The mobile facility location problem

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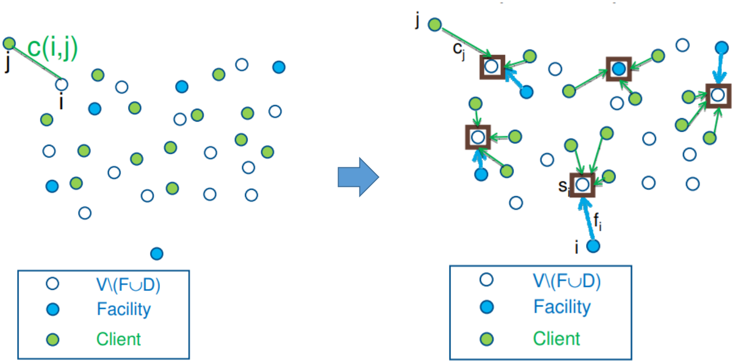
The problem

Consider the following scenario. There is a company with some manufacturing plants. There are also several retail stores (with different demands) to which the products must be shipped and we are interested in minimizing the cost of shipping. One possibility is to send the products to each retailer from its closest manufacturing plant. Another possibility is to set up a distribution center for each plant (perhaps somewhere else), send the products from that plant to the distribution center (in one shipment) and then for each retailer ship the products from the closest distribution center. This is called the minimum total movement Mobile Facility Location Problem (MFLP).

The problem



The MFLP in the literature

Demaine ED et al. Minimizing movement. ACM Trans Algorithms 2009, 5(3):28.

Friggstad Z and Salavatipour MR. Minimizing movement in mobile facility location problems. ACM Trans Algorithms 2011, 7(3):30.

Halper R et al. Local search heuristics for the mobile facility location problem. Computers & Operations Research 62 (2015), 210-223.

Raghavan S et al. The capacitated mobile facility location problem. Working paper, University of Maryland (2016).

Existing formulations

Let F be the set of facilities, C the set of customers and R the set of other locations $R = V \setminus (C \cup F)$.

$$y_{jv} = \begin{cases} 1 & \text{if facility } j \text{ is re-located at location } v, \\ 0 & \text{otherwise} \end{cases}$$

$$x_{iv} = \begin{cases} 1 & \text{if customer } i \text{ is re-allocated at location } v, \\ 0 & \text{otherwise} \end{cases}$$

$$z_v = \begin{cases} 1 & \text{if location } v \text{ is the destination of some facility} \\ 0 & \text{otherwise} \end{cases}$$

Existing formulations

$$
\begin{aligned}
(\text{P0}) \min \quad & \sum_{j\in F}\sum_{v\in V} d_{jv}y_{jv} + \sum_{i\in C}\sum_{v\in V} d_{iv}x_{iv} \\
\text{s.t.} \quad & \sum_{v\in V} x_{iv} = 1 \\
& \sum_{v\in V} y_{jv} = 1 \\
& \sum_{j\in F} y_{jv} \geq x_{iv} \\
& y_{jv}, x_{iv} \in \{0,1\}
\end{aligned}
$$

$$
\begin{aligned}
(\text{P1}) \min \quad & \sum_{j\in F}\sum_{v\in V} d_{jv}y_{jv} + \sum_{i\in C}\sum_{v\in V} d_{iv}x_{iv} \\
\text{s.t.} \quad & \sum_{v\in V} x_{iv} = 1 \\
& \sum_{v\in V} y_{jv} = 1 \\
& \sum_{j\in F} y_{jv} - z_v = 0 \\
& x_{iv} - z_v \leq 0 \\
& 0 \leq y_{jv}, x_{iv} \leq 1, z_v \in \{0,1\}
\end{aligned}
$$

Lp gap of the existing formulations

5 instances in http://www.math.nsc.ru/AP/benchmarks/UFLP/Engl/uflp_dg_eng.html

$\|F\|$	$\|C\|$	$\|R\|$	Lp gap(%)	time(s)
5	10	85	0	0
5	20	75	12	0
5	30	65	16	1
5	40	55	98	7
5	50	45	99	9
5	60	35	91	19
5	70	25	90	27
5	80	15	90	138
5	90	5	90	184
4	91	5	52	37
6	89	5	99	567
7	88	5	97	361

