Locating new stations and road links on a road-rail network

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Designing or extending railway networks is an important issue for many governments since trains:

1. reduce traffic congestion.
2. do not depend on petrol as much as road vehicles.
3. safety.
4. ...

A railway network must be attractive for passengers, otherwise nobody will use it and all the (huge) investment will be wasted!

Steps in railway network design

When designing a railway network one should (among others):

1. estimate potential trips (origin-destination matrix)
2. design the infrastructure: stations and tracks
3. propose lines
4. line frequencies
5. schedules
6. crew assignment
7. ...

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Motivation


AVE Madrid-Segovia-Valladolid
VALLADOLID
Valladolid
Medina del Campo
Olmedo
Coca
SEGOVIA
Sta. Mª La Real
La Granja
Segovia
Miraflores de la Sierra
Soto del Real
Tres Cantos
Colmenar Viejo
Madrid
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10
km
soymapas.com

Motivation

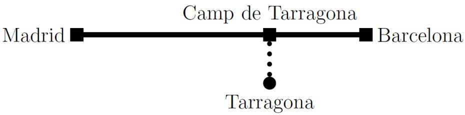

Figure 1: The Madrid-Barcelona line (solid line). The Camp de Tarragona station is 12 km away from the nearest large city, Tarragona. A road link (dotted line) joins these two points.

Input data

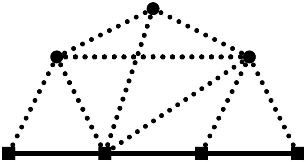

Figure 2: Input data to the problem. A set of cities (filled circles) and a set of train stations (filled squares) are joined by rail tracks (solid edges). Cities are linked among themselves and with the stations by means of road links (dotted edges).

Solution

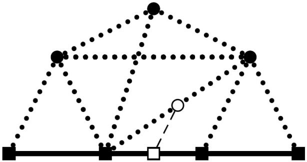

Figure 3: A possible solution to the problem depicted in Figure 2. The empty square represents the new station and the empty circle represents the new junction to be connected with the new station, by means of a new road link represented by dashed lines.

Input data

- Two train stations $T = \{n^1, n^k\} \subset \mathbb{R}^2$,
- A set of cities and junctions $R = \{n^2, ..., n^{k-1}\} \subset \mathbb{R}^2$,
- One rail link joining the two stations in T (set E_T),
- A set of road links E_R.
- $G = (R \cup T, E_R \cup E_T)$ is the road-rail network.
- n^1 is located at $(0, 0)$ and n^k is located at $(b, 0)$, $b > 0$.
- Let (n_1^i, n_2^i) be the coordinates of node $n^i \in R \cup T$.
- $A(E_R)$ and $A(E_T)$ are the arc sets associated to E_R and E_T.

Input data

- If d_{ij} denotes the Euclidean length of arc (i, j), then $t_{ij} = \alpha_1 d_{ij}$ if $(i, j) \in A(E_R)$ and $t_{ij} = \alpha_2 d_{ij}$ if $(i, j) \in A(E_T)$, with $\alpha_1 > \alpha_2 > 0$. (the train is faster than the road!).
- $t_{ij} = \alpha_1 d_{ij} + \psi$ if the new station is located at road edge (i, j), where $\psi \geq 0$ is a congestion parameter.
- g_{pq} denotes the number of potential trips of O/D pair (p, q), and u_{pq}^{ROAD} denotes the traveling time using the road network only.
- A stop time β at the new station.
- Construction costs of new station and new junction are c_1 and c_2, respectively.
- Construction cost of the new road edge proportional to Euclidean length, τ.
- Maximum budget equal to $C_{\max}$.

Objective

The problem consists of choosing:

- a location for the new station x on E_T at node $k+1$,
- a location for a new junction y on E_R at node $k+2$,
- and building a road segment linking nodes $k+1$ and $k+2$,

so that a certain objective function is optimized, without violating the budget constraint.

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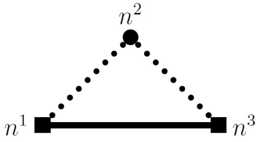
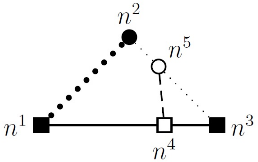
Solution strategy

- Trying to solve this problem by a unique MP model seems impossible.
- Fix the road link where the new junction will be.
- Solve all such $O(|E_R|)$ problems, and keep the best solution.

