.1	0.4%	22	41.1
l	76	11.7%	10	1263.1	0.6%	69	545.8
sum	134	10.8%	20	733.1	0.5%	123	317.4

We learneded from this:

On problems where good solutions have such a different structures (for example, such a different number of hubs), it can be convinient to keep part of the solution in our way to another good solution.

Do not begin from scratch.

Conclusions

- We have proposed a new metaheuristic based on Strategic Oscillation for the C-SA-HLP-wMLC
- The problem was introduced as an interesting variant of the classical HLP in which the cost of some edges is stepwise and the hubs are restricted in terms of transit capacities
- The computational experiments show that our algorithm is able to find good solutions in short computing times
- The proposed method outperforms the previous published tabu search metaheuristic procedure

Thank you very much for your attention!

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