DIAL-A-RIDE PROBLEMS IN PRESENCE OF TRANSSHIPMENTS

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SUMMARY

1. Dial-a-Ride Transit
2. Classical Pick-up and Delivery Problem
3. The Pick-up and Delivery Problem with Transfers
4. Formulations of PDP (without and with Transfers)
5. A comparative example
6. Geometric perspective of the PDP.
7. A real application.

Dial-a-Ride Transit

- Dial-a-Ride Transit is a transport modality, orientated to users of public transport systems, which is characterized by flexible routing and scheduling of small/medium vehicles operating in shared-ride mode between pick-up and drop-off locations according to the needs of their passengers.
- DRT systems typically provide a public transport service for
  - Areas of low passenger demand, such as rural areas, where a regular bus service would not be viable.
  - Alternative means of transportation for addressing disruptions in mass transit systems.
  - Management of ambulance fleet and disabled passengers.
  - Management of school bus routes
  - Shuttle bus to / from airports.
  - Sale of airline tickets and flight management
  - Shared taxis.

Where could be included Dial-a-Ride Transit?

- The Vehicle Routing Problem (VRP) arises when a dispatcher has a fleet of vehicles and a set of geographically scattered customers requiring some service from the vehicles. The objective of the VRP is often to minimize the travel distance (or travel costs) of the vehicles in operation, or to maximize the number of served customers.

- There exist several variations of the VRP. Between them:

  1. The Capacitated Vehicle Routing Problem (CVRP): the vehicles can only carry a limited number of goods or passengers.

  2. The Vehicle Routing Problem with Heterogeneous Vehicles (VRPHV): the vehicles are not necessarily alike in carrying capacity.

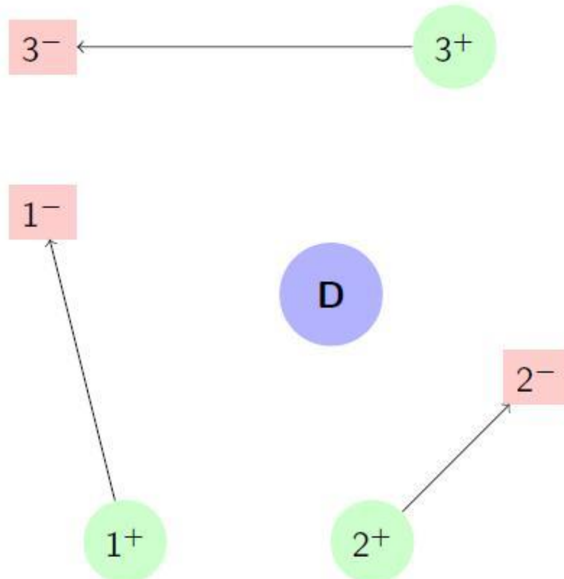
  3. The Vehicle Routing Problem with Time Windows (VRPTW): the vehicles can only serve customers in some specific part of the day, specified for each customer.

  4. The Pick-up and Delivery Problem (PDP): when serving a pick-up customer/location, the same vehicle later has to serve an associated delivery customer/location.

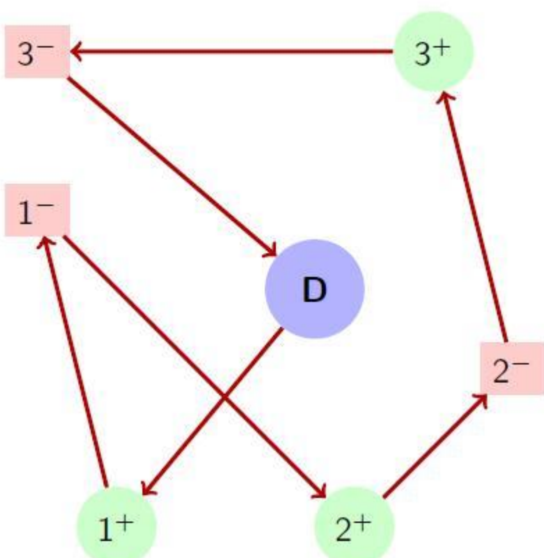
Classical Pick-up and Delivery Problem (i)

- The problem of pick up and drop passengers can be seen as a specific version of the VRP.
- Similarly, the DARP can be assumed as an instance of the Capacitated Pick-up and Delivery Problem with Time Windows and Heterogeneous Vehicles.
- In order to clarify the formulation we must distinguish two concepts: request and passenger.
- A transportation request consists of picking up a certain number of passengers from a predetermined pickup location and dropping them off at a predetermined delivery location. A transportation request is a mathematical object which is represented by an origin, a destination and the number of passengers to be transported.
- A request involves a set of passengers traveling from the same origin to the same destination. Usually, passengers belonging to the same request are never split into different vehicles.
- Each customer *i* is associated with a pair of nodes, denoted by (i^+, i^-) and called source and destination nodes of customer *i*, respectively. Customer *i* should be picked up at source i^+ by a vehicle and then, the same vehicle must delivery at its destination i^-.

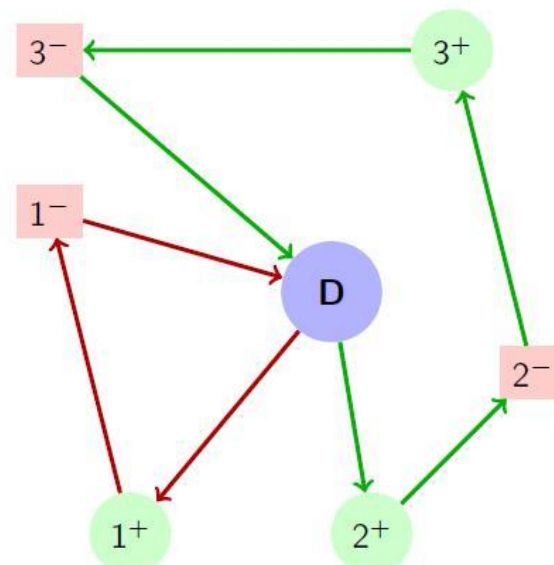
Classical Pick-up and Delivery Problem (example)



Solution for one vehicle



Solution for two vehicles



Classical Pick-up and Delivery Problem (ii)

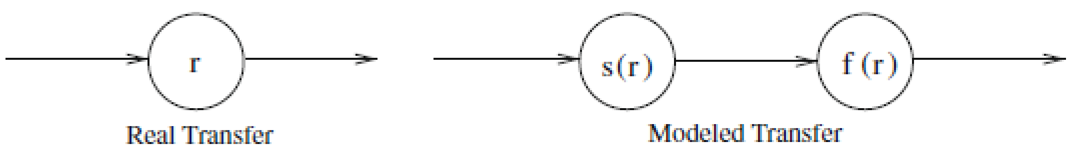
- In the classical formulation of the PDP, the demand for the service is known in advance (static version). However, numerous authors have explored its dynamic version with the purpose of solving the problem in real time by means of heuristic approaches. (Desrosiers et al., 1986; Psaraftis, 1988; Madsen et al., 1995; Gendreau et al., 1998).
- The objective f (x) can be stated as minimize certain amount (distance traveled by vehicles, travel/waiting times for customers, etc.) subject to serve the demand due to the *n* customers.
- The constraints of the static PDP can be classified (Desrosiers et al., 1995) as follows:
  - Visiting constraints (each pickup and delivery has to be visited exactly once)
  - Vehicle Capacity constraints
  - Depot constraints
  - Coupling constraints (stating that for a given request the same vehicle must visit the pickup and delivery stops)
  - Precedence constraints
  - resource constraints on the Availability of Drivers and Vehicles
  - and Time Window constraints to be satisfied at each stop (in case of time windows have been explicitly defined).