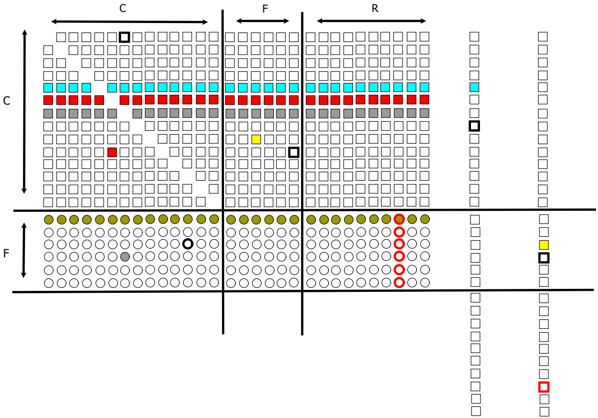
New valid inequalities for the existing formulations

Since there are not capacity constraints it holds that $z_i = x_{ii}$ for all $i \in C$, i.e., if a customer accommodates a re-located facility, the cheapest re-allocation for this customer is to stay at the same place. Moreover, any feasible solution of (P1) satisfies

$$x_{it} + \sum_{v \in V : v \neq t} x_{tv} \leq 1 \qquad \forall i, t \in C$$

$$y_{ji} + \sum_{v \in V : v \neq i} x_{iv} \leq 1 \qquad \forall j \in F, i \in C$$

New facets for the existing formulations



Defining $Z_v = 1 - z_v$, (P1) can be viewed as a set packing problem and the only new clique facet constraint is

$$\sum_{t \in F} y_{tv} + \sum_{t \in V : t \neq v} x_{vt} \leq 1 \quad \forall v \in C.$$

More valid inequalities for the existing formulations

Moreover, the following inequalities are valid inequalities of (P1)

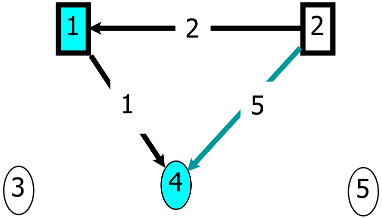
$$2x_{i_1 i_2} + x_{i_2 v_1} + x_{i_1 v_2} + 2y_{ji_2} + y_{jv_3} \leq 3$$

$$x_{i_{2a+1} i_1} + \sum_{t=1}^{2a} x_{i_t i_{t+1}} \leq a$$

$$\sum_{t=1}^{2a+1} (x_{ii_t} + x_{i_t v}) + (a-1)Z_v \leq a$$

$$\sum_{t=1}^{2a+1} (y_{ii_t} + x_{i_t v}) + (a-1)Z_v \leq a$$

New properties for Euclidean costs



If costs d_{jv} and d_{iv} for all $j \in F$, $v \in V$ and $i \in C$ satisfy the triangle inequality, optimal solutions of (P1) will verify that $z_j = y_{jj}$ for all $j \in F$. If costs do not satisfy the triangle inequality, it could happen that re-locate a facility at the location of another facility would be cheaper than keep the facility at the same location. If $F = \{1, 2\}$, $C = \{3, 4, 5\}$ and $V \setminus (F \cup C) = \emptyset$ and in the optimal solution the facilities are installed at nodes 1 and 4, then it is cheaper to move facility in node 1 than to keep facility in node 1.

New properties for Euclidean costs

If costs are euclidean and thus $z_j = y_{jj}$ for all $j \in F$, then a subset of variables can be fixed to the value of zero:

$$y_{jv} = 0 \qquad \forall j, v \in F : j \neq v.$$

And there is a new clique constraint

$$y_{jv} + x_{sj} + x_{vs} \leq 1 \qquad s, v \in C, j \in F$$

New formulation

$$w_{ijv} = \begin{cases} 1 & \text{if customer } i \text{ is served by falicity } j \text{ in location } v, \\ 0 & \text{otherwise} \end{cases}$$

$$
\begin{aligned}
\text{(P2) min} \quad & \sum_{i \in C} \sum_{j \in F} \sum_{v \in V} d_{iv} w_{ijv} + \sum_{j \in F} u_j \\
\text{s.t.} \quad & \sum_{j \in F} \sum_{v \in V} w_{ijv} = 1 && \forall i \in C \\
& w_{i_1 j v_1} + w_{i_2 j v_2} \leq 1 && \forall i_1, i_2, j, v_2 \neq v_1 \\
& \sum_{v \in V} d_{jv} w_{ijv} \leq u_j && \forall i \in C, j \in F \\
& w_{ijv} \in \{0, 1\} && \forall i \in C, j \in F, v \in V \\
& u_j \geq 0 && \forall j \in F
\end{aligned}
$$