Network extension





The new node set is $R \cup T \cup \{n^4, n^5\}$, and the new edge set is

$$E_{(2,3)} = \{(1,2),(1,4),(2,5),(3,4),(3,5),(4,5)\}.$$

Both the locations of the two new nodes and the lengths of the five new edges are variables.

Objective functions

- minimizing the total travel time,
- maximizing the number of travelers who will use the rail corridor (ridership),
- maximizing the number of users who are positively affected by the construction of the new station (winners).

According to some authors: “maximizing passenger attraction is the most appropriate objective to consider when planning transit systems”.

Variables

1. $x_1 \in [0, b]$: the first coordinate of the new station.
2. $\lambda \in [0, 1]$: the convex combination of (i^*, j^*) where the new junction is to be located.
3. y_1 and y_2: coordinates of the location of the new junction.
4. δ_{ij} define the travel times of the new arcs.
5. Binary variable f_{ij}^{pq}: O/D pair (p, q) uses arc (i, j).
6. Binary variable v_{pq}: O/D pair (p, q) stops at the new station.
7. u_{pq} is the travel time associated with O/D pair (p, q).

Some of these variables are now explicitly defined:

$$y_1 = \lambda n_1^{i^*} + (1-\lambda) n_1^{j^*},$$
$$y_2 = \lambda n_2^{i^*} + (1-\lambda) n_2^{j^*},$$
$$\delta_{i^*,k+2} = (1-\lambda) t_{i^*j^*}, \delta_{j^*,k+2} = \lambda t_{i^*j^*},$$
$$\delta_{1,k+1} = \alpha_2 x_1,\ \delta_{k,k+1} = \alpha_2 (b - x_1),$$
$$\delta_{k+1,k+2} = \alpha_1 \sqrt{(x_1 - y_1)^2 + (0 - y_2)^2},$$
$$\delta_{ij} = \delta_{ji},\ \forall\, (i,j) \in A(E_{(i^*,j^*)} \setminus E_R) : i > j,$$
$$u_{pq} = \sum_{(i,j)\in A(E_R \setminus (i^*,j^*))} t_{ij} f_{ij}^{pq} + \sum_{(i,j)\in A(E_{(i^*,j^*)} \setminus E_R)} \delta_{ij} f_{ij}^{pq} + \beta v_{pq},\ \forall\, (p,q)$$

Note that the definition of u_{pq} contains the quadratic terms $\delta_{ij} f_{ij}^{pq}$ which can easily be linearized. Unfortunately, the non-linearity in the definition of $\delta_{k+1,k+2}$ cannot be removed, which makes our model non-linear.

Budget constraint

$$c_1 + c_2 + \tau\sqrt{(x_1 - y_1)^2 + (0 - y_2)^2} \leq C_{max}. \quad (1)$$

This constraint ensures that the cost of building the new station c_1 plus the cost of building the new junction c_2 plus the cost of building the new road link does not exceed the available budget.

Constraints

$$\sum_{i:(i,p)\in A(E_{(i^*,j^*)})} f_{ip}^{pq} = 0,\ (p,q) \in W \qquad (2)$$

$$\sum_{j:(p,j)\in A(E_{(i^*,j^*)})} f_{pj}^{pq} = 1,\ (p,q) \in W \qquad (3)$$

$$\sum_{i:(i,q)\in A(E_{(i^*,j^*)})} f_{iq}^{pq} = 1,\ (p,q) \in W \qquad (4)$$

$$\sum_{j:(q,j)\in A(E_{(i^*,j^*)})} f_{ij}^{pq} = 0,\ (p,q) \in W \qquad (5)$$

$$\sum_{i:(i,r)\in A(E_{(i^*,j^*)})} f_{ir}^{pq} - \sum_{j:(r,j)\in A(E_{(i^*,j^*)})} f_{rj}^{pq} = 0,\ (p,q),\ r \notin \{p,q\} \qquad (6)$$

New stop constraints

$$f^{pq}_{1,k+1} + f^{pq}_{k+1,k} + f^{pq}_{k,k+1} + f^{pq}_{k+1,k} - 1 \leq v_{pq}, \ (p,q) \in W. \qquad (7)$$

$v_{pq} = 1$ if two rail arcs are used by the O/D pair (p,q), meaning that this O/D pair will incur a stop time at the new station n^{k+1}.