The Pick-up and Delivery Problem with Transfers (i)

In classical PDP, three kind of nodes can be distinguished: depots, origins and destinations.

- At depots, actually there is no load/unload of passengers since vehicles start and finish their routes empty.
- At every origin or destination node, we identify only one operation associated with vehicles, which is either boarding (origin) or alighting (destination) of passengers, but not both.

However, when visiting transfer nodes, vehicles may either load or unload passengers. To distinguish between both operations, Cortes et al. (2010) propose to split every transfer node r, into two separate nodes, s(r) (start node) and f(r) (finish node), so that only one operation associated with vehicles load/unload is allowed.



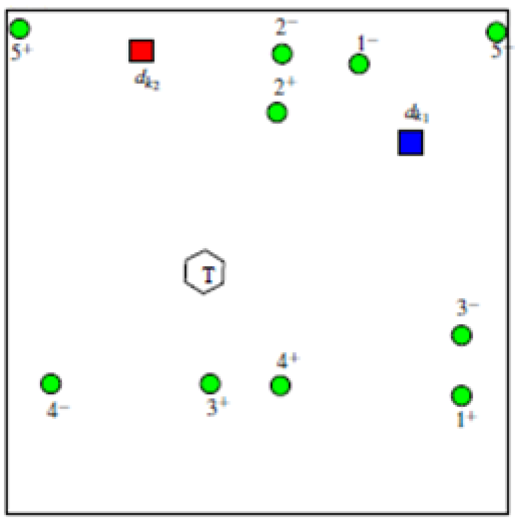
The Pick-up and Delivery Problem with Transfers (ii)

Assume a transportation demand located in different parts of the city that wants to travel from source nodes to destination points.

What is the best way of routing *m* vehicles of capacity *Q* in order to satisfy all customer demand while minimizing some objective function ?.

Suppose that there are parts of the city in which the vehicle can interact with each other, such that passengers can be transferred.

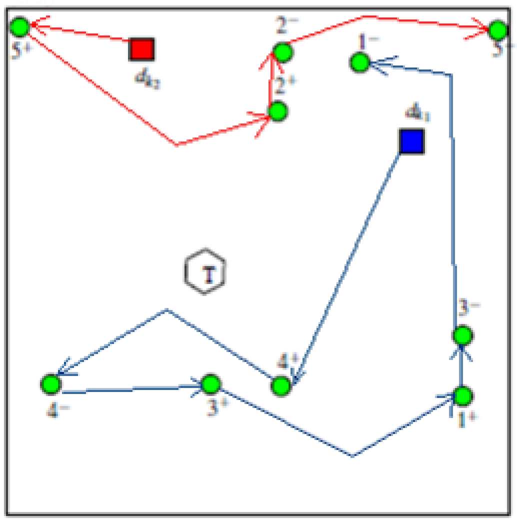
Could we improve the solutions obtained in the classic PDP?

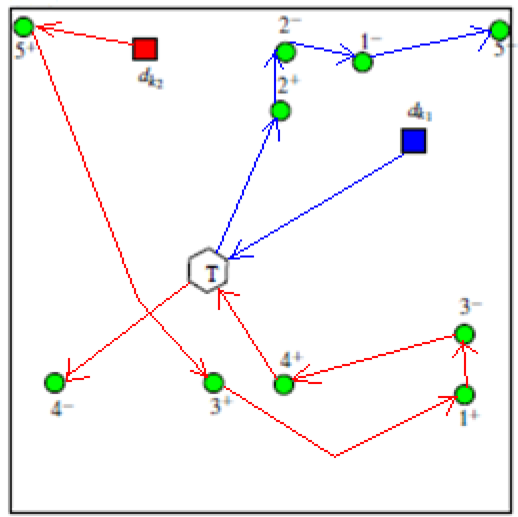


- 5 pickup requests of customers (superscript +) and their corresponding delivery locations (subscript -).
- 2 depots of vehicles from which the routes must start .
- 1 Transshipment point for optional use.

The Pick-up and Delivery Problem with Transfers (iii)

Solution without considering transfers





Solution including transfer options (the objective improves 2,3%)

Formulation: Sets (i)

Among others, Savelsbergh and Sol (1995) mathematically formulate the classical PDP as a Mixed Integer Programming (MIP).

Sets:

C Customer set.

C^+, C^- Sets of nodes for picking up and dropping customers.

$$N = C^+ \cup C^- = \{1, \ldots n\}; C^+ \cap C^- = \varnothing$$

$M = \{1, \ldots, m\}$ Vehicle set

M^+, M^- Sets of nodes for starting and ending routes, respectively.

If **transfers were not considered**, the graph which represents the solution space can be denoted by $G = (V, E)$, where

$$V = N \cup M^+ \cup M^-$$

$$E = \{(i, j) : i \in M^+, j \in C^+\} \cup \{(i, j) : i, j \in N, i \neq j\} \cup \{(i, j) : i \in C^-, j \in M^-\}$$

If transshipments were considered, the previous set of nodes should be expanded by incorporating new pickup and delivery points for customers at points $r \in T$.
Since each of these new nodes have a double functionality (picking up and dropping), their representation must be duplicated in the graph.

$$\forall r \in T \Rightarrow \text{ new vertices } \; s(r), f(r); \;\; \text{new arcs } \; (s(r), f(r))$$

Therefore, the graph containing the solution space $G = (V, E)$ must now be defined as follows:

$$V = N \cup M^{+} \cup M^{-} \cup s(T) \cup f(T)$$

$$\begin{aligned} E = {} & \left\{(i,j) : i \in M^{+}, j \in C^{+}\right\} \cup \left\{(i,j) : i, j \in N, i \neq j\right\} \cup \left\{(i,j) : i \in C^{-}, j \in M^{-}\right\} \cup \\ & \left\{(i, s(r)) : i \in M^{+} \cup N, r \in T\right\} \cup \left\{(s(r), f(r)) : r \in T\right\} \cup \\ & \left\{(f(r), j) : r \in T, j \in N \cup M^{-}\right\} \cup \left\{(f(r), s(r')) : r, r' \in T, r \neq r'\right\}. \end{aligned}$$

