OPTIMAL ALLOCATION OF FLEET FREQUENCY FOR “SKIP-STOP" STRATEGIES IN TRANSPORT NETWORKS

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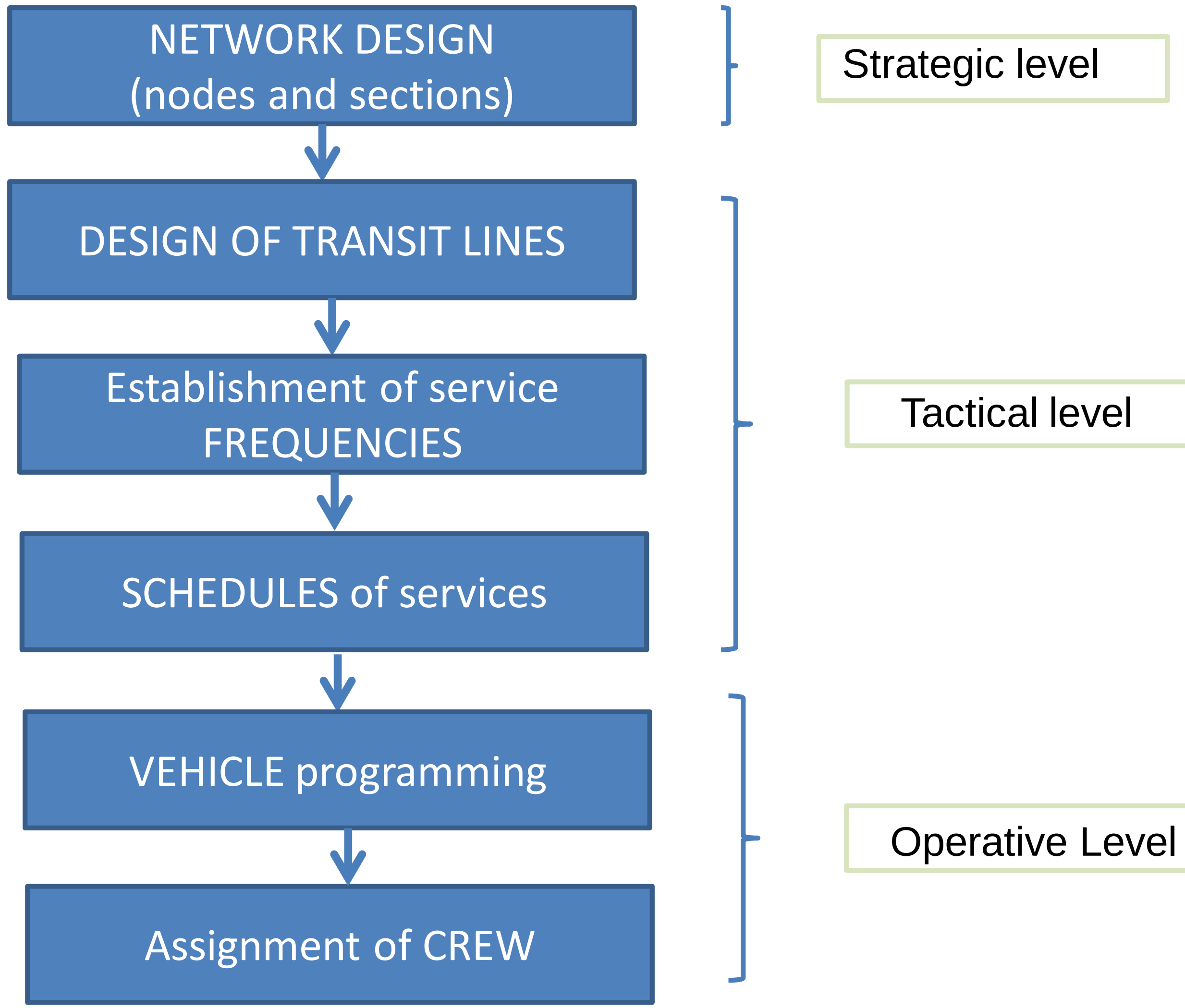
Introduction



Introduction

Occasional incidents in the system operation are generally not considered in this initial planning stage

Introduction

The strategies most commonly used for this purpose:

- The express service.
- Short cycle service (short-turning) or non-stop return (deadheading) .
- The combination of different control actions.