Objective 1

Minimizing the total travel time of the road-rail network:

$$\text{minimize } z_{TTT} := \sum_{(p,q)\in W} g_{pq} u_{pq}, \qquad (8)$$

Minimizing (8) subject to constraints (1) to ((7)) yields model $TTT_{(i^*,j^*)}$.

Objective 2

Maximizing ridership. Assume that the proportion of travelers in (p, q) who will use the rail corridor is given by

$$\psi(u_{pq}^{ROAD} - u_{pq}) = \frac{1}{1 + \gamma_1 e^{-\gamma_2 (u_{pq}^{ROAD} - u_{pq})}},$$

where $\gamma_1, \gamma_2 > 0$ are two parameters to be calibrated depending on the instance. In this case the objective is

$$\text{maximize } z_{RID} := \sum_{(p,q) \in W} g_{pq} \frac{1}{1 + \gamma_1 e^{-\gamma_2 (u_{pq}^{ROAD} - u_{pq})}}. \qquad (9)$$

Unfortunately, this function is neither convex nor concave, which makes the problem difficult to solve. Maximizing (9) subject to constraints (1) to (7) yields model $RID_{(i^*, j^*)}$.

Objective 3

Maximizing the number of users who are positively affected by the construction of the new station and the new road link (winners, if $u_{pq} < u_{pq}^{ROAD}$. Define $s_{pq} = 1$ if $u_{pq} < u_{pq}^{ROAD}$. The objective is modeled as follows:

$$\text{maximize } z_{WIN} := \sum_{(p,q)\in W} g_{pq} s_{pq}. \tag{10}$$

Add the following constraints:

$$u_{pq} - u_{pq}^{ROAD} + \varepsilon \leq (1 - s_{pq}),\ (p,q) \in W, \tag{11}$$

where ε is a small positive number. Maximizing (10) subject to (11) and constraints (1) to (7) yields model $WIN_{(i^*,j^*)}$.

MINLP algorithm

Data: A road-rail network with two train stations.
Set $z^*_{TTT} = \infty$;
for $(i,j) \in E_R$ **do**
  Set $t_{ij} = \alpha_1 d_{ij} + \psi$.
  Solve $TTT_{(i,j)}$ by means of a MINLP solver.
  Let $x^{(i,j)}, y^{(i,j)}$ be the resulting optimal locations for the new station and the new junction, and let $z_{TTT}(i,j)$ be the total travel time of the corresponding network;
  if $z_{TTT}(i,j) < z^*_{TTT}$ **then**
    $(x^*, y^*) = (x^{(i,j)}, y^{(i,j)}),\ z^*_{TTT} = z_{TTT}(i,j)$
  end
  Set $t_{ij} = \alpha_1 d_{ij}$.
end
Result: Locations for the new station and the junction, (x^*, y^*), yielding a locally minimal total travel time z^*_{TTT}.

Algorithm 1: Local optimal algorithm for the location of a new station on the rail corridor and a junction on the road network minimizing the total travel time.

Enumerative algorithm

Data: A road-rail network with two train stations.
Set $z^*_{TTT} = \infty$;
for $(i, j) \in E_R$ **do**
  Set $t_{ij} = \alpha_1 d_{ij} + \psi$.
  Set $z_{TTT}(i, j) = \infty$;
  for *feasible* $\bar{x} \in F^x$ *and* $\bar{y} \in F^y(i, j)$ **do**
    Compute $z_{TTT}(\bar{x}, \bar{y})$ of the corresponding network;
    if $z_{TTT}(\bar{x}, \bar{y}) < z_{TTT}(i, j)$ **then**
      $(\bar{x}^{(i,j)}, \bar{y}^{(i,j)}) = (\bar{x}, \bar{y}), z_{TTT}(i, j) = z_{TTT}(\bar{x}, \bar{y})$
    end
  end
  if $z_{TTT}(i, j) < z^*_{TTT}$ **then**
    $(\bar{x}^*, \bar{y}^*) = (\bar{x}^{(i,j)}, \bar{y}^{(i,j)}), z^*_{TTT} = z_{TTT}(i, j)$
  end
  Set $t_{ij} = \alpha_1 d_{ij}$.
end
Result: Locations for the new station and the new junction $(\bar{x}^*, \bar{y}^*)$ yielding a total travel time equal to z^*_{TTT}.