Formulation: Variables and parameters (i)

To formulate the programming model four types of variables and three sets of parameters are used:

$$x_{ij}^{k} \in \{0,1\} \quad \forall (i,j) \in E, k \in M$$

where *E* is the set of nodes in the graph and M is the set of vehicles.
These **variables** are binary, and equal to 1 if route of vehicle *k* uses arc (i, j), 0 otherwise.

$$z_{i}^{k} \in \{0,1\} \quad \forall i \in C, k \in M$$

These **variables** are also binary, and equal to 1 if customer *i* is assigned to vehicle k, 0 otherwise.

$$D_i \geq 0 \quad \forall i \in V$$

These **variables** are real and represent the time when a vehicle arrives to node *i*.

$$y_i \geq 0 \quad \forall i \in V$$

These **variables** are real and represents the load of vehicle when arrives to node *i*.

$$t_{ij} \geq 0 \quad \forall (i,j) \in E \qquad Q_k \geq 0 \quad \forall k \in M \qquad q_i \geq 0 \quad \forall i \in C$$

These **parameters** are real and represent costs (time) relative to the use of the arcs in the network, the capacity of the k-th vehicle and load of the request of the i-th customer, respectively.

Model Without Transfers: PDP Model by Savelsbergh and Sol (1995) [i]

$$\min_{x} \quad f(x)$$

s. t.

$$\sum_{k\in M} z_i^k = 1; \quad \forall i \in C$$

Each customer is assigned to an unique vehicle

$$\sum_{j\in V; j\neq l} x_{ij}^k = \sum_{j\in V; j\neq l} x_{jl}^k = z_i^k; \quad \forall i \in C; l \in \{i^+, i^-\}; k \in M$$

Vehicle k associated to customer i must visit nodes i^+ and i^-

$$\sum_{j/(k^+,j)\in E} x_{k^+ j}^k = 1; \quad \forall k \in M$$

$$\sum_{j/(j,k^-)\in E} x_{jk^-}^k = 1; \quad \forall k \in M$$

$$\sum_{m\in M^+} \sum_{j\in V/(m,j)\in E} x_{mj}^k = 1; \quad \forall k \in M$$

$$\sum_{j\in V/(i,j)\in E} x_{ij}^k - \sum_{j\in V/(j,i)\in E} x_{ji}^k = 0; \quad \forall i \in N; k \in M$$

Vehicles depart and arrive to / from their depots. They can not start two routes from different depots. The flow in the nodes must be preserved.

Model Without Transfers: PDP Model by Savelsbergh and Sol (1995) [ii]

$$\ldots \min_x \quad f(x) \quad \text{s. t.} \quad \ldots$$

$$D_{k^+} = 0; \quad \forall k \in M$$

$$D_{i^+} \le D_{i^-}; \quad \forall i \in C$$

$$x_{ij}^k = 1 \Rightarrow D_i + t_{ij} \le D_j; \quad \forall (i, j) \in E, k \in M$$

Precedence in the visit to the nodes and consistency over time visiting nodes.

$$y_{k^+} = 0; \quad \forall k \in M$$

$$y_l \le \sum_{k \in M} Q_k \, z_i^k; \quad \forall i \in C; l \in \{i^+, i^-\}$$

$$x_{ij}^k = 1 \Rightarrow y_i + q_i \le y_j; \quad \forall (i, j) \in E, k \in M$$

The vehicle starts the route at depot with zero load. The load in the vehicle must be consistent with its capacity.

$$x_{ij}^k \in \{0,1\}; \quad \forall (i, j) \in E, k \in M$$

$$z_i^k \in \{0,1\}; \quad \forall i \in C, k \in M$$

$$D_i, y_i \ge 0; \quad \forall i \in V$$

Nature of the variables

PDP With Transfers : New properties [i]

In order to adapt the classic model of PDP such that transfers can be incorporated, we must bear in mind that the new formulation must meet the following properties:

(Contardo, 2005; Cortés et al., 2010)

a) For each vehicle *k,* there will exist an unique route that, without containing cycles, starts from node k^+ and finishes at node k^-.
b) Every request must be served; that is, its origin and destination nodes must be visited exactly once.
c) For every request, its origin node must be visited before its destination node.

d) If a customer reaches transfer *r*, in order to move from vehicle k_1 to vehicle k_2, then vehicle k_2 must leave the transfer point r after the vehicle k_1 has arrived.
e) Vehicles can not exceed their capacity.

PDP With Transfers : New properties [i]

ADAPTATION 1: Vertices i^+ and i^- must be visited, although not necessarily by the same vehicle. Change the second constraint:

$$\sum_{j \in V;\, j \neq l} x_{ij}^k = \sum_{j \in V;\, j \neq l} x_{jl}^k = z_i^k; \quad \forall i \in C; l \in \{i^+, i^-\}; k \in M$$

by

$$\left.\begin{array}{l} \displaystyle\sum_{k \in M} \sum_{j \in N/(j,i^-) \in E} x_{ji^-}^k = 1; \quad \forall i \in C \\[2ex] \displaystyle\sum_{k \in M} \sum_{j \in N/(i^+,j) \in E} x_{i^+j}^k = 1; \quad \forall i \in C \end{array}\right\}$$

Now, a customer may be served by two different vehicles.

ADAPTATION 2: Flow conservation must also be satisfied at the transfer nodes. We will expand this set of constraints by including this pair:

$$\left.\begin{array}{l} \displaystyle\sum_{j \in V/(j,s(r)) \in E} x_{js(r)}^k = x_{s(r)f(r)}^k; \quad \forall r \in T; k \in M \\[2ex] \displaystyle\sum_{j \in V/(f(r),j) \in E} x_{f(r)j}^k = x_{s(r)f(r)}^k; \quad \forall r \in T; k \in M \end{array}\right\}$$

ADAPTATION 3: To satisfy properties a) and b), the number of variables associated with the time when the nodes are visited must be increased, in order to force transshipments are consistent. Thus there will be three types of variables and new restrictions:

D_i Time when node i is served [equal that before]