Introduction

- We can assume from the outset that the clients plan their trips according to **a known chronogram** and that they may **feel affected** if the railway services do not arrive or leave at the scheduled time.
- We developed a methodology to implement a redistribution of services along a line of railway traffic.
- The objective is to **minimize the loss of users**, who could perceive a worsening in the quality of the service that until now they had been receiving.

Introduction

- The **Skip-Stop** operation consists of privileging a larger number of passengers by offering **shorter travel times**, as a result of having previously selected a group of low-activity stations, where trains **wouldn’t stop** to pick up or let off passengers.
- In railway systems, where have no extra track for a faster train can pass a slower train, a skip-stop mechanism may be used either during busier travel hours to reduce travel time of particular trains by not stopping (skipping) at less densely populated stations.

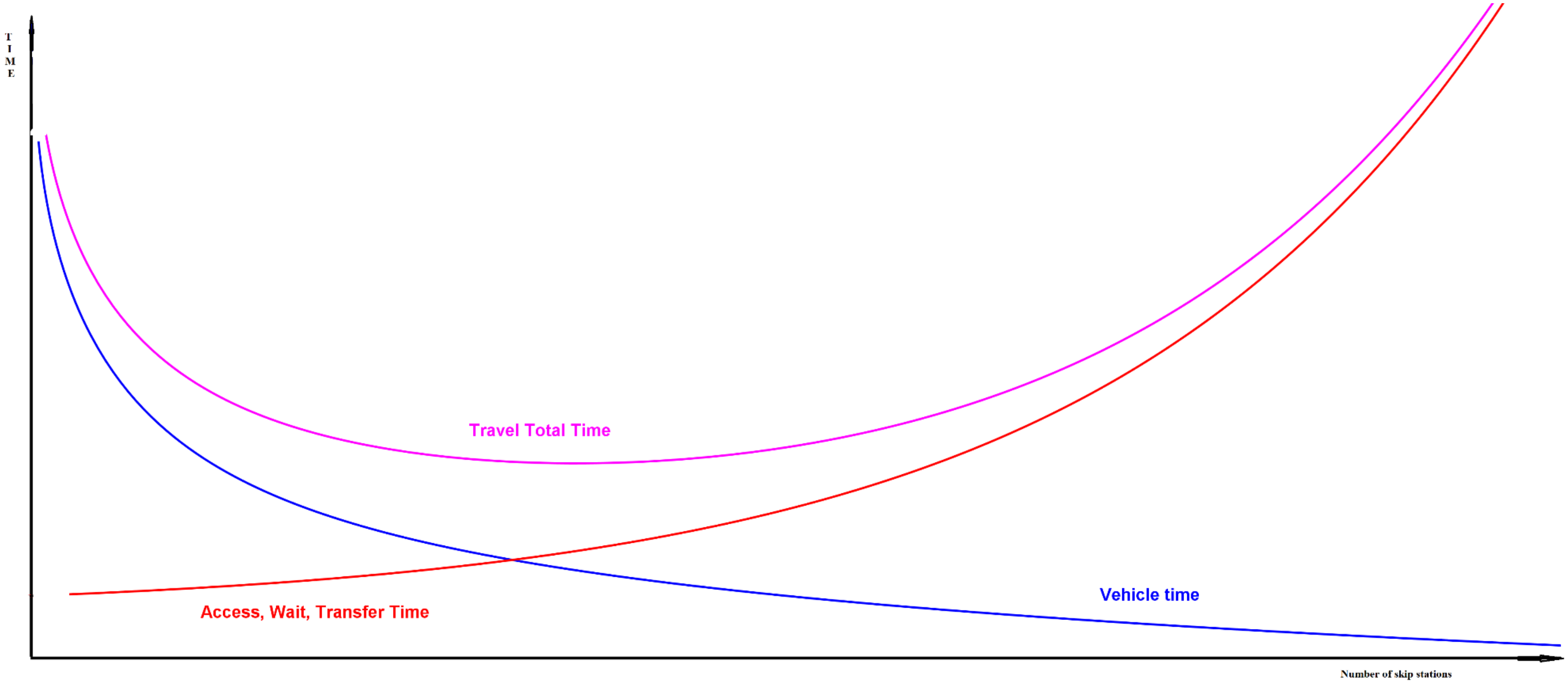
Introduction

- The travel time between stations along a railway line consists of five components, usually identified as phases of **acceleration**, **constant speed**, **inertia**, **braking** and **downtime**.
- Obviously, the operation of omitting stops reduces the travel time for the users within the vehicule and increases the speed of operation in the provision of new transit services.
- However, other users will experience, if this strategy were applied, a longer time of waiting, accesing, exiting and, possibly, transferring.