Algorithm 2: Enumerative algorithm for the station-junction location problem minimizing the total travel time.

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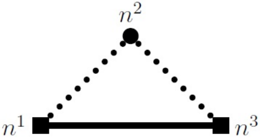
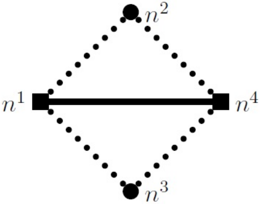
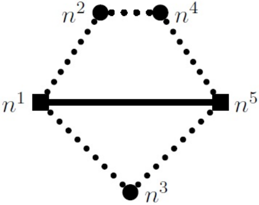
Instances



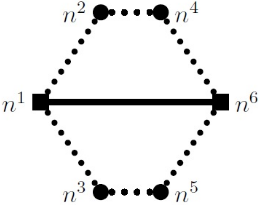
a) First configuration.



b) Second configuration.



d) Third configuration.



e) Fourth configuration.

Data generation

- $W = E_R \cup E_T$.
- $g_{pq} = \sqrt{\frac{w_p w_q}{d_{pq}}}$, where $w_p \sim \mathcal{U}(5, 15)$ d_{pq} is the Euclidean distance between n^p and n^q.
- $x_1 \in [0.5, 1.5]$ in order to avoid locating the new station too close to the existing ones (n^k is in (0,2)).
- $\alpha_1 = 1$, $\alpha_2 = 0.25$
- $c_1 = 1.5, c_2 = 0.5, c_{ij} = d_{ij}, C_{\max} = 2.8$.
- One hour of CPU time.

Results MINLP

5 instances for each configuration and, on average:

	z_{TTT}	z_{RID}	z_{WIN}	Seconds
TTT	74.8	30.9	28.5	236.0
RID	75.1	30.8	24.8	1162.2
WIN	83.8	28.9	33.3	1275.2

Table 1: Average value for each objective and CPU time (columns) obtained by the MINLP-based algorithms (rows).

Results MINLP interpretation

- Model TTT yields better total travel times.
- Model TTT yields better riderships.
- Model WIN yields better number of winners.
- Model TTT is significantly more efficient than the other two.

Conclusion: keep model TTT !

Comparison with enumerative

- Model TTT versus Enumerative.
- Three step sizes: 0.1, 0.05, 0.025
- 50 instances for each configuration and, on average:

Algorithm	MINLP	$ENUM_{0.1}$	$ENUM_{0.05}$	$ENUM_{0.025}$
Avg. TTT	77.21	76.36	76.25	76.21
Avg. CPU time (sec.)	149.20	0.20	0.74	3.01
Avg. %gap	–	−0.5	−0.68	−0.75

Table 2: Average results when comparing the MINLP-based algorithm and the enumerative algorithms.

Comparison with enumerative

- Enumerative yields better TTT.
- Enumerative is faster.
- The smaller the step size, the slower the enumerative.

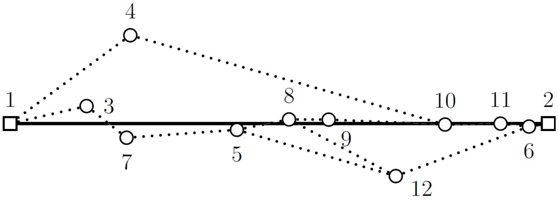
Conclusion: enumerative works well !

Application: Input data

City	Id	Population	Latitude	Longitude	x	y
Madrid	1	5098322*	40.42	−3.70	0.0	0.0
Valladolid	2	309714	41.66	−4.73	179.3	0.0
Colmenar Viejo	3	46955	40.66	−3.77	25.5	5.8
Collado Villalba - Galapagar	4	95207	40.63	−4.01	40.0	29.3
Segovia	5	54309	40.95	−4.12	75.5	−2.1
Laguna de Duero	6	22590	41.58	−4.72	172.9	−1.0
Miraflores de la Sierra	7	5907	40.81	−3.77	38.8	−4.7
Garcillán	8	477	40.98	−4.27	92.9	1.4
Santa María la Real de Nieva	9	993	41.07	−4.40	106.1	1.3
Olmedo	10	3776	41.29	−4.68	144.9	−0.3
Matapozuelos	11	1032	41.41	−4.79	163.4	−0.0
Cuéllar	12	9861	41.40	−4.31	128.5	−17.4

Table 3: Cities considered in the case study. *The population of Madrid includes its metropolitan area.

