# A Kernel Search for the Inventory Routing Problem

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# Outline

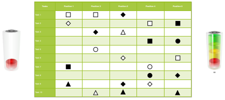
1. The Inventory Routing Problem
2. The Kernel Search framework
3. Computational experiments
4. Conclusions and future direction

# The Inventory Routing Problem (IRP)

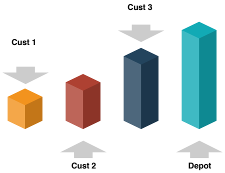
# Inventory Routing Problems

A class of problems, introduced by [Bell et al., 1983], which integrates:

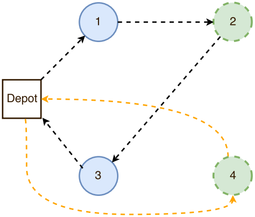
Scheduling



Inventory



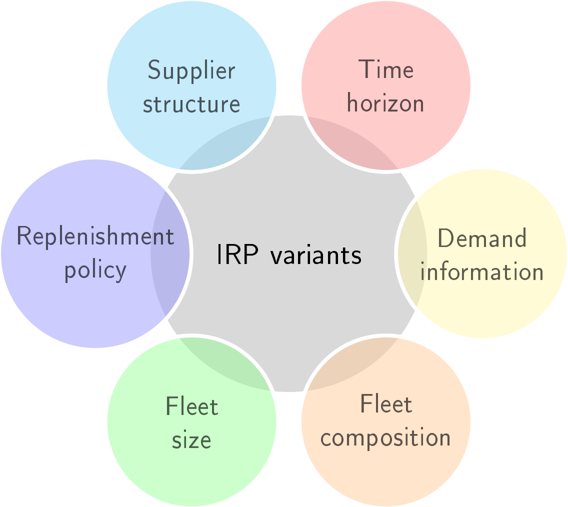
Routing



**The objective** is to determine the optimal distribution plan, i.e., that minimizes the total distribution costs: routing plus holding costs.

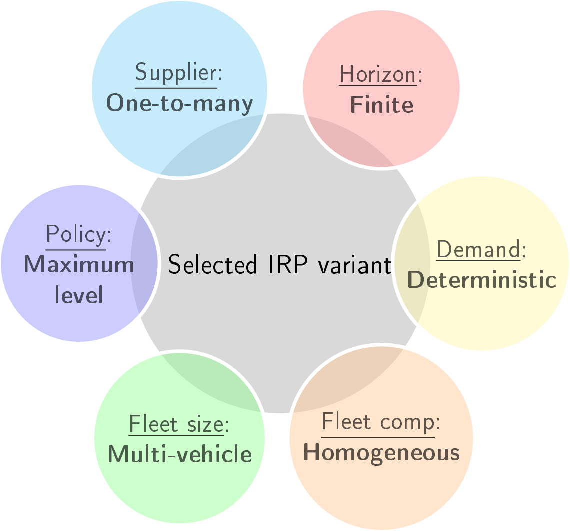
# IRP variants

IRP variants can be classified according to [Coelho et al., 2013]:



# IRP variants

The IRP variant studied in this work:



Selected IRP variant

Supplier: **One-to-many**

Horizon: **Finite**

Demand: **Deterministic**

Fleet comp: **Homogeneous**

Fleet size: **Multi-vehicle**

Policy: **Maximum level**

# Related work

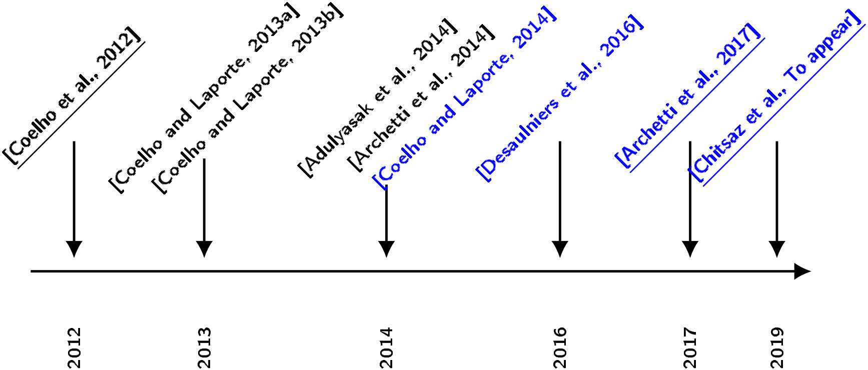
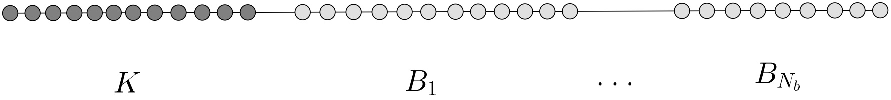