Introduction

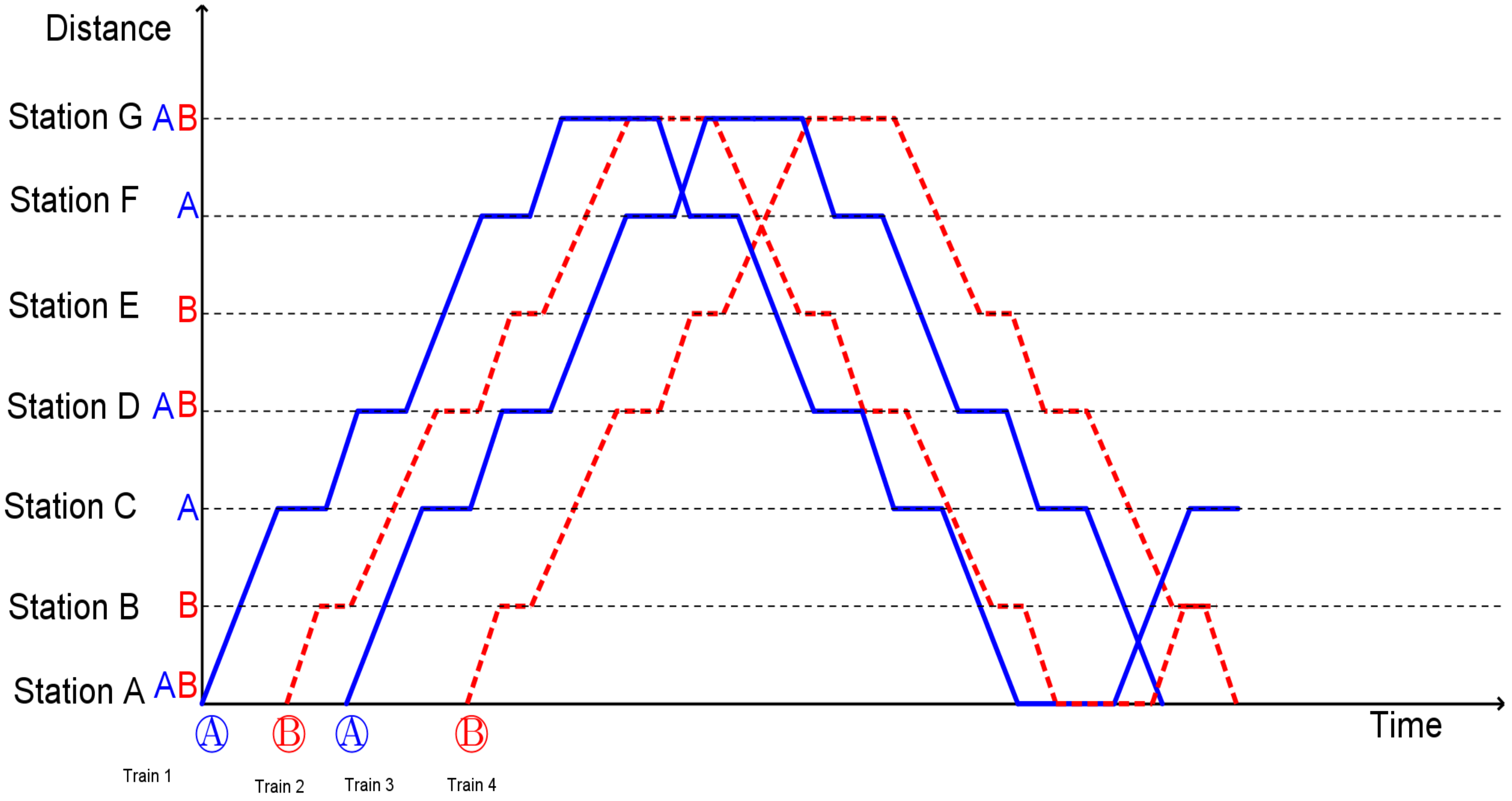
- Regarding the stop-skipping patterns for a one-way single track, the fundamental approaches are divided into deterministic (see, for instance, Mesa et al. [2009], Freyss et al [2013]), and stochastic forms (Sun and Hickman [2005]).
- The deterministic form is derived from the description and analisis given by Vuchic [1973] in which stations along a line are classified into three groups A, B and AB. The trains in line A stop at the A and AB stations, while the trains belonging to line B stop at the B and AB stations. When they intend to alight at a B station, passengers boarding at an A station will need to transfer at an AB station onto line B. Thus, this disadvantage might affect the attractiveness of stop-skipping schedules.
- The main drawbacks of this form are the determination of the skipped stations and the potential to fall behind the estimated demand with respect to historical statiscal data.