$D^k_{s(r)}$ Time when vehicle k arrives to transfer node r

$D^k_{f(r)}$ Time when vehicle k leaves from transfer node r

$$x^k_{ij} = 1 \Rightarrow D_i + t_{ij} \le D_j \ ; \quad \forall k \in M; \forall i, j \in N$$

$$x^k_{i\ s(r)} = 1 \Rightarrow D_i + t_{is(r)} \le D^k_{s(r)} \ ; \quad \forall k \in M; \forall i \in N; \forall r \in T$$

$$x^k_{f(r)j} = 1 \Rightarrow D^k_{f(r)} + t_{f(r)j} \le D_j \ ; \quad \forall k \in M; \forall j \in N; \forall r \in T$$

$$x^k_{k^+i^+} = 1 \Rightarrow t_{k^+i^+} \le D_{i^+} ; \quad \forall k \in M; \forall i \in C$$

$$x^k_{k^+s(r)} = 1 \Rightarrow t_{k^+s(r)} \le D^k_{s(r)} \ ; \quad \forall k \in M; \forall i \in C$$

$$x^k_{s(r)f(r)} = 1 \Rightarrow D^k_{s(r)} + t_{s(r)f(r)} \le D^k_{f(r)} \ ; \quad \forall k \in M; \forall r \in T$$

$$x^k_{f(r)s(r')} = 1 \Rightarrow D^k_{f(r)} + t_{f(r)s(r')} \le D^k_{s(r')} \ ; \quad \forall k \in M; \forall r, r' \in T \ (r \ne r')$$

New constraints

PDP With Transfers : New properties [iv]

ADAPTATION 4: To satisfy properties c) y d), we must adapt the variables that take value 1 if the customer *i* is assigned to the vehicle *k*, 0 otherwise: $z_i^k \in \{0,1\} \quad \forall i \in C, k \in M$.

The new variables can be denoted by: $z_j^{ki} \in \{0,1\} \quad \forall i \in C, k \in M, j \in V$.
and take the value 1 if customer *i* is in vehicle *k* when it reaches node j, 0 otherwise.

Constraints to necessarily incorporate into the model are:

$$z_{k^+}^{ki} = z_{k^-}^{ki} = 0 \quad \forall k \in M, \forall i \in C$$

$$x_{lj}^k = 1 \Rightarrow z_l^{ki} = z_j^{ki}; \quad \forall i \in C; \forall (l, j) \in E \setminus T, l \neq i^+, i^-$$

$$x_{i^+ j}^k = 1 \Rightarrow z_j^{ki} = 0; \quad \forall i \in C; \forall j / (i^+, j) \in E$$

$$x_{i^- j}^k = 1 \Rightarrow z_j^{ki} = 0; \quad \forall i \in C; \forall j / (i^-, j) \in E$$

$$\sum_{k \in M} z_{s(r)}^{ki} - \sum_{k \in M} z_{f(r)}^{ki} = 0; \quad \forall i \in C; \forall r \in T$$

$$\sum_{r \in T} \sum_{k \in M} z_{s(r)}^{ki} \leq p; \quad \forall i \in C$$

$$\sum_{l \in V / (l, j) \in E} x_{lj}^k = 0 \Rightarrow \sum_{i \in C} z_j^{ki} \leq 0; \quad \forall k \in M; \forall j \in V \setminus \{k^+, k^-\}$$

The combined action of these restrictions ensures the properties of precedence and temporal consistency of vehicle visits to the nodes.

Additionally, we can limit the maximum number of transfers by means parameter *p*.

ADAPTATION 5: Additional conditions:

- Establish a minimum standard *h* to allow the transfer of passengers:

$$z_{s(r)}^{ki} + z_{f(r)}^{vi} = 2 \Rightarrow D_{s(r)}^{k} + h \leq D_{f(r)}^{v}; \quad \forall r \in T; \forall k, v \in M; \forall i \in C$$

- Limit the vehicle load to its actual capacity (-property e)-):

$$\sum_{i \in C} z_j^{ki} \leq Q_k; \quad \forall k \in M; \forall j \in V$$

- Restrict the total of stops *P* that must do any of the customers:

$$\sum_{j \in V} \sum_{k \in M} z_j^{ki} \leq P; \quad \forall i \in C$$

- Incorporate time windows $[t_i^-, t_i^+], \forall i \in C,$ for carrying out the service to customer *i*:

$$t_i^- \leq D_i \leq t_i^+; \quad \forall i \in C$$

Among the objectives that can be applied in the decision model that incorporates transfers to the PDP, the most used in the literature are:

a) Minimize total ride time spent by vehicles (operators' viewpoint):

$$\min \sum_{k \in M} \sum_{(i,j) \in E} t_{ji} \, x_{ji}^{k}$$

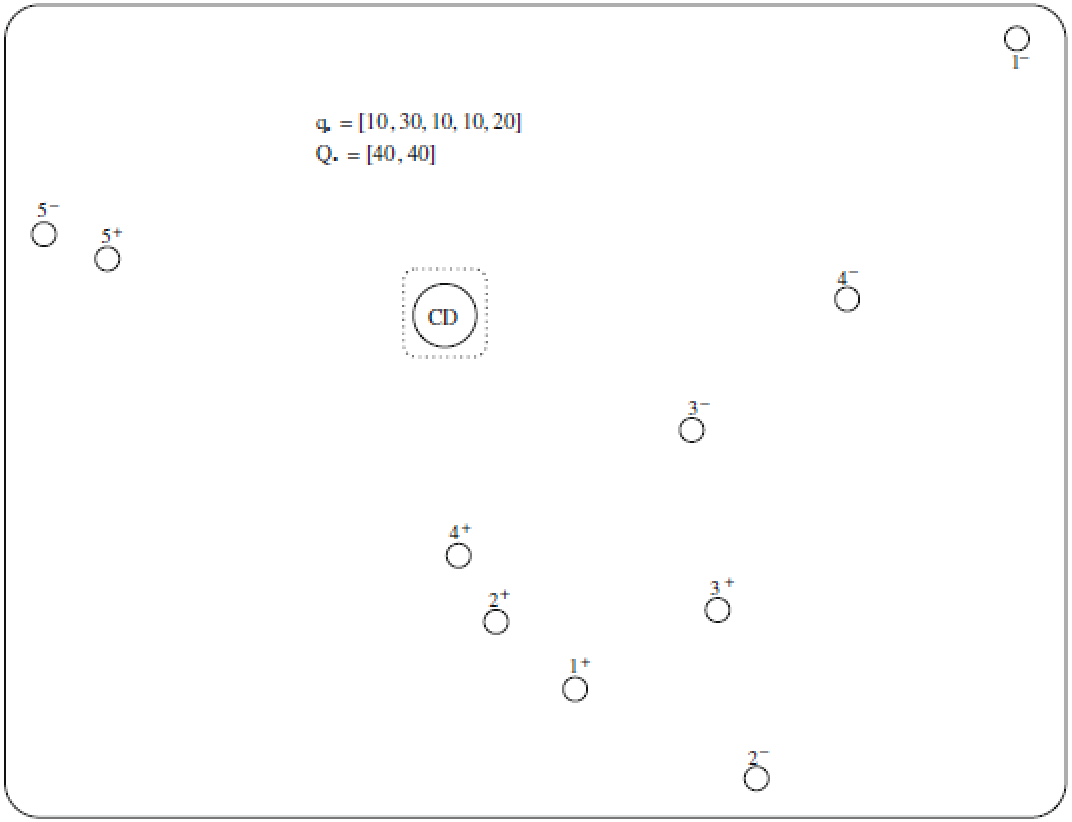
b) Minimize the travel time of customers:

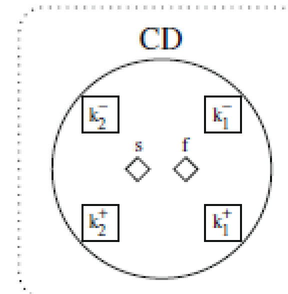
$$\min \sum_{i \in C} D_{i^{+}}$$

c) Minimize the waiting time of customers :

$$\min \sum_{i \in C} (D_{i^{-}} - D_{i^{+}})$$

PDP With Transfers: Example(i)

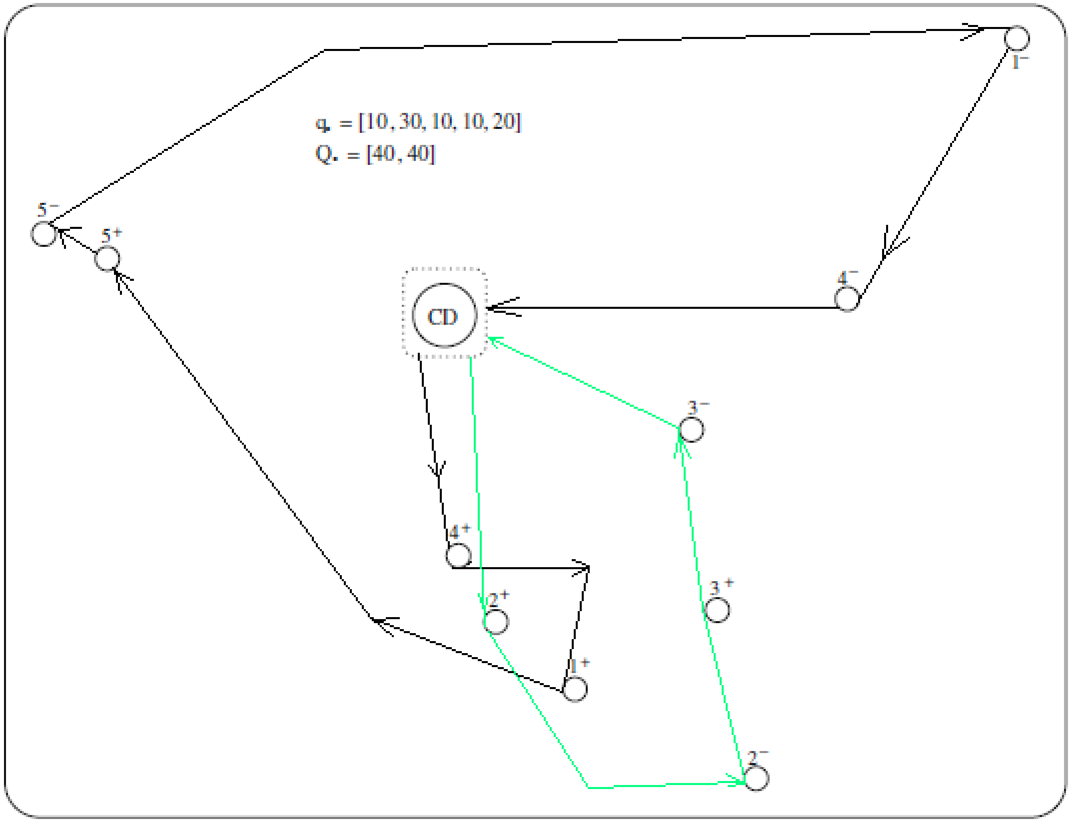


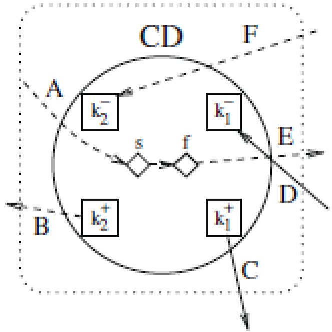


Central Depot

Example for 5 requests, 2 vehicles and 1 transfer point

PDP Without Transfers: Example(ii)