[Coelho et al., 2012]
[Coelho and Laporte, 2013a]
[Coelho and Laporte, 2013b]
[Adulyasak et al., 2014]
[Archetti et al., 2014]
[Coelho and Laporte, 2014]
[Desaulniers et al., 2016]
[Archetti et al., 2017]
[Chitsaz et al., To appear]
2012
2013
2014
2016
2017
2019

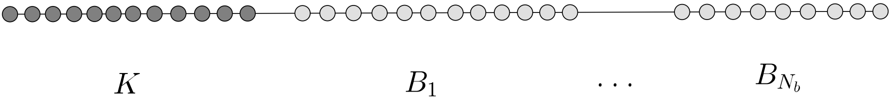
Figure 1: IRP papers (heuristics are underlined)

# Kernel Search

# Kernel Search (KS) heuristic



# Kernel Search (KS) heuristic



### Advantages

- Flexible structure
- Easy to implement
- Reduced number of parameters

### Drawback

- Solution quality depends on subsets

# KS heuristic

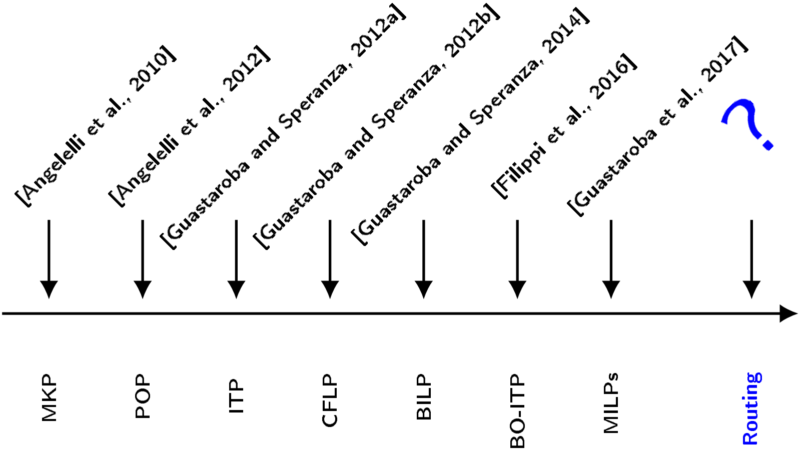
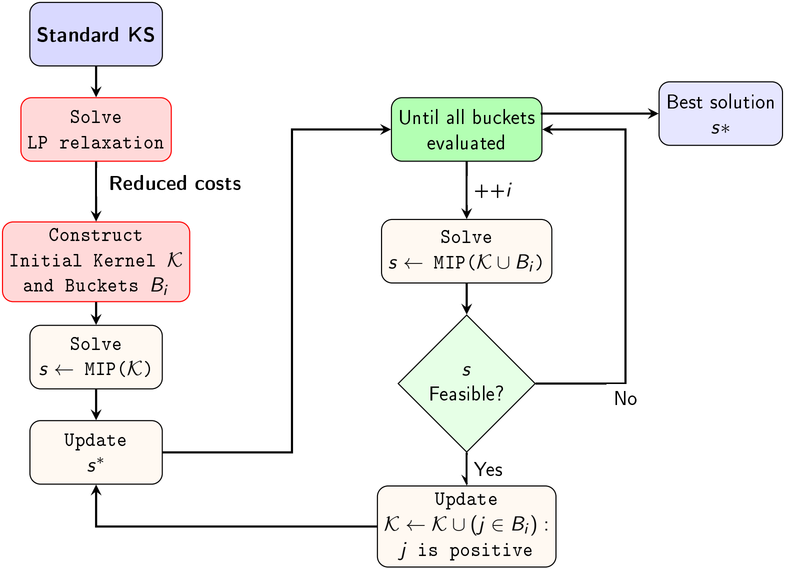

[Angelelli et al., 2010]
[Angelelli et al., 2012]
[Guastaroba and Speranza, 2012a]
[Guastaroba and Speranza, 2012b]
[Guastaroba and Speranza, 2014]
[Filippi et al., 2016]
[Guastaroba et al., 2017]
?
MKP
POP
ITP
CFLP
BILP
BO-ITP
MILPs
Routing