Skip-Stop strategy



In the cities of Chicago, Philadelphia, New York, Santiago de Chile, Seoul (among others) this strategy has been applied in recent years.

Skip-Stop strategy



Skip-Stop strategy

The stations are classified as type A, B and AB. Trains are named in two ways: those of type A, which will stop at stations A and AB, and type B trains, which will stop at stations B and AB. Consequently, origin-destination trips are classified into 9 groups.

Type OD	Orig.	Dest.	Decision
Type I	AB	AB	Take any train
Type II	A	A	Take only trains type A
Type II	A	AB	Take only trains type A
Type II	B	B	Take only trains type B
Type II	B	AB	Take only trains type B
Type II	AB	A	Take only trains type A
Type II	AB	B	Take only trains type B
Type III	A	B	Take trains type A and transfer to trains type B at AB station
Type III	B	A	Take trains type B and transfer to trains type A at AB station

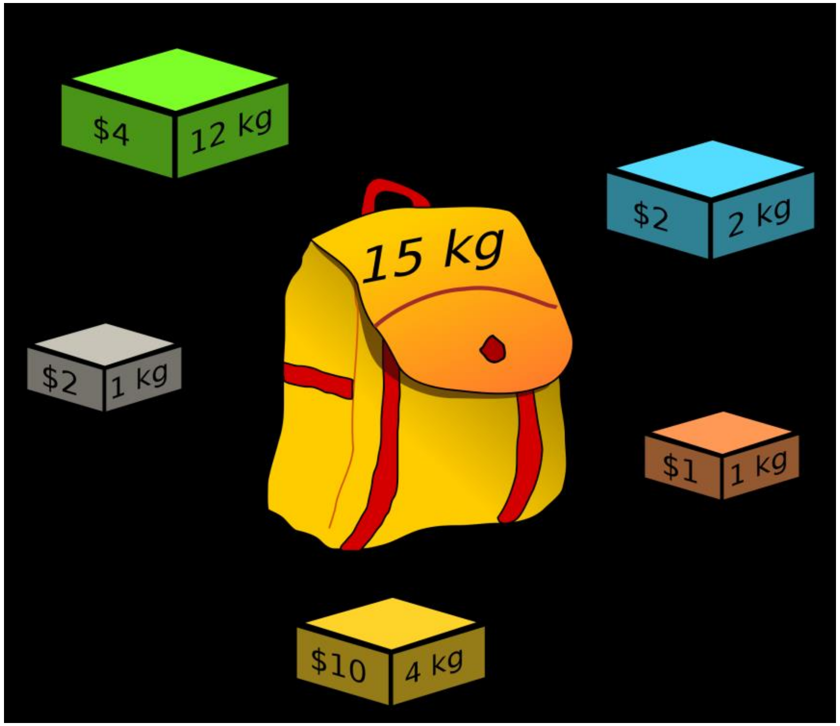
Knapsack Problem

The Knapsack Problem is inspired by the preparation of the necessary luggage that a walker places in his knapsack to make a trip. For this purpose, a selection among several possible objects that can provide the greatest benefit, without exceeding the storage capacity of the knapsack, must be carried out.

