Locating a new station/stop in a network based on trip coverage and times

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Outline

1. Motivation
2. Elements of the problem
3. Problem formulation
4. Solving the problem
5. Extensions

Motivation

Motivation: Spanish High-speed Railway network.

Motivation


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Motivation

The problem (general idea)

Given a high-speed line and a competitive/alternative mode of transportation, the problem is to locate one new station/stop in a tree network in order to add OD-pairs covered to the new rapid system without disturbing the demand already covered by the high-speed.

Applications: high-speed railway, metro, BRT, telecommunication, etc.

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Motivation

- Most of problems dealing with rapid transportation systems and coverage objectives in a continuous solution space, are devoted to cover demand points.
- However, in this work, we consider a covering location problem in a continuous solution space, focused on covering origin destination pairs (OD-pairs), instead of single points.

Elements of the problem

- The plane $\mathbb{R}^2$, with the Euclidean distance $||\cdot||$.
- A rapid transportation system represented by a tree $\mathcal{T}(V, E)$ embedded in the plane, with $|V| \geq 2$.
- $\Gamma(x, y) \subseteq \mathcal{T}$ denotes the path linking $x, y \in \mathcal{T}$.
- The nodes of V are either stations already located or junctions:

Let $\boxed{V_s \subseteq V}$ be the set of stations.

The stations: red nodes.

The leaves are always stations.

- A stopping time (dwell time) $\delta_k > 0$ spent at each station $v_k \in V_s$. (We assume $\delta_k = 0$, if $v_k \in V \setminus V_s$).

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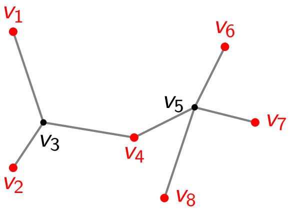

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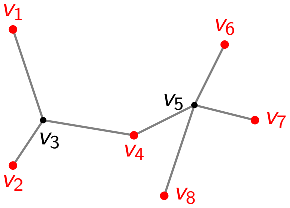

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Elements of the problem

- A set: $\mathcal{P} = \{P_i \in \mathbb{R}^2, i = 1, \ldots, m\}$ of m existing points in the plane. Each point could represent a settlement.
- Each two points P_i, P_j, give rise to two OD-pairs:

$$(P_i, P_j), \text{ and } (P_j, P_i), \quad (\text{or } (i, j) \text{ and } (j, i))$$

- Traveling between P_i, P_j, can be made by plane or by a planar-network mode.
- With each OD-pair (P_i, P_j) we associate
  - A weight $\tau_{ij} \geq 0$, (representing the potential traffic, or the amount of trips, from P_i to P_j).
  - A time acceptance level $0 \leq \widehat{t}_{ij} < ||P_i - P_j||$, meaning that (i, j) will chose the planar-network mode if the traveling time from this combined mode is not greater than $\widehat{t}_{ij}$.

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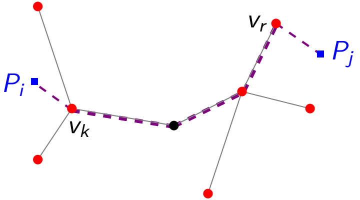
Elements of the problem

Given $v_k, v_r \in V_s$, there are two ways of traveling between P_i, P_j through $\Gamma(v_k, v_r)$:



Traveling time: $h_{ij}(v_k, v_r)$

(Only the stopping time at each intermediate station is considered):

$$h_{ij}(v_k, v_r) = ||P_i - v_k|| + \frac{d(v_k, v_r)}{\kappa} + ||v_r - P_j|| + \sum_{\substack{v_s \in V \cap \Gamma(v_k, v_r) \\ v_s \neq v_k, v_r}} \delta_s$$

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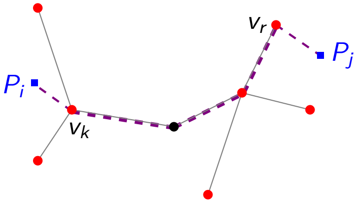

