Rationalizing capacities in the facility location problem

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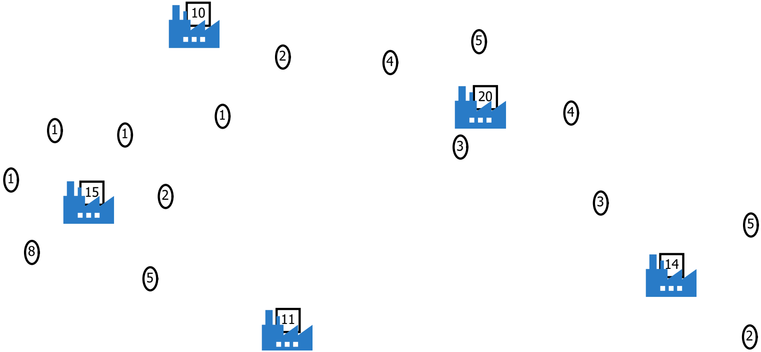
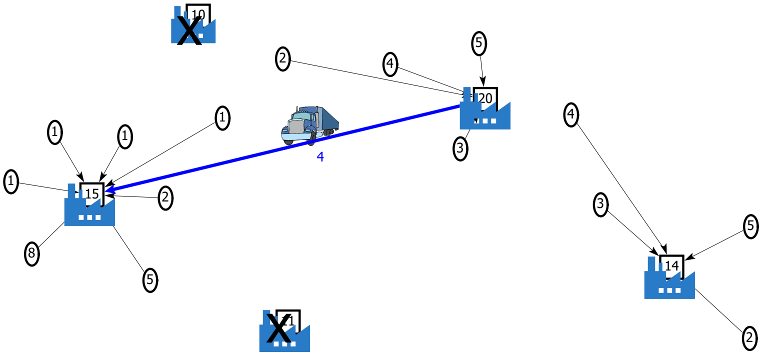
Outline

The problem

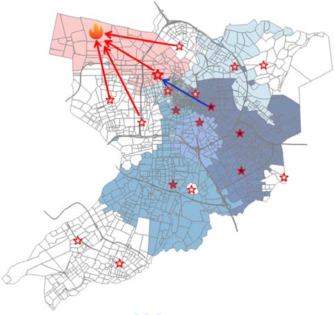


The problem



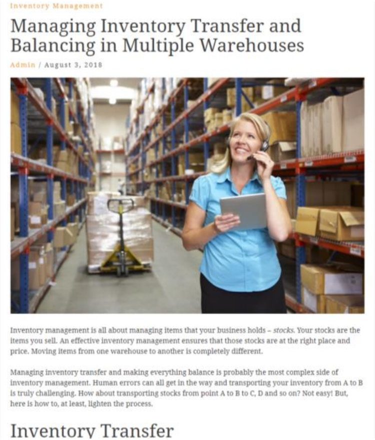
Related problems

- Emergency models (ambulances, fire trucks),



Related problems

- Emergency models (ambulances, fire trucks),
- Inventory transfer.



Inventory Management

Managing Inventory Transfer and Balancing in Multiple Warehouses

Admin / August 3, 2018

Inventory management is all about managing items that your business holds – *stocks*. Your stocks are the items you sell. An effective inventory management ensures that those stocks are at the right place and price. Moving items from one warehouse to another is completely different.

Managing inventory transfer and making everything balance is probably the most complex side of inventory management. Human errors can all get in the way and transporting your inventory from A to B is truly challenging. How about transporting stocks from point A to B to C, D and so on? Not easy! But, here is how to, at least, lighten the process.

Inventory Transfer

Notation

- d_j, demand of customer $j \in J$.
- q_i, capacity of a facility located at $i \in I$.
- f_i, fixed cost for facility $i \in I$.
- c_{ij}, unitary cost for supplying customer $j \in J$ from facility $i \in I$.
- g_{ik} unitary transfer cost between facility $i \in I$ and $k \in I, i \neq k$.

Let d_{ij} be the distance among i and j. Supply and transfer costs may depend on these distances.

- $c_{ij} = d_{ij}$ and $g_{ik} = \alpha d_{ij}$.
- $c_{ij} = d_{ij}$ and
  $g_{ik} \in \{max\{min_j d_{ij}, min_j d_{kj}\}, min\{max_j d_{ij}, max_j d_{kj}\}\}$

Conversely, It might happen that g_{ij} is SMALL or that g_{ij} is LARGE.

Variables

$$y_i = \begin{cases} 1, & \text{if a facility is installed at } i \in I; \\ 0, & \text{otherwise.} \end{cases}$$

x_{ij} = demand of customer j supplied from facility $i \in I$.

w_{ik} = amount of commodity sent from facility i to facility k, $i, k \in I,\ i \neq k$.