$$\text{(KP)}\ \max \quad \sum_{j=1}^{n} p_j x_j \qquad (1)$$

$$\text{s.t.} \quad \sum_{j=1}^{n} w_j x_j \leq c, \qquad (2)$$

$$x_j \in \{0,1\}, j = 1, \ldots, n. \qquad (3)$$



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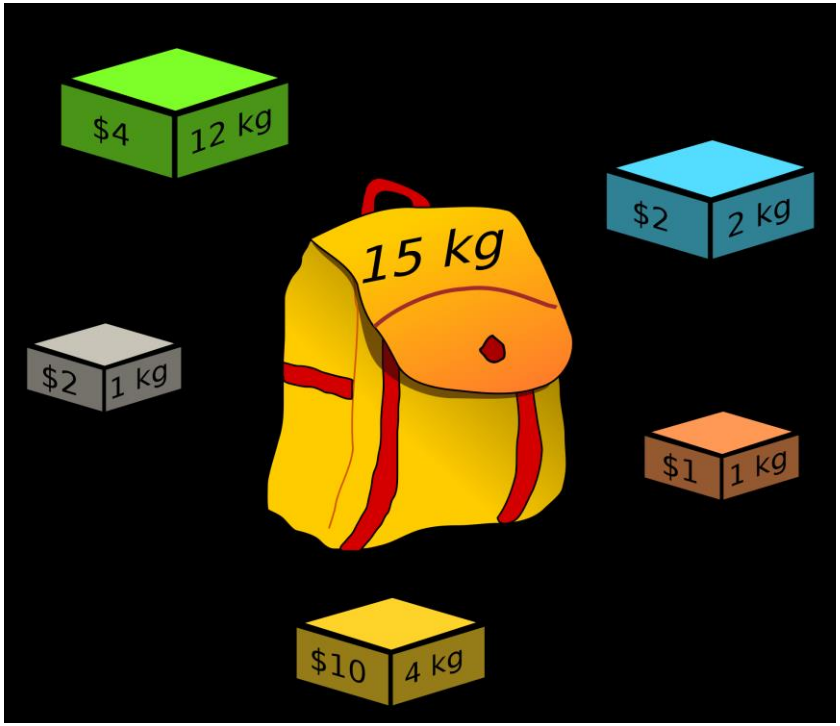


We propose to model the Skip-Stop problem through the KP taking advantage of the large amount of available contributions in the Literature.

Description problem and formulation

Indices and Sets

$s \in S$ — index identifying train services of set *S.*

$i, j, k, l \in I$ — indices identifying stations of set *I.*

Parameters

p_{ij}^{s} — population available to board train *s* at station *i* with destination *j* (if the number of intermediate stops were 0).

Variables

w_{ij}^{s} — real population available to board train *s* at station *i* with destination *j* (including intermediate stops).

n_{ij}^{s} — number of intermediate stops between stations *i* and *j* for train service *s.*

y_{i}^{s} — binary variable equals to 1 if train *s* stops at station *i*;
0, otherwise.

x_{ij}^{s} — binary variable equals to 1 if train *s* stops at stations *i* and *j*;
0, otherwise.

Mathematical programming model

$$\max \quad \sum_{s \in S} \sum_{\substack{i,j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s$$

Maximize the number of passengers taking trains in stations of S.

s.t.:

[1] $$w_{ij}^s = \frac{p_{ij}^s}{n_{ij}^s + 1} \qquad i,j \in I,\ j > i;\ \ s \in S,$$

Identify the actual demand according to the number of intermediate stations.

[2] $$\left(\sum_{\substack{j \in I \\ j > i}} w_{ij}^s - \sum_{\substack{k \in I \\ k < i}} w_{ki}^s \right) \cdot y_i^s \leq c^s \qquad i \in I;\ \ s \in S$$

Avoid exceeding train capacity when stopped at each station i.

[3] $$x_{ij}^s \leq y_i^s;\quad x_{ij}^s \leq y_j^s \qquad i,j \in I,\ j > i;\ \ s \in S$$

If you choose to pick up travelers from a origin-destination pair, you will have to stop at both stations.

[4] $$\sum_{s \in S} y_i^s \geq 1 \qquad i \in I,$$

All stations have at least one train that stops in them

[5] $$\sum_{i \in I} y_i^s \geq 2 \qquad s \in S,$$

Guarantees train configurations that at least stop at two stations

[6] $$x_{ij}^s, y_i^s \in \{0,1\},\ i \in I;\ \ n_{ij}^s \in \mathrm{Z}^+,\ i,j \in I,\ i > j;\ \ s \in S.$$

SOLVING THE MODEL: GREEDY ALGORITHM (1st Phase)