Application: Graph



Application: Solution

	TTT	Station	Connection		Seconds
Heuristic	27963495.69	75	75.65334116	−2.102538069	201.54
MINLP	28042844.56	74.7828814	75.54361877	−2.070662348	19244.40

In both cases, very close to the actual station Segovia-Guiomar.

Input data (the same)

- Two train stations $T = \{n^1, n^k\} \subset \mathbb{R}^2$,
- A set of cities and junctions $R = \{n^2, ..., n^{k-1}\} \subset \mathbb{R}^2$,
- One rail link joining the two stations in T (set E_T),
- A set of road links E_R.
- $G = (R \cup T, E_R \cup E_T)$ is the road-rail network.
- n^1 is located at $(0, 0)$ and n^k is located at $(b, 0)$, $b > 0$.
- Let (n_1^i, n_2^i) be the coordinates of node $n^i \in R \cup T$.
- $A(E_R)$ and $A(E_T)$ are the arc sets associated to E_R and E_T.

Input data (the same)

- If d_{ij} denotes the Euclidean length of arc (i,j), then $t_{ij} = \alpha_1 d_{ij}$ if $(i,j) \in A(E_R)$ and $t_{ij} = \alpha_2 d_{ij}$ if $(i,j) \in A(E_T)$, with $\alpha_1 > \alpha_2 > 0$. (the train is faster than the road!).
- $t_{ij} = \alpha_1 d_{ij} + \psi$ if the new station is located at road edge (i,j), where $\psi \geq 0$ is a congestion parameter.
- g_{pq} denotes the number of potential trips of O/D pair (p,q), and u_{pq}^{ROAD} denotes the traveling time using the road network only.
- A stop time β at the new station.
- Construction costs of new station and new junction are c_1 and c_2, respectively.
- Construction cost of the new road edge proportional to Euclidean length, τ.
- Maximum budget equal to $C_{\max}$.

Objective

The problem consists of choosing:

- a location for two new stations x_1, x_2 on E_T at nodes $k+1, k+3$,
- a location for two new junctions y_1, y_2 on E_R at nodes $k+2, k+4$,
- and building a road segment linking nodes $k+1$ and $k+2$, and a road segment linking nodes $k+3$ and $k+4$

so that a certain objective function is optimized, without violating the budget constraint.

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Solution strategy

- Trying to solve this problem by a unique MP model seems impossible.
- Fix the **two road links** where the new junctions will be.
- Solve all such $\mathbf{O}(|\mathbf{E_R}|^2)$ problems, and keep the best solution.

Variables

1. **New** $x_1 < x_2 \in [0, b]$: the first coordinates of the new stations.
2. **New** $\lambda_1, \lambda_2 \in [0, 1]$: the convex combinations of (i_1^*, j_1^*) and (i_2^*, j_2^*) where the new junctions are to be located.
3. **New** $y_1 = (y_1^1, y_1^2)$ and $y_2 = (y_2^1, y_2^2)$: coordinates of the location of the new junction.
4. δ_{ij} define the travel times of the new arcs.
5. Binary variable f_{ij}^{pq}: O/D pair (p, q) uses arc (i, j).
6. Binary variable v_{pq}^1: O/D pair (p, q) stops at the first new station.
7. **New** Binary variable v_{pq}^2: O/D pair (p, q) stops at the second new station.
8. u_{pq} is the travel time associated with O/D pair (p, q).