A non-linear model

$$\text{minimize} \quad \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} d_j x_{ij} + \sum_{i\in I}\sum_{k\in I} g_{ik} w_{ik},$$

$$\text{subject to} \quad \sum_{i\in I} x_{ij} = d_j, \qquad j \in J, \tag{1}$$

$$\sum_{j\in J} x_{ij} \le \left(q_i + \sum_{k\in I \mid k\neq i} (w_{ki} - w_{ik})\right) y_i, \qquad i \in I, \tag{2}$$

$$\sum_{k\in I \mid k\neq i} w_{ik} \le (q_i + \sum_{k\in I \mid k\neq i} w_{ki})\, y_i, \qquad i \in I, \tag{3}$$

$$w_{ik} \le \left(q_i + \sum_{r\in I \mid r\neq i,k} (w_{ri} - w_{ir})\right) y_k, \qquad i,k \in I,\ i\neq k, \tag{4}$$

$$y_i \in \{0,1\}, \qquad i \in I, \tag{5}$$

$$x_{ij} \ge 0, \qquad i \in I,\ j \in J, \tag{6}$$

$$w_{ik} \ge 0, \qquad i,k \in I,\ i\neq k. \tag{7}$$

A linear model

$u_{ik} = 1$ iff facility i transfers capacity to facility $k, k \neq i$.

$$
\begin{array}{lll}
\text{minimize} & \displaystyle\sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J} c_{ij} d_j x_{ij} + \sum_{i\in I}\sum_{k\in I} g_{ik} w_{ik}, & \\
\text{subject to} & (1), (5) - (7), & j \in J, \\
& \displaystyle\sum_{j\in J} x_{ij} \leq q_i y_i + \sum_{k\in I \mid k\neq i} (w_{ki} - w_{ik}), & i \in I, \quad (8) \\
& \displaystyle q_i y_i + \sum_{k\in I \mid k\neq i} (w_{ki} - w_{ik}) \geq 0, & \forall i \in I \quad (9) \\
& w_{ik} \leq M u_{ik}, & i, k \in I,\ i \neq k \quad (10) \\
& \displaystyle\sum_{k\in I \mid k\neq i} (u_{ik} + u_{ki}) \leq M' y_i, & i \in I, \quad (11) \\
& u_{ik} + u_{ki} \leq 1, & i, k \in I,\ i \neq k \quad (12) \\
& u_{ik} \in \{0, 1\}, & i, k \in I,\ i \neq k, \quad (13)
\end{array}
$$

Improvements of the linear model

Original constraint	Improvement
$q_i y_i + \sum_{k \in I \mid k \neq i} (w_{ki} - w_{ik}) \geq 0$	$\sum_{k \in I \mid k \neq i} w_{ik} \leq q_i y_i$
$w_{ik} \leq M u_{ik}$	$w_{ik} \leq q_i u_{ik}$
$\sum_{k \in I \mid k \neq i} (u_{ik} + u_{ki}) \leq M' y_i$	$(\lvert I \rvert - 1) y_i$
$u_{ik} + u_{ki} \leq 1$	$u_{ik} + u_{mi} \leq 1$

We make the assumption that costs satisfy the triangle inequality and thus that transshipment points are not allowed.
If costs do not satisfy the triangle inequality, constraints $u_{ik} + u_{mi} \leq 1$ avoid transshipment points

Valid inequalities for the linear model

$$
\begin{aligned}
x_{ij} &\leq d_j y_i, \quad i \in I,\ j \in J \\
w_{ik} &\leq q_i y_k, \quad i, k \in I \\
u_{ik} &\leq w_{ik} \quad i, k \in I,
\end{aligned}
$$

The first one is an *effective capacity inequality* for the Capacitated Plant Location Problem.

Valid inequalities for the linear model

Optimal solutions to the linear model satisfy:

$$x_{tj} \leq d_j(1 - u_{ti}), \qquad \forall i, t \in I, \quad \forall j \in J \ : \ g_{ti} + c_{ij} < c_{tj}.$$

$$x_{tj} \leq d_j(1 - u_{it}), \qquad \forall i, t \in I, \quad \forall j \in J \ : g_{it} + c_{tj} > c_{ij}.$$

Computational experiments: the instances

We have used some of instances in

- M. Fischetti. I. Ljubi, M. Sinnl. *Benders decomposition without separability: A computational study for capacitated facility location problems.* EJOR 253 (2016) 557-569.

We have randomly generated g_{ik}.
In our preliminary computational experiment, we have solved 20 instances with 700 customers and 700 potential facilities and 20 instances with 1500 customers and 300 potential facilities.

Preliminary results

For the 20 instances with 700 customers and 700 facilities:

Gap	# Cplex	# B&C	Time*	Nodes Cplex	Nodes B&C
$< 1\%$	0	18	2381	142	296

For the 20 instances with 1500 customers and 300 facilities:

Gap	# Cplex	# B&C	Time*	Nodes Cplex	Nodes B&C
$< 1\%$	0	17	1836	528	287

Conclusions

- We deal with a new capacitated facility location problem.
- The provided linear model has a small LP gap.
- The introduced model enables to solve instances with up to 700 facilities and 1500 clients.
- Most of the extensions for the capacitated plant location problem make sense for this problem.

Thank you!