For each $s \in S$ **do**

- **Set** $Y^s = \left(y_i^s\right) = (1)$
- **Read** $\text{matrix}\left(p_{ij}^s\right)$
- **Compute** $\text{matrix}\left(w_{ij}^s\right)$
- **Set** $Q(s) = \sum_{i \in I} \sum_{\substack{j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s$
- **For each** $l \in I$ **do**
  - **While** $\sum_{s \in S} y_i^s \geq 1$ **and** $\sum_{i \in I} y_i^s \geq 2$
    - **Set** $y_l^s = 0$
    - **Compute** $\text{matrix}\left(w_{ij}^s\right)$
    - **Set** $R(s) = \sum_{i \in I} \sum_{\substack{j \in I \\ j > i}} w_{ij}^s \cdot x_{ij}^s$
    - **If** $R(s) > Q(s)$ **then** $Q(s) := R(s)$ **else** $y_l^s := 1$

ROUTINE Calculate $\mathrm{matrix}\left(w_{ij}^{s}\right)$ (1st Phase)

For each $i, j \in I\,(i < j)$ **do**

- **If** $y_i^s = 1$ and $y_j^s = 1$ **then** $x_{ij}^s := 1$ **else** $x_{ij}^s := 0$
- **Set** $par = 0$
- **For** $k = i + 1$ **to** $k = j - 1$ **do**
  - **If** $y_k^s = 1$ **then** $par := par + 1$
- **Set** $n_{ij}^s = par$
- **Set** $w_{ij}^s = \dfrac{p_{ij}^s}{n_{ij}^s + 1}$

SOLVING THE MODEL: GREEDY ALGORITHM (2nd Phase)

- Once the model is solved, we will obtain an optimal solution that will indicate the stops that each train must make in its service, in order to maximize the number of passengers.

$$\text{Train } s = (\ldots y_i^s \ldots) = (0/1, \ldots, 0/1, \ldots, 0/1),\ \ s \in S.$$

- From this solution, the trains should be classified according to a concept of «proximity» between binary chains of 0/1 (not stop / stop) from the point of view of an ad hoc defined metric.
- This proximity is one-dimensional in nature. Therefore, we can construct a W matrix of inter-distances (from Hamming, or Rectangular, or Euclidean) and, based on the method published by Hall in 1970 (where a spatial interpretation of maximum eigenvectors of the matrix B = D-W is made), we will obtain the relative position on the OX axis of the representative points. This relative position will allow us to establish a classification of trains and stations in types A and B.

Hall Method (2nd Phase)

- W = (w_{ij}) with $w_{ij} = y_i^j$
- Compute D = (d_{ij}) diagonal matrix

  $$d_{ij} = 0 \; if \; i \neq j$$

  $$d_{ij} = \sum_{k=1}^{n} w_{ki}$$ if i=j
- Compute B = D – W.
- Compute the eigenvalues of B and take de máximum α_{max}.
- Compute v_{max} the eigenvector of α_{max}. It has asociated distribution in [-1,1] of points (the coordinates of v_{max}).

Hall, K. M. (1970). An r-Dimensional Quadratic Placement Algorithm. Management Science, vol. 17 Issue 3, p219-229.

3rd Phase

- We denote trains *i, j* the points furthest from each other of the previous distribution and we set train *i* like type A and train *j* like type B.
- **For each station *k***, do
  - If train *i* (type A) stop (=1 in *k*) and train *j* don't stop (=0 in *k*) do station k = station type A.
  - If train *i = 0* and train *j*=1 in *k* do station k = station type B.
  - If train *i = 1* and train *j*=1 in *k* do station k = station type AB.

Freyss M., Giesen R. and Muñoz J.C. (2013) Continuous approximation for skip-stop operation in rail transit. Transportation Research Part C-Emerging Technologies 36 419-433

3rd Phase

- **For each intermediate train *m*** do
  - **Compute** coincidences with *i* and *j.*
  - **Choose** the most coincident with *m* (for example *i)*.
  - In the no coincident stations, we change the train which doesn't stop *(0 → 1)* and change the type of the station if required.

Freyss M., Giesen R. and Muñoz J.C. (2013) Continuous approximation for skip-stop operation in rail transit. Transportation Research Part C-Emerging Technologies 36 419-433

Example

We have 4 trains and 5 stations in the railway corridor.

As a result of a previous optimization process, the sequence of operations stop (1) -skip (0) in the 5 stations for the 4 trains are represented vectorially:

1: (1,1,0,0,1)

2: (0,1,0,1,0)

3: (0,1,1,1,0)

4: (0,0,1,1,1)

Example

We constructed the distance matrix W (from Hamming, or Rectangular, or Euclidean) between each pair of sequences.