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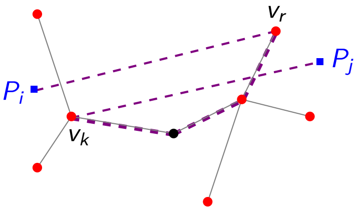

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Elements of the problem

Minimum traveling time of (i, j) through the path $\Gamma(v_k, v_r)$:

$$H_{ij}(v_k, v_r) = \min\{h_{ij}(v_k, v_r), h_{ij}(v_r, v_k)\}$$

Definition of covered pair

(i, j) is covered by $\mathcal{T}$ if there exists some stations $v_k, v_r \in V_s$, such that

$$H_{ij}(v_k, v_r) \leq \widehat{t}_{ij}$$

Γ_{ij}: Set of paths from which (i, j) is covered by $\mathcal{T}$:

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- H_{ij}: Minimum traveling time of (i, j) by using $\mathcal{T}$:

$$H_{ij} = \min_{\Gamma(v_k, v_r) \in \Gamma_{ij}} \Big\{ H_{ij}(v_k, v_r) \Big\}$$

- $C = \{(i, j) : H_{ij} \leq \widehat{t}_{ij}\}$: Set of covered pairs by $\mathcal{T}$.
- $\overline{C}$: Set of not covered pairs.

Total weighted OD-pairs covered by $\mathcal{T}$:

$$F = \sum_{(i,j) \in C} \tau_{ij}$$

Total time of such weighted pairs:

$$H = \sum_{(i,j) \in C} \tau_{ij} H_{ij}$$

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Elements of the problem

Assume we add a new station at point x, with stopping time δ_x:

$$\mathcal{T}_x = (V_x, E_x), \quad \text{with} \quad V_x = V \cup \{x\}$$

By adapting the previous concepts to $\mathcal{T}_x$, we have:

- $\Gamma_{ij}(x) \subseteq \mathcal{T}_x$: Set of paths from which (i,j) is covered by $\mathcal{T}_x$:

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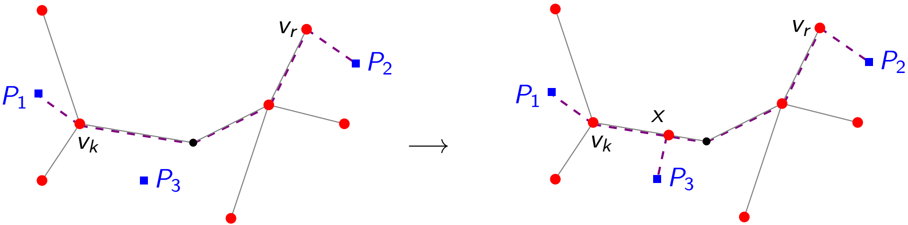
Problem formulation

The new station could cause two opposite effects:

- The accessibility of the network could be increased. The OD-pairs not covered by $\mathcal{T}$ could be *captured* by $\mathcal{T}_x$.
- The OD-pairs already covered by $\mathcal{T}$ may be *lost* by $\mathcal{T}_x$, since the stopping time at x increases the traveling time of such pairs.



$(P_1, P_2) \in C$,
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$(P_1, P_2) \in \overline{C}(x)$,
$(P_1, P_3), (P_2, P_3) \in C(x)$

Problem formulation

Main goal

To locate a new station x maximizing the total weighted pairs covered by $\mathcal{T}_x$:

$$x \in \mathcal{T}: \quad \text{Maximizing } \; F(x) := \sum_{(i,j)\in C(x)} \tau_{ij}$$

The total time of weighted pairs covered by $\mathcal{T}_x$:

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Problem formulation

Taking into account the time penalization

- $C \cap C(x)$ is the set of permanent pairs: pairs which do no change their combined transportation mode, since they are covered by both $\mathcal{T}$ and $\mathcal{T}_x$.
- The variation in the overall time of these pairs:

$$\Delta H(x) = \sum_{(i,j)\in C\cap C(x)} \tau_{ij} \left(H_{ij}(x) - H_{ij} \right)$$