New valid inequalities for the new formulation

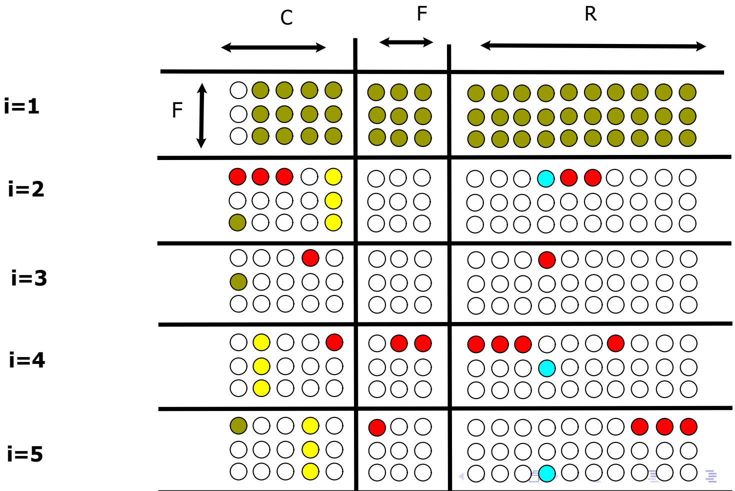
Although (P2) gives the solution to the MFLP, there are some properties of the problem that can be added to (P2) in terms of valid inequalities. First of all, if customer i_1 is assigned to location $v \neq i_1$ (i.e. $w_{i_1 j_1 v} = 1$ for some j_1), then no customer will be re-allocated to i_1 :

$$w_{i_1 j_1 v} + w_{i_2 j_2 i_1} \leq 1 \quad \forall i_1, i_2 \in C, j_1, j_2 \in F, v \neq i_1 \in V.$$

On the other hand, if a customer i_1 is re-allocated at location v which is supplied by facility j_1, then all the customers (i_2) re-allocated to location v will be supplied by facility j_1 :

$$w_{i_1 j_1 v} + w_{i_2 j_2 v} \leq 1 \quad \forall i_1, i_2 \in C : i_1 \neq i_2, j_1, j_2 \in F : j_1 \neq j_2, v \in V.$$

The subset of set packing constraints can be improved.



New valid inequalities for the new formulation

The following inequalities

$$\sum_{v \in V} w_{f(v)jv} \leq 1 \quad \forall j \in F, f : V \to C$$

$$\sum_{j \in F} \sum_{v \in V : v \neq i} w_{ijv} + \sum_{j \in F} w_{g(j)ji} \leq 1 \quad \forall i \in C, g : F \to C$$

$$\sum_{j \in F} (w_{i_1 j i_2} + w_{i_2 j i_3} + w_{i_3 j i_1}) \leq 1 \quad \forall i_1, i_2, i_3 \in C : PWD$$

$$w_{i_1 j_1 i_2} + w_{i_2 j_2 i_3} + w_{i_3 j_1 i_4} + w_{i_4 j_2 i_1} + w_{i_1 j_2 i_4} + w_{i_3 j_2 i_2} + \sum_{j \neq j_2} (w_{i_1 j i_3} + w_{i_3 j i_1}) \leq 1$$

are clique facets of the associated set packing polyhedron.

Euclidean costs in (P2)

If costs are proportional to distances, a facility will never move if the location where it is allocates customers. Consequently,

$$w_{ij_1j_2} = 0 \quad \forall i \in C, j_1, j_2 \in F.$$

In the intersection graph all the nodes with columns in F at any box will be removed. In total, $|C| \times |F|^2$ nodes will be removed.

Conclusion

- A new formulation has been proposed.
- A branch and cut algorithm will illustrate the performance of all the new properties.