Central Depot

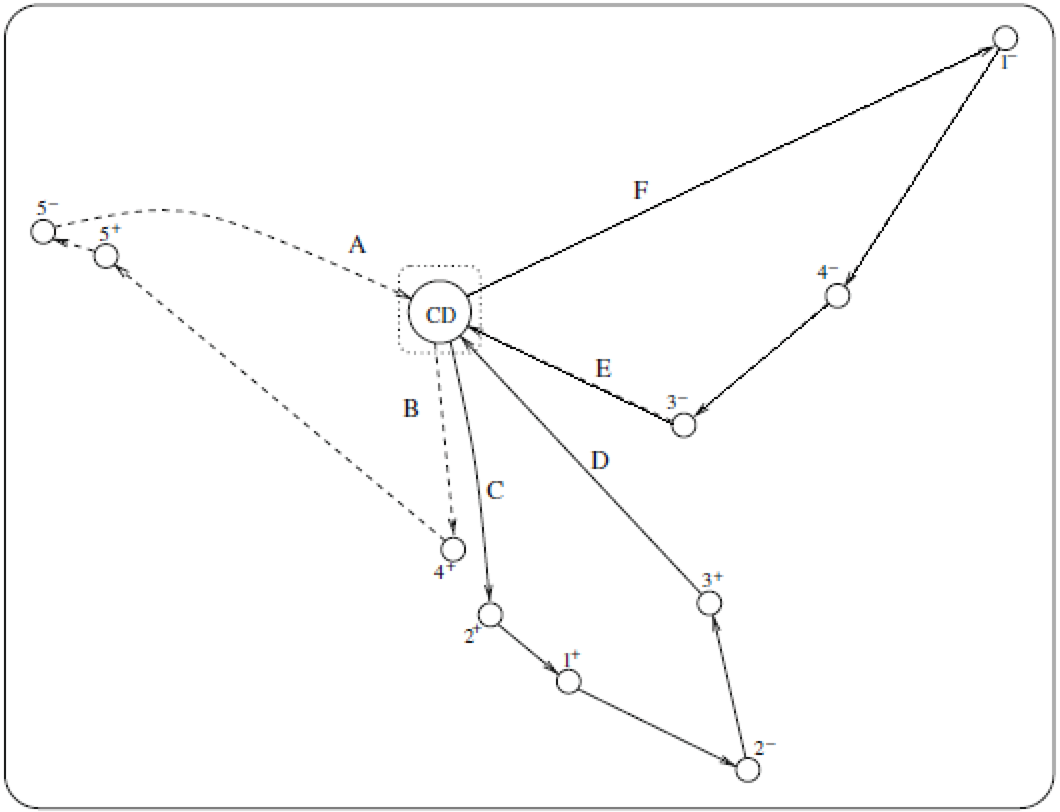
—— Arcs used by vehicle 1

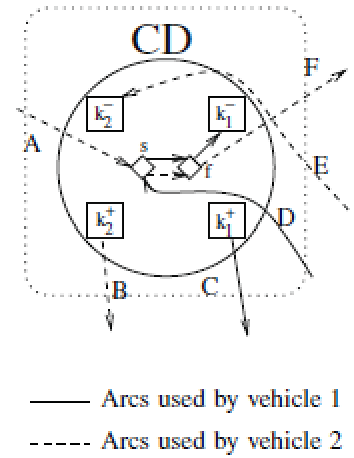
- - - - Arcs used by vehicle 2

An instance of the PDP without transfers (objective value = 1204):
Optimal routes for 5 requests, 2 vehicles and 1 central depot

$$\text{Route 1}: k_1^+ \xrightarrow{15} 4^+ \xrightarrow{104} 1^+ \xrightarrow{138} 5^+ \xrightarrow{96} 5^- \xrightarrow{166} 1^- \xrightarrow{113} 4^- \xrightarrow{122} k_1^-$$

$$\text{Route 2}: k_2^+ \xrightarrow{20} 2^+ \xrightarrow{113} 2^- \xrightarrow{103} 3^+ \xrightarrow{104} 3^- \xrightarrow{110} k_2^-$$





Central Depot

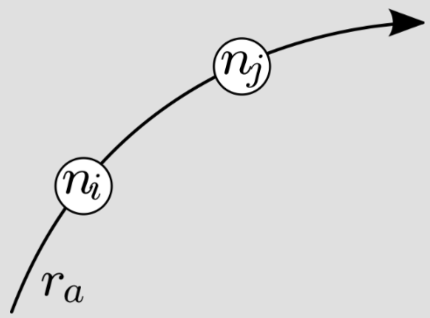
An instance of the PDP with transfers (objective value = 1187):
Optimal routes for 5 requests, 2 vehicles and 1 central depot

$$\text{Route 1}: k_1^+ \xrightarrow{20} 2^+ \xrightarrow{98} 1^+ \xrightarrow{104} 2^- \xrightarrow{103} 3^+ \xrightarrow{126} s \xrightarrow{1} f \xrightarrow{51} 1^- \xrightarrow{113} 4^- \xrightarrow{105} 3^- \xrightarrow{110} k_1^-$$

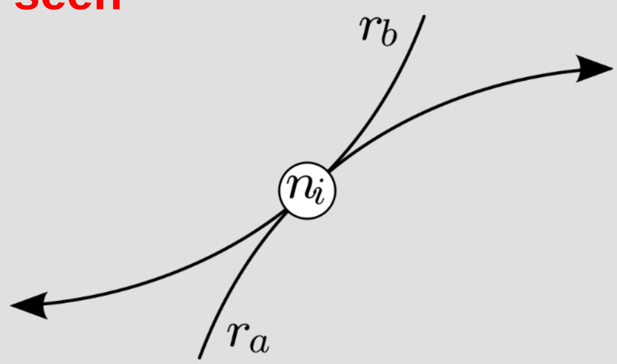
$$\text{Route 2}: k_2^+ \xrightarrow{15} 4^+ \xrightarrow{124} 5^+ \xrightarrow{96} 5^- \xrightarrow{120} s \xrightarrow{1} f \xrightarrow{0} k_2^-$$

Bouros et al. (2011) consider the possibility of including detours in the Static and Dynamic Pickup and Delivery Problems with Transfers. Three new scenarios can be identified:

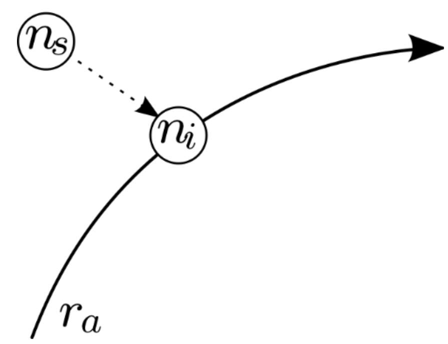
Already seen



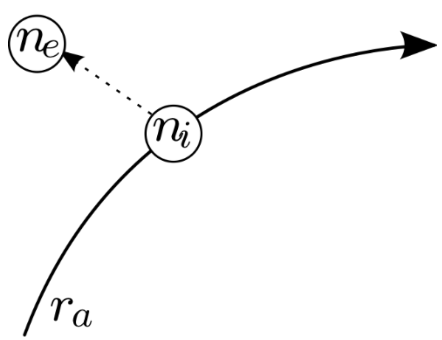
Transport without transfers



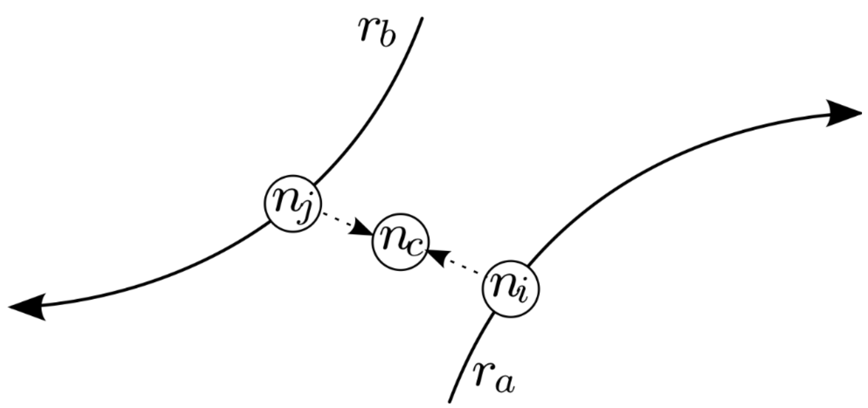
Transfers without detour



1. Pickup with detour

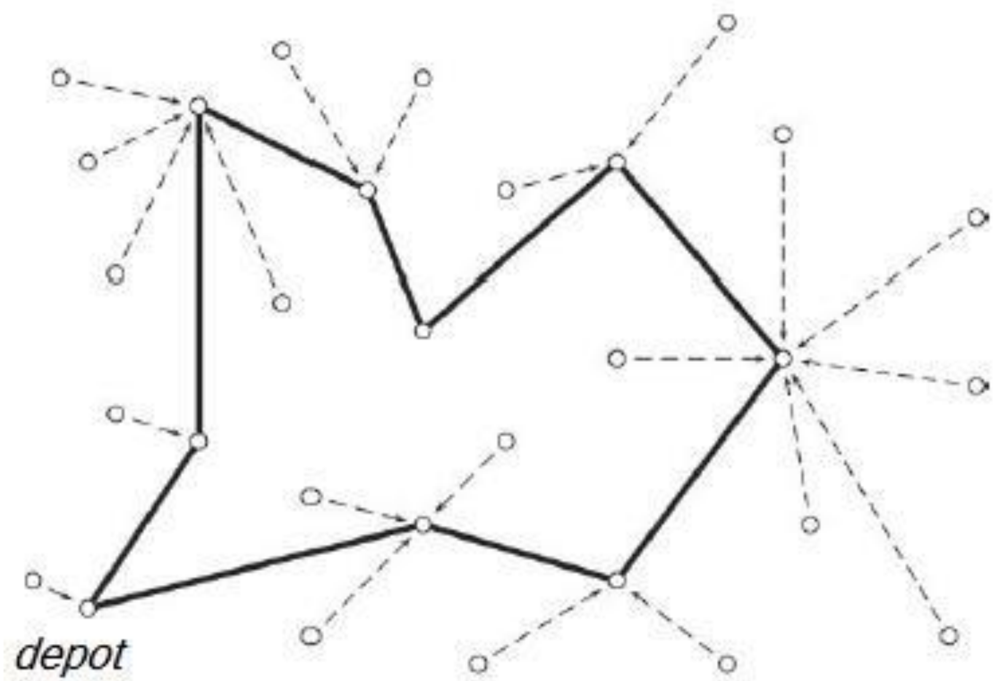


2. Delivery with detour



3. Transfer with detour

These scenarios can be geometrically interpreted as cyclic routes with branches which visit heterogeneous nodes.
The Ring Star Problem (RSP; Labbé et al., 2004) consists of determining a solution that minimizes the sum of the length of a main cycle along a subset of nodes and the cost of allocating points of access to this cycle from the remainder nodes.

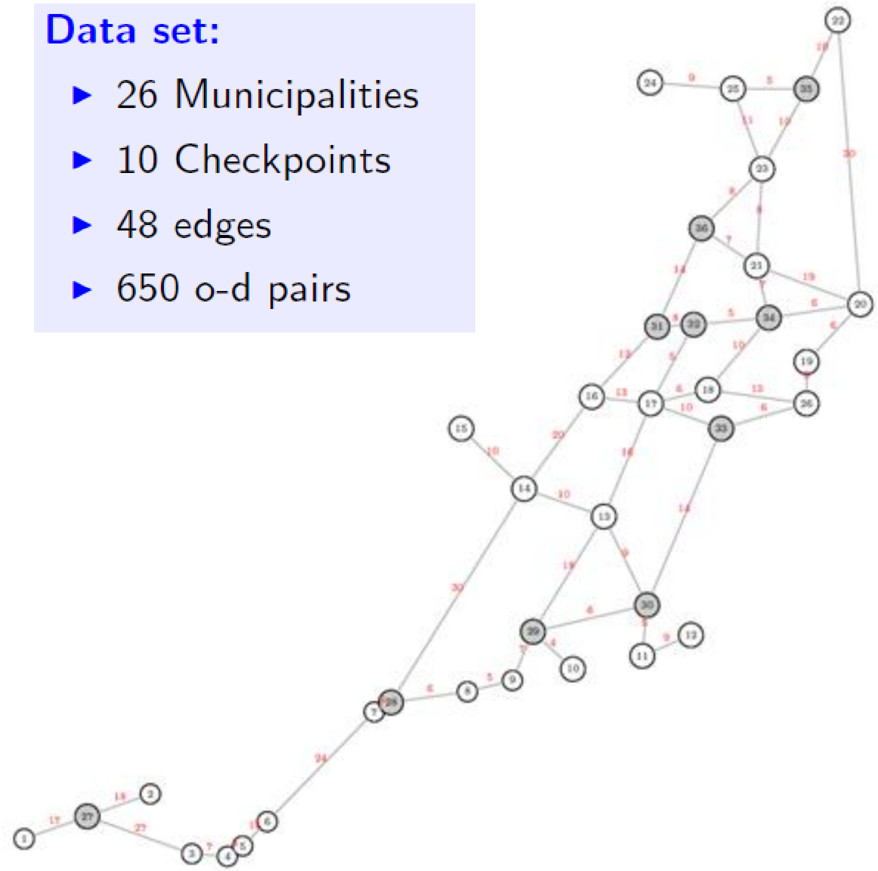


PDP: A geometric perspective (Laporte et al., 2012)

The multi Ring Star Problem with Pickups and Deliveries (m-RSPPD) consists of designing a set of minimal cost ring stars in such a way that each node shipping/receiving one/several commodities is connected to a ring star and the number of satisfied precedences is maximized (Laporte et al. 2012).

