INFRASTRUCTURE RAPID TRANSIT NETWORK DESIGN MODEL solved by BENDERS DECOMPOSITION

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1. Rapid Transit Design Problem

Rapid Transit Design Problem

In recent years, new rail transit systems have been built, expanded or are being planned for construction.

Reasons:

- house spreading
- enlargement of urbanised areas
- traffic problem in the centres or in entrances of the cities
- reduction on average ground traffic speed

2. Infrastructure Network Design Model

Data:

- Let $N = \{i| \quad i = 1, 2, \ldots, n\}$ and $E = \{e_1, \ldots, e_m\}$ two given sets of potential sites (stations) and edges (links), respectively.
- Each station i and edge e has an associated construction cost, $c_i, c_e \in \mathbb{R}^+$. Let C_{max} the available budget.
- Every feasible arc $a \in A = \{a_1, \ldots, a_{2m}\} \subset N \times N$ has an associated length d_a.
- Travel patterns are given by the origin-destination matrix $G = (g^p)$, where g^p is the demand of the pair $p = (p^s, p^t) \in P = \{p_1, \ldots, p_v\} \subset N \times N$ and P is the set of pairs of demand.
- Let u^p_{PRIV} the time needed to go from $p^s \in N$ to $p^t \in N$.

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Infrastructure Network Design Model

- Objective function: maximize trip coverage

$$\max \sum_{p \in P} g^p z^p$$

- Budget constraint

$$\sum_{e \in E} c_e x_e + \sum_{i \in N} c_i y_i \leq C_{max}$$

- Alignment location constraints

$$x_e \leq y_i, \quad e \in E, \quad i \in \{e^s, e^t\}$$

- Routing demand conservation constraints

$$\sum_{a \in \delta^+(p^s)} f_a^p = z^p, \quad p = (p^s, p^t) \in P$$

$$\sum_{a \in \delta^-(p^s)} f_a^p = 0, \quad p = (p^s, p^t) \in P$$

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$$\sum_{a\in\delta^+(p^t)} f_a^p = 0, \quad p = (p^s, p^t) \in P$$

$$\sum_{a\in\delta^-(k)} f_a^p - \sum_{a\in\delta^+(k)} f_a^p = 0, \quad k \notin \{p^s, p^t\}, \quad p = (p^s, p^t) \in P$$

- Location-Allocation constraints

$$f_a^p + f_{a'}^p \le x_e, \quad p = (p^s, p^t) \in P, \quad e = a \text{ or } e = a'$$

- Splitting demand constraints

$$\sum_{a\in A} d_a f_a^p \le u_{PRIV}^p z^p, \quad p = (p^s, p^t) \in P$$

- Binary constraints

$$x_e, \quad y_i, \quad f_a^p, \quad z^p \quad \in \{0,1\}.$$

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Infrastructure Network Design Model

Observation

Continuous Budget Constraint

$$\sum_{e \in E} c_e\, x_e + \sum_{i \in N} c_i\, y_i \leq C_{max}$$

can be replaced by the set of Discrete Budget Constraints:

$$\sum_{i \in N} y_i \leq N_{max}, \quad \sum_{e \in E} x_e \leq M_{max},$$

where N_{max} and M_{max} are de the maximum number of stations and edges that can be built. These polyhedrons are related by the equality:

$$C_{max} = \sum_{i=1}^{N_{max}} (L_{stations})_i + \sum_{j=1}^{M_{max}} (L_{edges})_j,$$

where $L_{stations}$ and L_{edges} are the costs lists in non-creasing order of the stations and edges sets respectively.

4. Benders Decomposition for INDM

Master Problem:

$$
\begin{aligned}
\max \quad & \sum_{p\in P} g^p\, z^p + \mathbf{q}(x,y,z) \\
\text{s.t.} \quad & \sum_{e\in E} c_e\, x_e + \sum_{i\in N} c_i\, y_i \le C_{max}, \quad e \in E, \quad i \in N \\
& x_e \le y_i, \quad e \in E, \quad i \in \{e^s, e^t\} \\
& x_e, \quad y_i, \quad z^p \quad \in \{0,1\}
\end{aligned}
\tag{1}
$$

Subproblem:

$$
\begin{aligned}
\mathbf{q}(x,y,z) = \max \quad & 1 \\
\text{s.t.} \quad & \sum_{a\in\delta^+(p^s)} f_a^p = \overline{z}^p, \quad p=(p^s,p^t)\in P \\
& \sum_{a\in\delta^-(p^s)} f_a^p = 0, \quad p=(p^s,p^t)\in P \\
& \sum_{a\in\delta^+(p^t)} f_a^p = 0, \quad p=(p^s,p^t)\in P \\
& \sum_{a\in\delta^-(k)} f_a^p - \sum_{a\in\delta^+(k)} f_a^p = 0, \quad k\notin\{p^s,p^t\}, \quad p=(p^s,p^t)\in P \\
& f_a^p + f_{a'}^p \le \overline{x}_e, \quad p=(p^s,p^t)\in P, \quad e=a \text{ or } e=a' \\
& \sum_{a\in A} d_a f_a^p \le u_{PRIV}^p \overline{z}^p, \quad p=(p^s,p^t)\in P \\
& f_a^p \quad \in [0,1]
\end{aligned}
\tag{2}
$$