- If $\Delta H(x) > 0$, the passenger-time of some permanent pairs is penalized, since it increases.
- Constraint: To limit such penalization, a time increasing of permanent pairs is acceptable if it does not exceed a given percent on the previous time:

$$\Delta H(x) \leq \lambda \sum_{(i,j)\in C\cap C(x)} \tau_{ij} H_{ij}, \quad \text{with} \quad \lambda \geq 0$$

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Problem formulation

The time-constrained trip covering 1-location problem

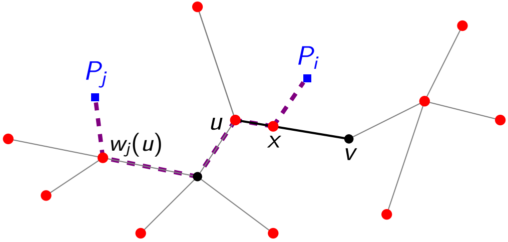
$$\max_{x \in \mathcal{T}} \quad F(x) := \sum_{(i,j) \in C(x)} \tau_{ij}$$

$$\text{s.t.} \quad \Delta H(x) \leq \lambda \sum_{(i,j) \in C \cap C(x)} \tau_{ij} H_{ij}, \quad \lambda \geq 0$$

where $\Delta H(x) = \sum_{(i,j) \in C \cap C(x)} \tau_{ij} \left(H_{ij}(x) - H_{ij} \right)$

Solving the problem

We solve the restricted problem on each edge $\boldsymbol{e} = [u, v]$:



We first consider P_i directly connects x, and the path between P_i, P_j contains u. The traveling time can be decomposed into two terms:

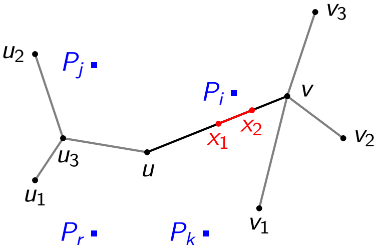
$$f_i(x; u) + K_j(u), \quad \text{for} \quad x \in \boldsymbol{e}$$

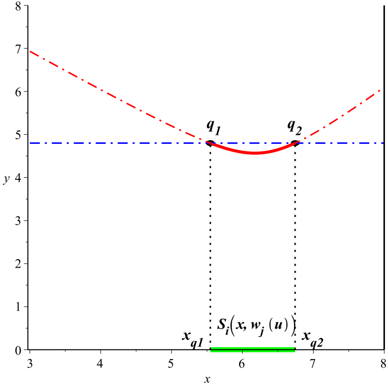
- $f_i(x, u) = ||P_i - x|| + \dfrac{d(x, u)}{\kappa}$, convex
- $K_j(u)$, constant

Solving the problem

The sublevel curve associated with such traveling time consists of the set of points x such that (i, j) is covered by a path in which x is an access/exit point:

$$S_i(x, w_j(u)) = \{x \in \boldsymbol{e} : f_i(x; u) + K_i(u) \leq \widehat{t}_{ij}\}$$

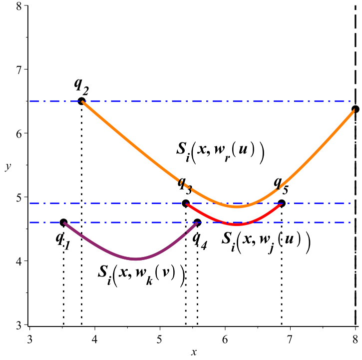




The subinterval generated by this curve corresponds to the subedge $[x_1, x_2]$ in $\boldsymbol{e}$.

Solving the problem

- Each pair (i,j) generates (at most) 4 sublevel curves.
- The abscissas of endpoints $q_1,\ldots,q_{\ell-1}$ of the sublevel curves obtained from all pairs induce a partition of edge $\boldsymbol{e}$.