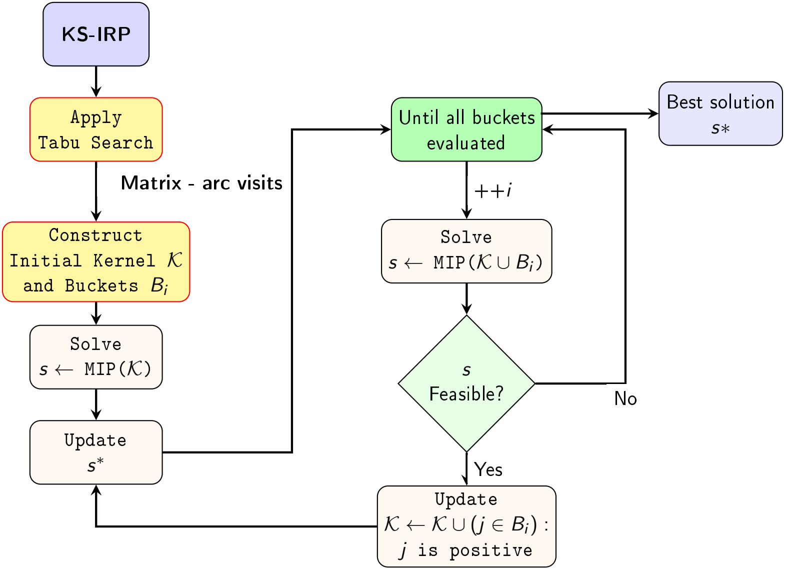
Figure 2: KS papers

# KS: Standard framework



Standard KS
Solve LP relaxation
Reduced costs
Construct Initial Kernel $\mathcal{K}$ and Buckets $B_i$
Solve $s \leftarrow \texttt{MIP}(\mathcal{K})$
Update $s^*$
Until all buckets evaluated
Best solution $s*$
$++i$
Solve $s \leftarrow \texttt{MIP}(\mathcal{K} \cup B_i)$
$s$ Feasible?
No
Yes
Update $\mathcal{K} \leftarrow \mathcal{K} \cup (j \in B_i)$ : $j$ is positive

# KS for the IRP (KS-IRP)


KS-IRP
Apply Tabu Search
Matrix - arc visits
Construct Initial Kernel $\mathcal{K}$ and Buckets $B_i$
Solve $s \leftarrow \texttt{MIP}(\mathcal{K})$
Update $s^*$
Until all buckets evaluated
Best solution $s*$
$++i$
Solve $s \leftarrow \texttt{MIP}(\mathcal{K} \cup B_i)$
$s$ Feasible?
No
Yes
Update $\mathcal{K} \leftarrow \mathcal{K} \cup (j \in B_i)$ : $j$ is positive

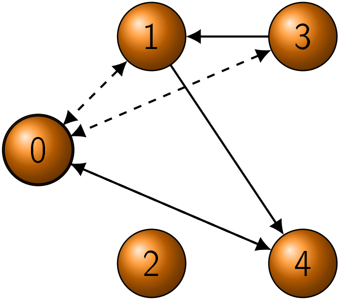
# Initial Kernel and Buckets

$$\underbrace{\begin{array}{cc} & \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{pmatrix} - & 26 & 7 & 19 & 86 \\ 9 & - & 0 & 21 & 219 \\ 0 & 0 & - & 42 & 0 \\ 19 & 81 & 28 & - & 41 \\ 32 & 51 & 30 & 0 & - \end{pmatrix} \end{array}}_{\text{TS Matrix for } t=1} \longrightarrow \underbrace{\begin{bmatrix} \color{blue}{(1,4) = 219} \\ \color{red}{(0,4) = 86} \\ \color{brown}{(3,1) = 81} \\ (4,1) = 51 \\ (2,3) = 42 \\ (3,4) = 41 \\ \dots \\ (4,3) = 0 \end{bmatrix}}_{\text{Sorted arcs}} \longrightarrow \underbrace{\begin{bmatrix} \color{blue}{y^1_{14}} \\ \color{blue}{y^1_{04}} \\ \color{blue}{y^1_{40}} \\ \color{blue}{y^1_{01}} \\ \color{blue}{y^1_{10}} \\ \color{brown}{y^1_{31}} \\ \color{brown}{y^1_{03}} \\ \color{brown}{y^1_{30}} \end{bmatrix}}_{\text{Initial kernel } \mathcal{K}} \quad \underbrace{\begin{bmatrix} y^1_{41} \\ y^1_{23} \\ y^1_{34} \\ y^1_{40} \\ y^1_{42} \\ y^1_{32} \\ \dots \\ y^1_{43} \end{bmatrix}}_{\text{Bucket list}} \Big\downarrow$$

All $y^t_{ij} = 0$
sorted
according
to their
closeness
to $\mathcal{K}$

Constructing the initial kernel and the bucket list.

