THE URBAN TRANSIT NETWORK DESIGN PROBLEM

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LOGRO

IX International Workshop on Locational Analysis and Related Problems

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OUTLINE

- Network design
- Motivation
- Assumptions and Problem description
- Modelling approach
- Illustration
- Conclusions and further research

NETWORK DESIGN

Line planning process

The subproblems that are tackled during the planning of a public transportation system based on buses, according to the division proposed in [Ceder & Wilson, 1986] are:

1. **Design of routes**: to decide the number of lines and the itinerary of each according to a given demand.
2. **Frequency setting**, where the time interval between buses of each line is decided.
3. **Timetable construction**, to determine the exact starting and ending time of each bus.
4. **Fleet assignment**. determining the sequence of trips assigned to each bus, respecting constraints of available fleet and depot location.
5. **Crew assignment**. Drivers and other staff needed to operate each bus should be assigned, respecting the working rules.

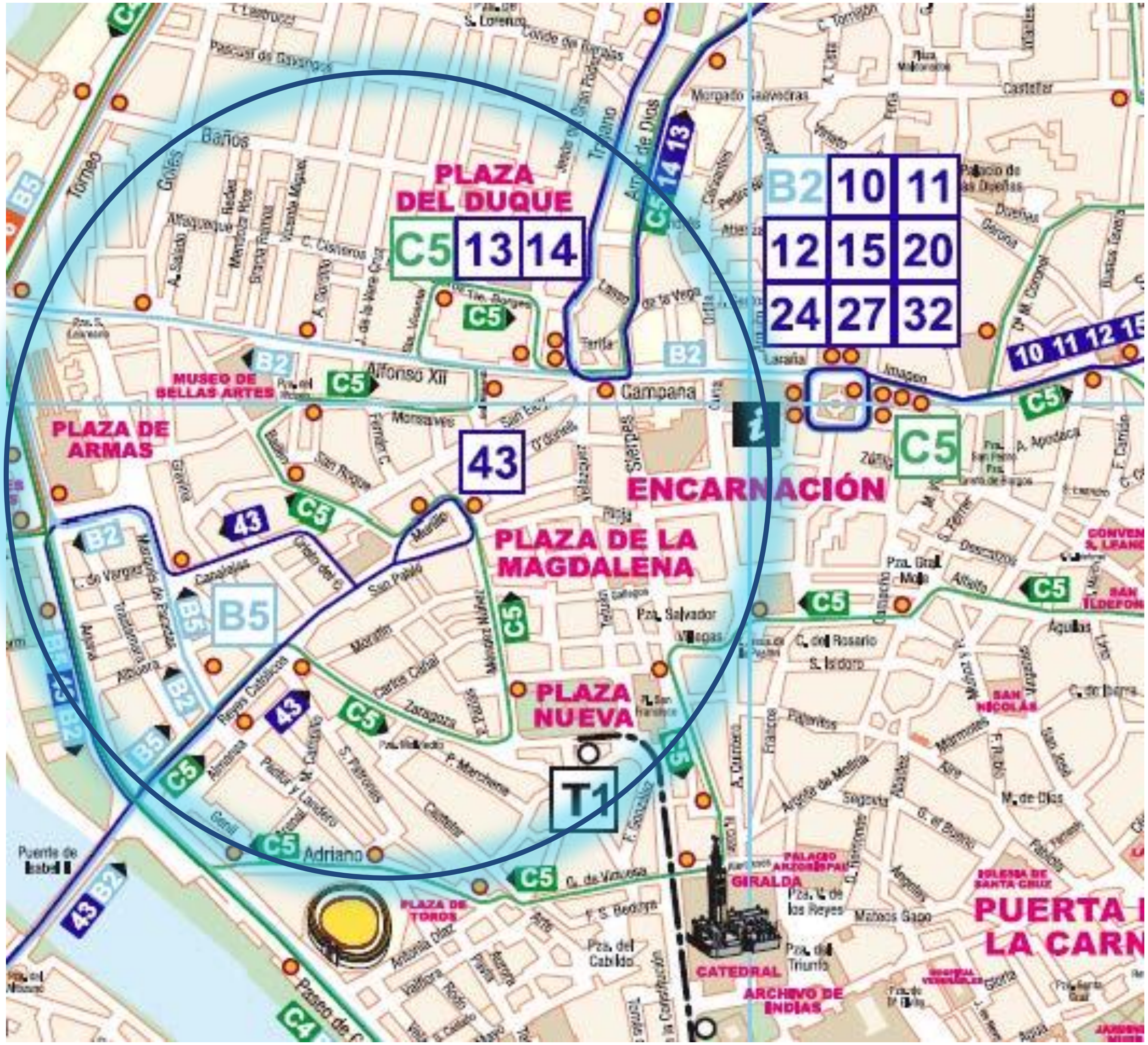
Design of routes

- Mathematical models
  - Cancela et.all 2015, Mathematical programming formulation for transit network design. *Transportation Research B: Methodological*, 77(C):17-37.
    - Predefined line pool
    - Frequency
    - Passenger assignment based on strategy concept as in Florian 1989
  - Wan and Lo 2013, A mixed integer formulation for multiple route transit network design. *Journal of mathematical modelling and algorithms*, 2(4):299-308.
    - Frequency
    - Without passenger behaviour
  - Bordörfer et all 2007, A column-generation approach to line planning in public transport. Transportation Science, 41(1):123-132.
    - Frequency
    - Consider the set of feasible lines associated to a set of terminal nodes.
- Heuristics
  - Feng et all. 2018, A new transit network design study in consideration of transfer time composition. *Transportation Research Part D: Transport and environment*
    - *Genetic algorithm*
    - *Transfer time decomposition*

Florian 1989, Optimal strategies: a new assignment model for transit networks, *Transportation Research B: Methodological*, 23(2), 83-102

MOTIVATION

Map of Seville



Map of Seville

Map of Seville

Possibles transfers in the pedestrian network

- From the previous representation we can define two layers:
  - The pedestrian layer, according to the passenger movement.
  - The road-infrastructure layer, according to the bus circulation.

  - In this work we are interested in introducing two types of transfers: **transfers in the same stop** and **transfers between different stops** (if it has sense).