Some of these variables are now explicitly defined(**New**):

$$
\begin{aligned}
y_1^1 &= \lambda_1 n_1^{i_1^*} + (1-\lambda_1) n_1^{j_1^*},\\
y_1^2 &= \lambda_1 n_2^{i_1^*} + (1-\lambda_1) n_2^{j_1^*},\\
y_2^1 &= \lambda_2 n_1^{i_2^*} + (1-\lambda_2) n_1^{j_2^*},\\
y_2^2 &= \lambda_2 n_2^{i_2^*} + (1-\lambda_2) n_2^{j_2^*},
\end{aligned}
$$

Some of these variables are now explicitly defined(**New**):

$$\delta_{i_1^*,k+2} = (1-\lambda_1)t_{i_1^* j_1^*}, \delta_{j_1^*,k+2} = \lambda_1 t_{i_1^* j_1^*},$$
$$\delta_{i_2^*,k+4} = (1-\lambda_2)t_{i_2^* j_2^*}, \delta_{j_2^*,k+4} = \lambda_2 t_{i_2^* j_2^*},$$
$$\delta_{1,k+1} = \alpha_2 x_1,\ \delta_{k,k+3} = \alpha_2(b-x_2), \delta_{k+1,k+3} = \alpha_2(x_2-x_1),$$
$$\delta_{k+1,k+2} = \alpha_1\sqrt{(x_1-y_1^1)^2+(0-y_1^2)^2},$$
$$\delta_{k+3,k+4} = \alpha_1\sqrt{(x_2-y_2^1)^2+(0-y_2^2)^2},$$
$$\delta_{ij} = \delta_{ji},\ \forall\,(i,j)\in A(E_{(i^*,j^*)}\setminus E_R): i>j,$$
$$u_{pq} = \sum_{(i,j)\in A(E_R\setminus(i^*,j^*))} t_{ij} f_{ij}^{pq} + \sum_{(i,j)\in A(E_{(i^*,j^*)}\setminus E_R)} \delta_{ij} f_{ij}^{pq} + \beta v_{pq},\ \forall\,(p,q)$$

Budget constraint

$$2(c_1 + c_2) + \tau(\sqrt{(x_1 - y_1^1)^2 + (0 - y_1^2)^2} + \sqrt{(x_2 - y_2^1)^2 + (0 - y_2^2)^2})$$
$$\leq C_{max}.$$

Constraints

$$\sum_{i:(i,p)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{ip}^{pq} = 0,\ (p,q)\in W \qquad (12)$$

$$\sum_{j:(p,j)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{pj}^{pq} = 1,\ (p,q)\in W \qquad (13)$$

$$\sum_{i:(i,q)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{iq}^{pq} = 1,\ (p,q)\in W \qquad (14)$$

$$\sum_{j:(q,j)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{ij}^{pq} = 0,\ (p,q)\in W \qquad (15)$$

$$\sum_{i:(i,r)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{ir}^{pq} - \sum_{j:(r,j)\in A(E_{(i_1^*,j_1^*),(i_2^*,j_2^*)})} f_{rj}^{pq} = 0,\ (p,q),\ r\notin \{p,q \qquad (16)$$

New stop constraints

$$f^{pq}_{1,k+1} + f^{pq}_{k+1,k+3} + f^{pq}_{k+3,k+1} + f^{pq}_{k+1,k} - 1 \leq v^1_{pq},\ (p,q) \in W. \quad (17)$$

$$f^{pq}_{k+3,k+1} + f^{pq}_{k+1,k+3} + f^{pq}_{k+3,k} + f^{pq}_{k,k+3} - 1 \leq v^2_{pq},\ (p,q) \in W. \quad (18)$$

$v^1_{pq} = 1$ if a stop time at the new station n^{k+1}. $v^2_{pq} = 1$ if a stop time at the new station n^{k+3}

Objectives

- Same objectives as before.
- Minimizing TTT yields the best results in terms of ridership and CPU time.

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MINLP algorithm

- For all $(i_1^*, j_1^*), (i_2^*, j_2^*) \in E_R$ solve the model before.
- Keep the best solution

Main drawback: you need to solve $O(|E_R|^2)$ MINLP problems, which might be too much.

Greedy strategy

- Locate one station and update the railroad network.
- Locate the second station.

Main drawback: the first station is kind of centered, the second one is kind of in the middle of one of the new rail segments.

Enumerative algorithm

- For all $(i_1^*, j_1^*), (i_2^*, j_2^*) \in E_R$ find (near) optimal locations using the enumerative algorithm before.
- Keep the best solution.

Seems the most efficient approach

End

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Announcement

15th International Conference on Project Management and Scheduling
Valencia from 19 to 22 April 2016
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