Benders Master Problem:

$$
\begin{aligned}
\max \quad & \phi + \sum_{p \in P} g^p z^p \\
\text{s.t.} \quad & \phi \le \sum_{p \in P} \left(z^p \left(\alpha^p_{p^s} + U^p_{PRIV} \sum_{a \in A} \omega^p_a \right) + \sum_{e \in E} x_e \beta^p_e + \sum_{a \in A} \gamma^p_a \right) \\
& 0 \le \sum_{p \in P} \left(z^p \left(\alpha^p_{p^s} + U^p_{PRIV} \sum_{a \in A} \omega^p_a \right) + \sum_{e \in E} x_e \beta^p_e + \sum_{a \in A} \gamma^p_a \right) \\
& \sum_{e \in E} c_e x_e + \sum_{i \in N} c_i y_i \le C_{max}, \quad e \in E, \quad i \in N \\
& x_e \le y_i, \quad e \in E, \quad i \in \{e^s, e^t\} \\
& x_e, \quad y_i, \quad z^p \quad \in \{0, 1\}
\end{aligned} \tag{3}
$$

Subproblem for each commodity p:

$$
\begin{aligned}
\mathbf{q}_p(x,y,z) = \max \quad & 1 \\
\text{s.t.} \quad & \sum_{a\in\delta^+(p^s)} f_a^p = \overline{z}^p, \\
& \sum_{a\in\delta^-(p^s)} f_a^p = 0, \\
& \sum_{a\in\delta^+(p^t)} f_a^p = 0, \\
& \sum_{a\in\delta^-(k)} f_a^p - \sum_{a\in\delta^+(k)} f_a^p = 0, \quad k \notin \{p^s, p^t\}, \\
& f_a^p + f_{a'}^p \leq \overline{x}_e, \quad e = a \text{ or } e = a' \\
& \sum_{a\in A} d_a f_a^p \leq u_{PRIV}^p \overline{z}^p, \\
& f_a^p \quad \in [0,1]
\end{aligned}
\tag{4}
$$

Benders Master Problem:

$$
\begin{aligned}
\max \quad & \sum_{p\in P} \phi^p + \sum_{p\in P} g^p z^p \\
\text{s.t.} \quad & \phi^p \leq z^p \left(\alpha^p_{p^s} + U^p_{PRIV} \sum_{a\in A} \omega^p_a \right) + \sum_{e\in E} x_e \beta^p_e + \sum_{a\in A} \gamma^p_a, \quad p = (p^s, p^t) \in P \\
& 0 \leq z^p \left(\alpha^p_{p^s} + U^p_{PRIV} \sum_{a\in A} \omega^p_a \right) + \sum_{e\in E} x_e \beta^p_e + \sum_{a\in A} \gamma^p_a, \quad p = (p^s, p^t) \in P \\
& \sum_{e\in E} c_e x_e + \sum_{i\in N} c_i y_i \leq C_{max}, \quad e \in E, \quad i \in N \\
& x_e \leq y_i, \quad e \in E, \quad i \in \{e^s, e^t\} \\
& x_e, \quad y_i, \quad z^p \quad \in \{0,1\}
\end{aligned} \tag{5}
$$

4. Preliminary computational results

Preliminary computational results

Network	Nmax	Mmax	ObjValue	BB	BD	DBD
N20-65	10	20	2386	2079.12	169.509	165.307
	15	30	4734	512.24	98.437	89.462
	18	40	6482	106.63	52.377	51.068
	20	65	7698	1.13	2.840	2.305
N25-116	13	35	4131	*1d	1473.71	1407
	19	53	8022	82251.03	484.461	486.83
	23	71	11029	6446.85	371.05	703.96
	25	116	12557	6.66	33.68	32.86
N30-166	15	50	5446	*4d	145340.58	143785.65
	23	76	11673	*4d	5079.85	4791.26
	27	101	15391	*3d	1409.71	1616.58
	30	166	18448	13.60	45.42	45.49
N35-202	18	60	7571	*5d	*5d	354550.95
	26	93	14902	*3d	86966.48	81941.39
	32	123	21645	*3d	11365.28	9374.81
	35	202	24913	16.33	378.02	352.93
N40-250	20	75	-	*5d	*5d	*5d
	30	115	-	*3d	*3d	*3d
	36	153	18448	*3d	261.27	239.42
	40	250	32426	24.82	487.36	500.67

Preliminary computational results

- The more adjusted the upper bound, the bigger is the difference between the computational time of the three algorithms.
- In the majority of the cases, DBD is faster than BD but there are weird cases.
- DB constraints accelerates the resolution process.

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- Solve INDM by other exacts technics
- Use Benders Decomposition to solve others similar problems
  - Design and locate the lines
  - Take into account transfer lines and other parameters

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Thanks for your attention!