  - The transfer **time plays an important** role in the passenger decisions since transfers represent discomfort for the passenger.

ASSUMPTIONS AND PROBLEM DESCRIPTION

We present a mathematical model to determing the set of lines from scratch (no line pool),

incorporating passenger assignment and

considering two types of transfers.

- We assume all lines have enough capacity to transport all passengers in the bus transportation network.
- Each line is defined by a set of arcs with a determined direction.
- We assume lines are not bidirectional since in a bus network the set of stops belonging to main direction may be different to opposite direction. For this reason, we assume each line is a circular line in which we can identify two terminals stops.
- No predefined lines are given but lines are constructed from scratch.
- Each arc A in the road-infrastructure layer is associated a time corresponding to travel time by bus.
- Each edge E in the pedestrian layer is associated a time corresponding to walk time.
- Frequency is not considered at this step. No capacity constraints, no waiting time.
- Users move from their origins to their destinations using the shortest path taking into account transfers between lines (in the same stops or between different stops).

MODELLING APPROACH

Data and notation I

- $S = \{1, \ldots, n\}$, set of stops.
- $A = \{(i, j) : i, j \in S\}$, set of arcs linking the stops over the road-infrastructure layer.
- $E = \{\{i, j\} : i, j \in S\}$, set of edges representing the walking edges between stops where is reasonable to do a transfer (over the pedestrian layer).
- $G = G(S, A)$, directed graph representing the road-infrastructure layer.
- $G = G(S, E)$, graph representing the pedestrian layer.
- $G = G(S, E \cup A)$, directed graph representing the road-infrastructure and the walking arcs where have sense to do transfers.
- d_{ij}, distance in kilometre associated to each arc $(i, j) \in A$.
- d^p_{ij}, pedestrian distance between stops i and j.

Data and notation II

- $\Theta = \{\theta_1, \ldots, \theta_{|\Theta|}\} \subset S \times S$, the set of ordered Origin-Destination (OD) pairs. Each element $\theta = (\theta^o, \theta^d) \in \Theta$ is defined by the the origin stop θ^o and the destination stop θ^d.
- p_θ, number of passengers per hour for an average day travelling from origin θ^o to destination θ^d.
- ℓ, an index denoting each line.
- $\mathcal{L}$, a set used to enumerate the possible lines.
- α and β, two penalties to be considered in the objective function.
- $\bar{D}$ is a matrix of order $|S| \times |S|$, where each element $\bar{d}_{ij}$ is equal to one if it is possible to transfer between the stops i and j (that is, if $\{i, j\} \in E$), 0 otherwise.
- S_{min} a lower on the number of stops of each line.
- S_{max} an upper on the number of stops of each line.