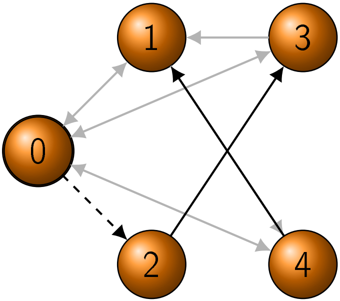
# Initial Kernel and Buckets



A graph considering only arcs in $\mathcal{K}$

$$\underbrace{\begin{bmatrix} y^1_{14} \\ y^1_{04} \\ y^1_{40} \\ y^1_{01} \\ y^1_{10} \\ y^1_{31} \\ y^1_{03} \\ y^1_{30} \end{bmatrix}}_{\text{Initial kernel } \mathcal{K}}$$

# Initial Kernel and Buckets



Adding arcs from Bucket $B_1$

$$\underbrace{\begin{bmatrix} y_{41}^{1} \\ y_{23}^{1} \\ y_{34}^{1} \\ y_{40}^{1} \\ y_{42}^{1} \\ y_{32}^{1} \\ \cdots \\ y_{43}^{1} \end{bmatrix}}_{\text{Bucket list}}$$

# Computational experiments

# Environment and benchmark instances

## Environment

- **Workstation:** HP Intel(R)-Xeon(R) at 3.5GHz/64GB RAM (Win10Pro, 64bits)
- **KS-IRP implementation:** C++ and ILOG Concert Technology API (CPLEX 12.6.0.0)

## Benchmark instances

- **Small-size:** 640 instances
  40 inst. $n = 5 - 50$, $H = 3, 6$, $|K| = 2 - 5$, Low & High inv. costs
- **Large-size:** 240 instaces
  80 inst. $n = 50, 100, 200$, $H = 6$, $|K| = 2 - 5$, Low & High inv. costs

# Parameters

- **Time limit KS-IRP:** 7,200 sec.
- **Small-size instances:**
  - $|\mathcal{K}| =$ All arcs visits $\leq$ Avg
  - $|B_i| =$ 70 arcs
- **Large-size instances:**
  - $|\mathcal{K}| =$ 40% of the visited arcs
  - $|B_i| =$ 100 arcs

# Evaluated solution methods

- **Exact solution methods:**
  - **LF:** Load-based formulation (CPLEX,MILP) [Archetti et al., 2014]
  - **CL:** Branch-and-cut method [Coelho and Laporte, 2014]
  - **DRC:** Branch-and-price-and-cut method [Desaulniers et al., 2016]
- **Approximate solution methods**
  - **ABS:** A three-phase matheuristic [Archetti et al., 2017]
  - **CCJ:** A three-phase decomposition matheuristic [Chitsaz et al., To appear]