Endpoints $\{u=q_0,q_1,\ldots,q_5,q_6=v\}$:

$$x_{q_0}<x_{q_1}<\ldots<x_{q_5}<x_{q_6}$$

Properties

- $C(x)$ is constant, for $x\in(x_{q_i},x_{q_{i+1}})$.
- $C(x)\subseteq C(x_{q_i})$ and $C(x)\subseteq C(x_{q_{i+1}})$, for $x\in(x_{q_i},x_{q_{i+1}})$.

- The maximum value of $F(x)$ is attained on the set $\{x_{q_i}, i=0,\ldots,6\}$.

Solving the problem

Theorem

Let $x \in \boldsymbol{e} = [u, v]$ be a new station, and let $Q_{ij}(\boldsymbol{e})$ be the set of endpoints of the sublevel curves generated by (i, j) on edge $\boldsymbol{e}$. Let $Q(\boldsymbol{e})$ be the set of endpoints obtained from all OD-pairs, together with the end vertices u, v, given by

$$Q(\boldsymbol{e}) = \left(\bigcup_{(i,j)} Q_{ij}(\boldsymbol{e}) \right) \cup \{u, v\}$$

Then $Q_x(\boldsymbol{e})$ (the abscissas of points in $Q(\boldsymbol{e})$) is a FDS for the restricted problem.

Solving the problem

Minimum traveling time of permanent pairs at the points of $Q_x(\boldsymbol{e})$

For $x_q \in Q_x(\boldsymbol{e}) \setminus V_s$, let $\Phi_{ij}(x_q)$ be the minimum traveling time among all (i,j)-paths in which x_q is an access/exit station, given by

$$\Phi_{ij}(x_q) = \min_{z \in \{u,v\}} \{f_i(x_q, z) + K_j(z), f_j(x_q, z) + K_i(z)\}$$

For each $(i,j) \in C \cap C(x_q)$, the minimum traveling time $H_{ij}(x_q)$ can be obtained as follows:

$$H_{ij}(x_q) = \min \left\{ \min_{\substack{\Gamma(v_k,v_r)\in\Gamma_{ij} \\ x_q\in\Gamma(v_k,v_r)}} \{H_{ij}(v_k, v_r)\} + \delta_{x_q}, \Phi_{ij}(x_q), \min_{\substack{\Gamma(v_k,v_r)\in\Gamma_{ij} \\ x_q\notin\Gamma(v_k,v_r)}} \{H_{ij}(v_k, v_r)\} \right\}$$

Moreover, $H_{ij}(x_q) = H_{ij}$, when $x_q \in V_s$.

Solving the problem

We assume all required data for solving the restricted problem have been computed in a preprocessing phase. Let F^* be the optimal value on $\mathcal{T}$.

Main procedure (idea) for the restricted problem

1. For each OD-pair (i, j), compute the endpoints of its sublevel curves.
2. Sort $Q_x(\boldsymbol{e})$ increasingly from the distance to the left endvertex.
3. Compute recursively $C(x_q)$, along $x_q \in Q_x(\boldsymbol{e})$, and the objective value $F(x_q)$.
4. For each $x_q \in Q_x(\boldsymbol{e})$ such that $F(x_q) > F^*$:
   For each $(i, j) \in C \cap C(x_q)$:
   Compute the minimum traveling time $H_{ij}(x_q)$.
   If x_q is a feasible point, then update F^* and the optimum point.

Solving the problem

Complexity

- The cardinal of $Q(\boldsymbol{e})$ is M, where M is the number of OD-pairs.
- The maximum complexity is given by Step 4, in which the minimum time of the permanent pairs must be computed. This requires $O(M^2)$.
- For the unconstrained problem, the complexity is $O(M \log M)$.

Extensions

Open problems and future work

- Different scenarios: Cyclic network (in which the distance is not convex), planar distance different than the Euclidean, etc
- Additional constraints.
- Locating $p > 1$ stations.
- Different speed factors in different parts of the network.
- Forbidden regions for locating stations.
- Etc.

Thanks for your attention

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