Variables

- $r_{ij}^{\theta\ell} = 1$, if the flow of pair θ traverses the arc (i, j) using line ℓ, 0 otherwise.
- $s_i^{\ell} = 1$, if the stop i is selected for line ℓ, 0 otherwise.
- $u_{ij}^{\ell} = 1$, if the arc (i, j) is activated for line ℓ, 0 otherwise.
- $t_{ij}^{\theta\ell\ell'} = 1$, if the flow of pair θ transfers from stop i of line ℓ to stop j of line ℓ', 0 otherwise. Note that if $i \neq j$, this variable represents transfers that require to walk between stops while if $i = j$ describes transfers at the same stop.

Objective function

The objective function is the total travel time taking into account passengers and transfers between lines.

$$\sum_{\theta\in\Theta}\sum_{\ell\in\mathcal{L}}\Big(\underbrace{\sum_{(i,j)\in A} d_{ij}\cdot r_{ij}^{\theta\ell}\cdot p_\theta}_{\text{Riding time}} + \underbrace{\alpha\sum_{\ell'\neq\ell}\sum_{i\in S} t_{ii}^{\theta\ell\ell'}\cdot p_\theta}_{\text{Time for transferring at same stop}} + \underbrace{\beta\sum_{\ell'\neq\ell}\sum_{i,j\in S} t_{ij}^{\theta\ell\ell'}\cdot d_{ij}\cdot p_\theta}_{\text{Time for transferring between different stops}}\Big).$$

Constraints

$$u_{ij}^{\ell} \leq s_i^{\ell},\ i,j \in S,\ \ell \in \mathcal{L} \quad (2)$$

$$u_{ij}^{\ell} \leq s_j^{\ell},\ i,j \in S,\ \ell \in \mathcal{L} \quad (3)$$

To select arcs

$$\sum_{j \in S} u_{ij}^{\ell} \leq 1,\ i \in S,\ \ell \in \mathcal{L} \quad (4)$$

$$\sum_{j \in S} u_{ji}^{\ell} \leq 1,\ i \in S,\ \ell \in \mathcal{L} \quad (5)$$

$$\sum_{(i,j) \in A} u_{ij}^{\ell} = \sum_{i \in S} s_i^{\ell},\ \ell \in \mathcal{L} \quad (6)$$

$$\sum_{\ell \in \mathcal{L}} s_i^{\ell} \geq 1,\ i \in S \quad (7)$$

$$S_{min} \leq \sum_{\ell \in \mathcal{L}} s_i^{\ell} \leq S_{max},\ i \in S \quad (8)$$

To define lines

$$\sum_{\ell \in \mathcal{L}} \sum_{\ell' \neq \ell} \sum_{i,j \in S} t_{ij}^{\theta \ell \ell'} \leq 2,\ \theta \in \Theta \quad (9)$$

Maximum number of transfers

Constraints

$$r_{ij}^{\theta\ell} \leq u_{ij}^{\ell},\ (i,j) \in A, \theta \in \Theta, \ell \in \mathcal{L} \quad (10)$$

To assign flow on arcs

$$t_{ij}^{\theta\ell\ell'} \leq s_i^{\ell},\ i,j \in S, \ell, \ell' \in \mathcal{L}, \ell \neq \ell', \theta \in \Theta \quad (11)$$

$$t_{ij}^{\theta\ell\ell'} \leq s_j^{\ell'},\ i,j \in S, \ell, \ell' \in \mathcal{L}, \ell \neq \ell', \theta \in \Theta \quad (12)$$

To assign transfer at stops

$$t_{ij}^{\theta\ell\ell'} \leq \bar{d}_{ij},\ i,j \in S, \ell, \ell' \in \mathcal{L}, \ell \neq \ell', \theta \in \Theta \quad (13)$$

Transfers between closer stops

Constraints

$$t_{ij}^{\theta\ell\ell'} \leq 1 - u_{ij}^{\ell},\ (i,j) \in A, \ell, \ell' \in \mathcal{L}, \ell \neq \ell', \theta \in \Theta \tag{14}$$

$$t_{ij}^{\theta\ell'\ell} \leq 1 - u_{ij}^{\ell},\ (i,j) \in A, \ell, \ell' \in \mathcal{L}, \ell \neq \ell', \theta \in \Theta \tag{15}$$

Transfers no allowed

$$\sum_{\ell \in \mathcal{L}} \sum_{\ell' \neq \ell} \sum_{j \in S} t_{\theta^o j}^{\theta\ell\ell'} + \sum_{\ell \in \mathcal{L}} \sum_{(\theta^o, j) \in A} r_{\theta^o j}^{\theta\ell} = 1,\ \theta = (\theta^o, \theta^d) \in \Theta \tag{16}$$

$$\sum_{\ell \in \mathcal{L}} \sum_{\ell' \neq \ell} \sum_{j \in S} t_{j\theta^d}^{\theta\ell\ell'} + \sum_{\ell \in \mathcal{L}} \sum_{(j, \theta^d) \in A} r_{j\theta^d}^{\theta\ell} = 1,\ \theta = (\theta^o, \theta^d) \in \Theta \tag{17}$$