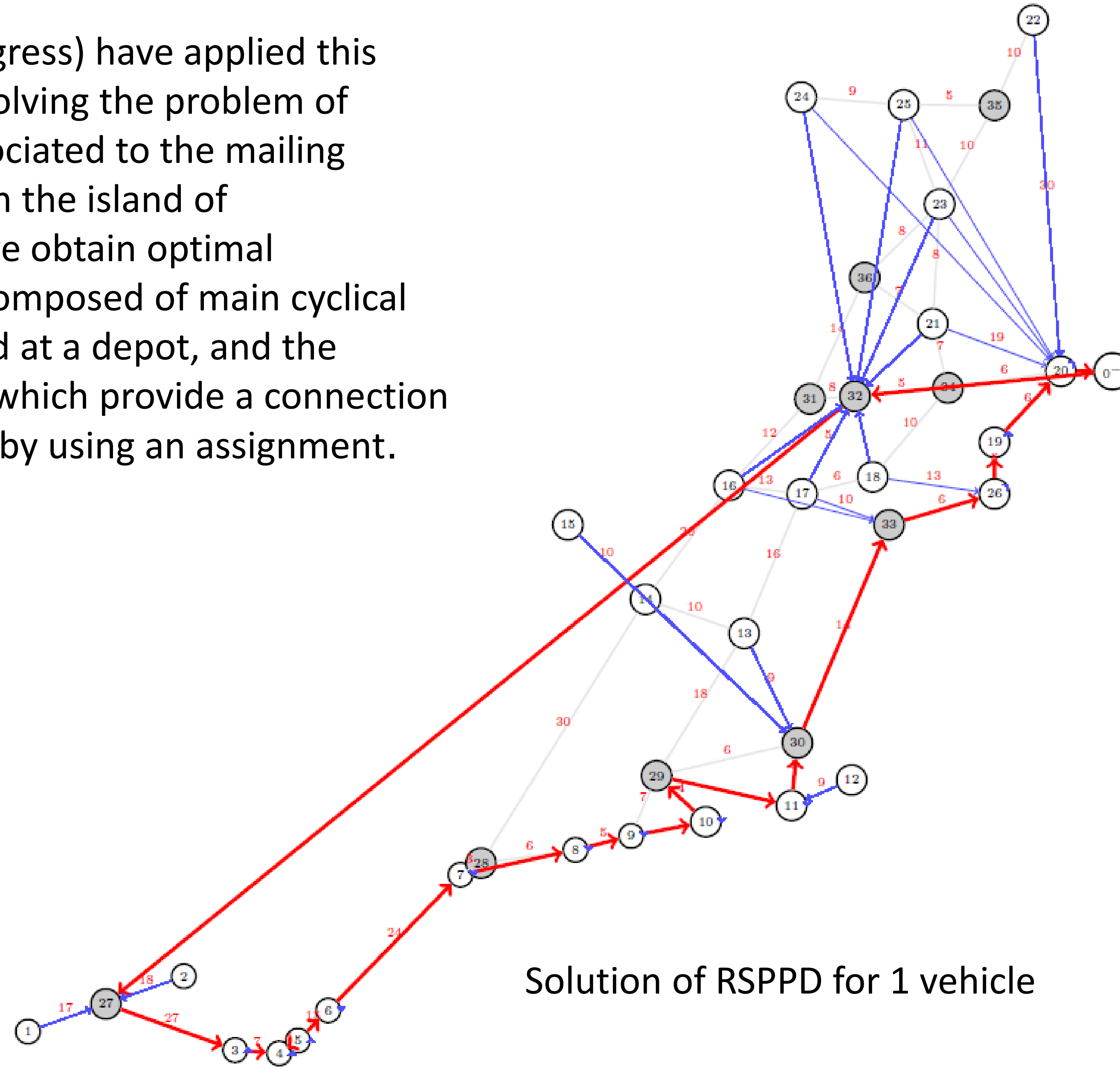
Data set:

- 26 Municipalities
- 10 Checkpoints
- 48 edges
- 650 o-d pairs



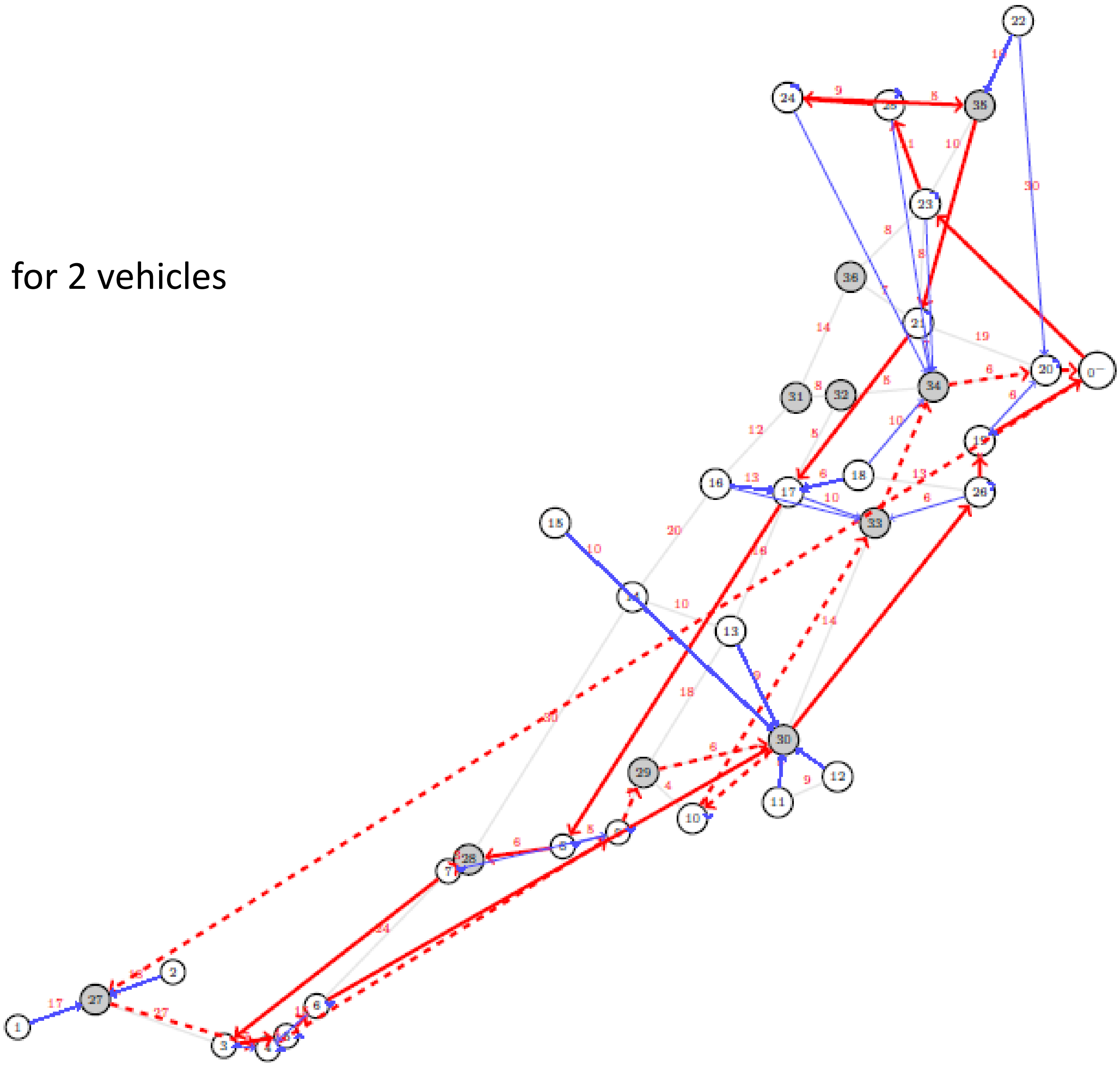
PDP: A geometric perspective (Pozo et al., 2015)

Pozo et al., 2015 (in progress) have applied this model of m-RSPPD for solving the problem of Pickup and Delivery associated to the mailing between postal offices in the island of Fuerteventura. They have obtain optimal distribution networks, composed of main cyclical routes that start and end at a depot, and the location of checkpoints which provide a connection with unvisited locations by using an assignment.



Solution of RSPPD for 1 vehicle

Solution of m-RSPPD for 2 vehicles



Thanks for your attention