# Computational results: Small-size

| | | LF | | | CL | | | DRC | | | KS-IRP | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *H* | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg |
| 5 | 3 | 36 | 0.06% | 18.08 | 40 | 0.00% | 0.63 | 40 | 0.00% | 0.02 | 36 | 0.06% | 16.75 |
| 10 | | 40 | 0.00% | 3146.23 | 40 | 0.00% | 96.05 | 40 | 0.00% | 3.37 | 35 | 0.04% | 114.08 |
| 15 | | 38 | 0.00% | 3431.88 | 40 | 0.00% | 809.60 | 40 | 0.00% | 350.97 | 28 | 0.11% | 306.90 |
| 20 | | 25 | 0.23% | 7197.43 | 31 | 0.29% | 2512.15 | 37 | 0.01% | 1422.59 | 20 | 0.36% | 1180.53 |
| 25 | | 23 | 0.18% | 7200.05 | 32 | 0.82% | 3280.10 | 35 | 0.03% | 2452.21 | 25 | 0.16% | 2306.28 |
| 30 | | 20 | 0.48% | 7200.03 | 21 | 2.57% | 3943.20 | 35 | 2.30% | 3122.49 | 18 | 0.78% | 3635.60 |
| 35 | | 15 | 0.65% | 6865.25 | 20 | 3.92% | 4508.70 | 35 | 4.41% | 3632.29 | 19 | 0.62% | 2931.65 |
| 40 | | 13 | 3.48% | 7020.03 | 16 | 8.28% | 4995.00 | 20 | 18.32% | 5464.43 | 14 | 3.46% | 2802.75 |
| 45 | | 17 | 0.97% | 7200.00 | 13 | 7.25% | 5569.98 | 18 | 23.44% | 6263.18 | 8 | 1.54% | 4019.58 |
| 50 | | 14 | 1.08% | 7200.00 | 8 | 6.60% | 6430.55 | 2 | 40.90% | 7078.65 | 13 | 1.43% | 4952.00 |
| **Avg** | | **(241)** | **0.71%** | **5647.90** | **(261)** | **2.97%** | **3214.60** | **(302)** | **8.94%** | **2979.02** | **(216)** | **0.86%** | **2226.61** |
| 5 | 6 | 30 | 0.10% | 5720.23 | 38 | 0.00% | 55.80 | 38 | 0.00% | 269.28 | 28 | 0.10% | 133.43 |
| 10 | | 28 | 0.20% | 7200.00 | 21 | 0.30% | 4671.23 | 27 | 2.99% | 4108.00 | 13 | 0.36% | 3157.28 |
| 15 | | 28 | 0.09% | 7200.53 | 17 | 0.88% | 5383.35 | 2 | 47.69% | 6709.32 | 5 | 0.54% | 5814.45 |
| 20 | | 23 | 0.18% | 7200.00 | 9 | 2.80% | 6394.63 | 1 | 52.85% | 7197.51 | 7 | 0.60% | 5691.15 |
| 25 | | 17 | 0.28% | 7200.08 | 7 | 3.30% | 6836.43 | 0 | 57.47% | 7200.06 | 18 | 0.31% | 6118.85 |
| 30 | | 17 | 0.94% | 7200.00 | 8 | 6.99% | 6826.70 | 0 | 57.62% | 7200.24 | 11 | 1.04% | 6049.73 |
| **Avg** | | **(143)** | **0.30%** | **6953.47** | **(100)** | **2.38%** | **5028.02** | **(68)** | **36.44%** | **5447.40** | **(82)** | **0.49%** | **4494.15** |

Table 1: Individual performance: KS-IRP and the **exact methods**.

# Computational results: Small-size

| | | ABS | | | CCJ | | | KS-IRP | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | $H$ | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg |
| 5 | 3 | 36 | 0.06% | 6.38 | 0 | 10.31% | 1.66 | 36 | 0.06% | 16.75 |
| 10 | | 38 | 0.01% | 392.78 | 0 | 12.75% | 4.24 | 35 | 0.04% | 114.08 |
| 15 | | 34 | 0.06% | 837.65 | 0 | 11.76% | 10.81 | 28 | 0.11% | 306.90 |
| 20 | | 23 | 0.72% | 1409.40 | 0 | 12.83% | 22.90 | 20 | 0.36% | 1180.53 |
| 25 | | 20 | 0.53% | 1595.60 | 0 | 13.33% | 49.07 | 25 | 0.16% | 2306.28 |
| 30 | | 9 | 1.47% | 2036.20 | 0 | 14.44% | 54.12 | 18 | 0.78% | 3635.60 |
| 35 | | 13 | 1.73% | 2548.90 | 0 | 14.23% | 71.64 | 19 | 0.62% | 2931.65 |
| 40 | | 8 | 3.92% | 3234.15 | 0 | 17.97% | 118.17 | 14 | 3.46% | 2802.75 |
| 45 | | 6 | 1.31% | 3728.13 | 0 | 15.34% | 143.61 | 8 | 1.54% | 4019.58 |
| 50 | | 13 | 0.67% | 5213.70 | 0 | 14.32% | 192.38 | 13 | 1.43% | 4952.00 |
| **Avg.** | | **(200)** | **1.05%** | **2100.29** | **(0)** | **13.73%** | **66.86** | **(216)** | **0.86%** | **2226.61** |
| 5 | 6 | 30 | 0.10% | 585.65 | 0 | 8.00% | 6.34 | 28 | 0.10% | 133.43 |
| 10 | | 12 | 1.62% | 1846.28 | 0 | 11.34% | 11.36 | 13 | 0.36% | 3157.28 |
| 15 | | 5 | 2.64% | 2440.18 | 0 | 10.61% | 52.46 | 5 | 0.54% | 5814.45 |
| 20 | | 0 | 3.25% | 3307.45 | 0 | 9.58% | 65.31 | 7 | 0.60% | 5691.15 |
| 25 | | 1 | 2.56% | 4365.38 | 0 | 8.84% | 121.95 | 18 | 0.31% | 6118.85 |
| 30 | | 5 | 1.72% | 7461.68 | 0 | 8.28% | 187.59 | 11 | 1.04% | 6049.73 |
| **Avg.** | | **(53)** | **1.98%** | **3334.43** | **(0)** | **9.44%** | **74.17** | **(82)** | **0.49%** | **4494.15** |