1: (1,1,0,0,1) → row 1: (0,3,4,4)

2: (0,1,0,1,0) → row 2: (3,0,1,3)

3: (0,1,1,1,0) → row 3: (4,1,0,2)

4: (0,0,1,1,1) → row 4: (4,3,2,0)

Example

Let's build from W, matrices D and B, following the article. The eigenvalues of the B child matrix (ordered from highest to lowest): 14.8482, 11.3273, 7.82446, 0.

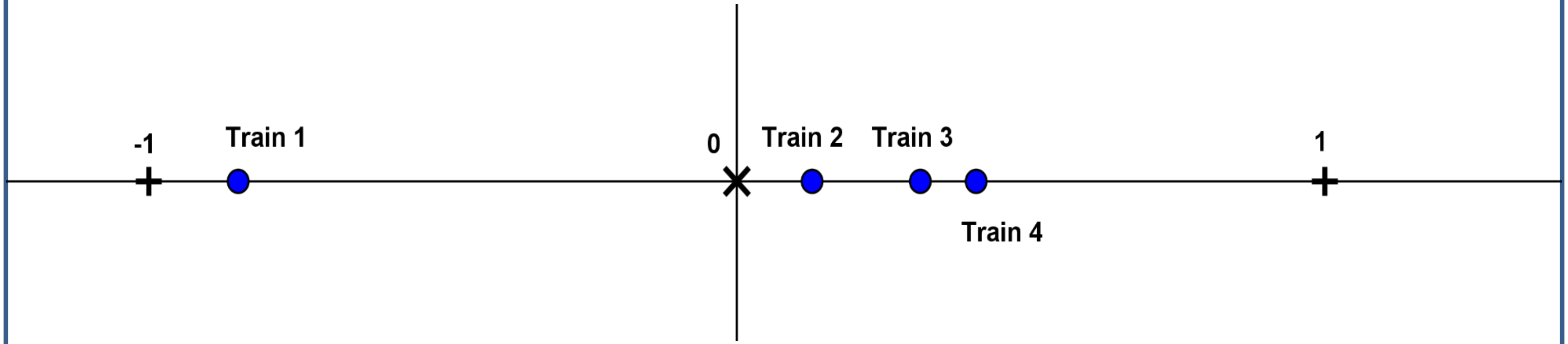
The eigenvalue 0 is always sold by the construction of the matrix B. The eigenvalue is equivalent to the value of the objective function, whose eigenvector gives us the position on the OX axis of the four points representative of the sequence vectors.

Example

The eigenvector corresponds to the highest eigenvalue **14.8482** is

-0.848468, 0.128783, 0.312184, 0.407501

which indicates the relative positions of sequences **1, 2, 3 and 4 on the OX** axis. If we graphically represent this 4 points,



Example

Initial assignment:

1: (1,1,0,0,1) → Type A

4: (0,0,1,1,1) → Type B

The stations

St. 1 → Type A

St. 2 → Type A

St. 3 → Type B

St. 4 → Type B

St. 5 → Type A/B

Example

For trains 2 and 3 we calculate the coincidences with 1 and 4.

- **Train 2:** coincidences(1,2) = 1; coincidences(2,4) = 2 → **Type B.**

Change train 2 = (0,1,0,1,0) and train 4 = (0,0,1,1,1) to **train 2'= train 4' = (0,1,1,1,1),** so we have to change Station 2 = Type A to **Station 2 = Type AB.**

- **Train 3:** coincidences(1,3) = 1; coincidences(3,4') = 4 → **Type B.**

Change train 3 = (0,1,1,1,0) to **train 3' = (0,1,1,1,1).**

Example

Finally,

Trains

1: (1,1,0,0,1) → Type A

2: (0,1,1,1,1) → Type B

3: (0,1,1,1,1) → Type B

4: (0,1,1,1,1) → Type B

The stations

St. 1 → Type A

St. 2 → Type AB

St. 3 → Type B

St. 4 → Type B

St. 5 → Type AB

There are 4 modifications, 4 new stops.

Conclusion

The Skip-Stop operation represents a low cost approach to improve the operation speed of the transaction, since it does not necessarily require additional investments in infrastructure. We have proposed a two-phase methodology to optimize both the classification of stations and trains based on passenger travel demand that varies according to stops that let's eliminate. As resolution tools we propose an entire linear programming model based on the model of the multiple knapsack and a heuristic based on the Hall method.

Future Work

- Improve the mathematical model to take into account the possibility of transshipment.
- Study the case that no train stops at a station.
- Study the efficiency of the model.

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