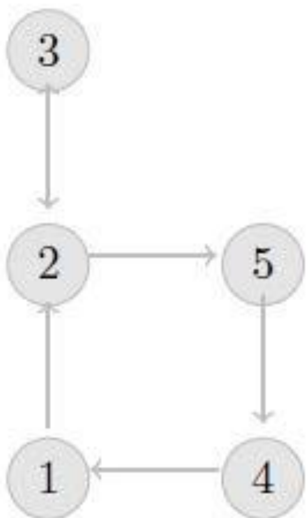
$$\sum_{(i,k) \in A} r_{ik}^{\theta\ell} + \sum_{\ell' \in \mathcal{L} \setminus \{\ell\}} \sum_{i \in S} t_{ik}^{\theta\ell'\ell} = \sum_{(k,j) \in A} r_{kj}^{\theta\ell} + \sum_{\ell' \in \mathcal{L} \setminus \{\ell\}} \sum_{j \in S} t_{kj}^{\theta\ell\ell'}, \tag{18}$$

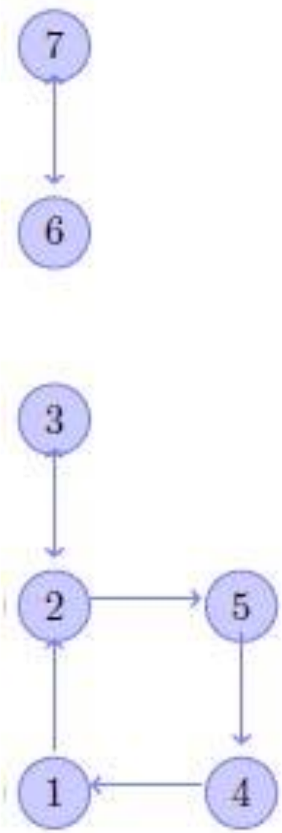
$$\ell \in \mathcal{L},\ \theta = (\theta^o, \theta^d) \in \Theta,\ k \neq \{\theta^o, \theta^d\},\ k \in S.$$

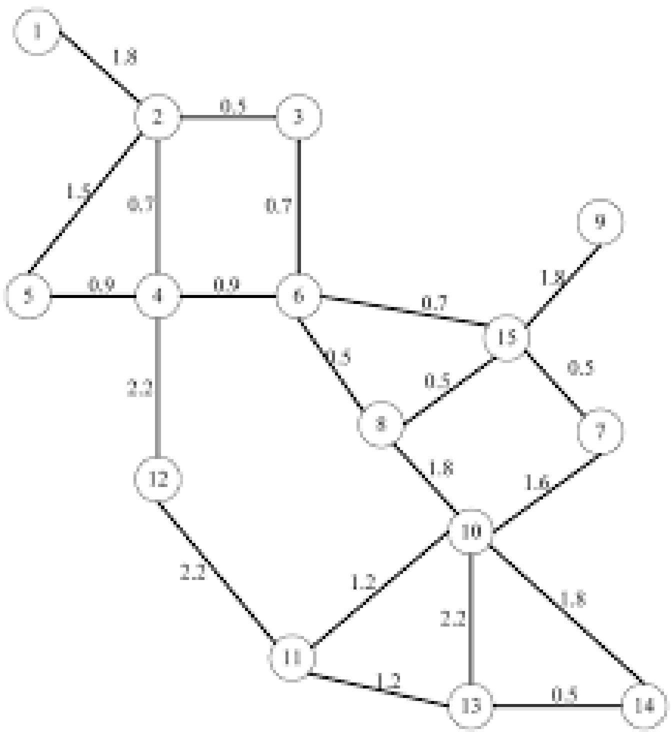
Flow conservation

ILLUSTRATION

Testing networks







CONCLUSIONS AND FURTHER RESEARCH

- We have presented a **mathematical model** for solving the network design in the bus transportation context, in which the lines are defined from square one without considering a predefined line pool, the passenger assignment is taken into account and the transfer time is presented in a realistic way.
- We are testing the model on several networks.
- Further research:
  - Next step will be test the Seville bus network.
  - To incorporate the frequency in the model as a decision variable.

THANK YOU!!

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