Table 2: Individual performance: KS-IRP and the **heuristic methods**.

# Computational results: Large-size

| | | LF | | | CL | | | DRC | | | KS-IRP | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *H* | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg |
| 50 | 6 | 2 | 13.21% | 7200.00 | — | — | — | — | — | — | 18 | 1.57% | 5496.39 |
| 100 | | — | — | — | — | — | — | — | — | — | 53 | 0.68% | 4528.33 |
| 200 | | — | — | — | — | — | — | — | — | — | 32 | 2.05% | 3325.15 |
| **Avg** | | **(2)** | **13.21%** | **7200.00** | — | — | — | — | | — | **(103)** | **1.43%** | **4449.95** |

Table 3: Individual performance: KS-IRP and the **exact methods**.

| | | ABS | | | CCJ | | | KS-IRP | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *n* | *H* | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg | #Best | G-Avg | T-Avg |
| 50 | 6 | 54 | 0.42% | 11193.45 | 6 | 6.89% | 518.51 | 18 | 1.57% | 5496.39 |
| 100 | | 12 | 3.26% | 5582.79 | 15 | 5.72% | 2395.91 | 53 | 0.68% | 4528.33 |
| 200 | | 16 | 3.22% | 7800.63 | 33 | 5.53% | 12167.48 | 32 | 2.05% | 3325.15 |
| **Avg.** | | **(82)** | **2.30%** | **8192.29** | **(54)** | **6.05%** | **5027.30** | **(103)** | **1.43%** | **4449.95** |

Table 4: Individual performance: KS-IRP and the **heuristic methods**.

# Conclusions and future direction

- A KS heuristic is proposed to solve a common IRP variant.
- The KS-IRP differs from the standard version in the way to construct initial Kernel and Buckets.
- The KS-IRP is competitive to the state-of-the-art algorithms.
- Future direction: Apply the proposed algorithm to other VRP variants.

# References

Adulyasak, Y., Cordeau, J.-F., and Jans, R. (2014).
Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems.
*INFORMS Journal on Computing*, 26(1):103–120.

Angelelli, E., Mansini, R., and Speranza, M. G. (2010).
Kernel search: A general heuristic for the multi-dimensional knapsack problem.
*Computers & Operations Research*, 37(11):2017 – 2026.

Angelelli, E., Mansini, R., and Speranza, M. G. (2012).
Kernel search: A new heuristic framework for portfolio selection.
*Computational Optimization and Applications*, 51(1):345–361.

Archetti, C., Bianchessi, N., Irnich, S., and Speranza, M. G. (2014).
Formulations for an inventory routing problem.
*International Transactions in Operational Research*, 21(3):353–374.

Archetti, C., Boland, N., and Speranza, M. G. (2017).
A matheuristic for the multivehicle inventory routing problem.
*INFORMS Journal on Computing*, 29(3):377–387.

Bell, W.J., Dalberto, L.M., Fisher, M.L., Greenfield, A.J., Jaikumar, R., Kedia, P., Mack, R.G., and Prutzman, P.
Improving the distribution of industrial gases with an on-line computerized routing and scheduling optimizer.
*Interfaces*, 13(6):4–23.

Chitsaz, M., Cordeau, J.-F., and Jans, R. (To appear).
A unified decomposition matheuristic for assembly, production and inventory routing.
*INFORMS Journal on Computing*.

Coelho, L.C., Cordeau, J.-F., and Laporte, G. (2012).
Consistency in multi-vehicle inventory-routing.

Thank You.