A stochastic multi-period covering model

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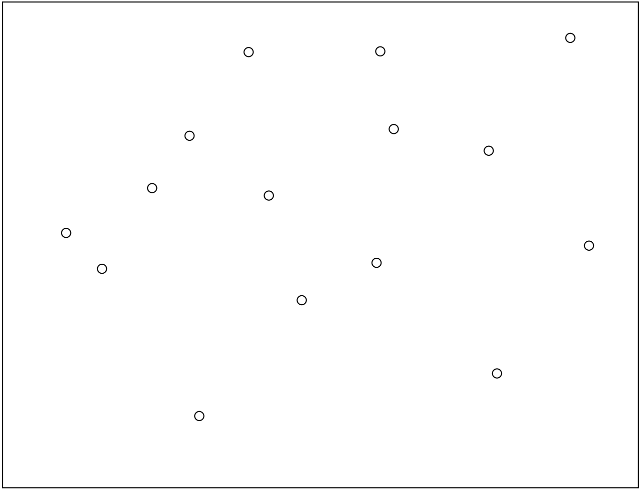
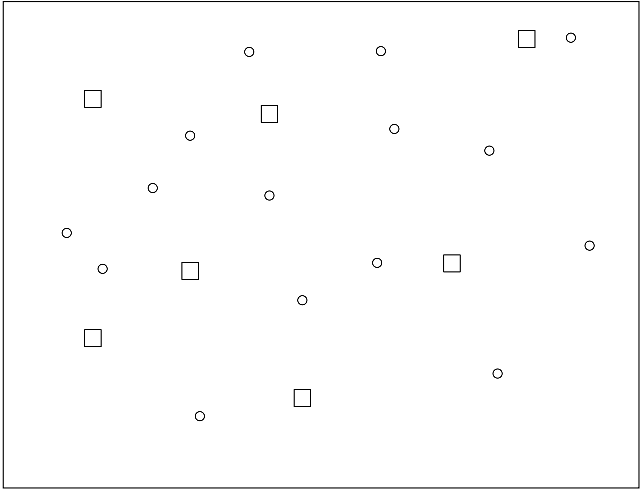
Outline

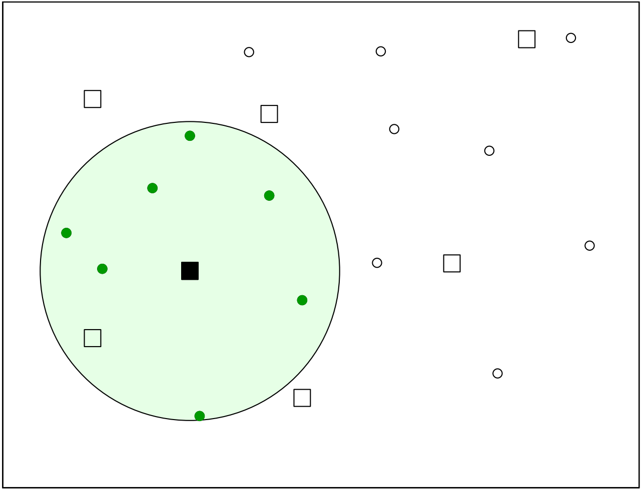
Introduction: Discrete covering location problems



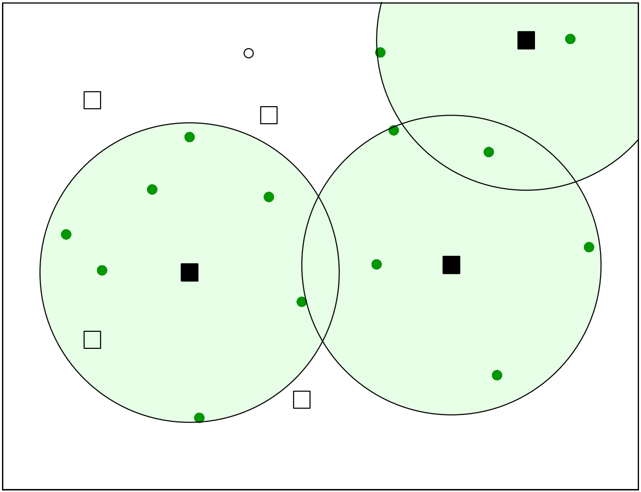
Introduction: Discrete covering location problems



Introduction: Discrete covering location problems



Introduction: Discrete covering location problems



- Set covering location problem (SCP)

  C. Toregas, A. Swain, C. ReVelle, and L. Bergman.
  The location of emergency service facilities.
  Operations Research, 19:1363–1373, 1971.

- Maximal covering location problem (MCLP)

  R. Church and C. ReVelle.
  The maximal covering location problem.
  Papers of the Regional Science Association, 32(1):101–118, 1974.

A general covering location model

S. García and A. Marín.
Covering location problems.
In G. Laporte, S. Nickel, and F. Saldanha da Gama, editors, *Location Science*, chapter 5, pages 93–113. Springer, 2015.

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Notation:

$$
\begin{aligned}
I &= \text{Set of potential locations for facilities,}\\
J &= \text{set of demand points,}\\
f_i &= \text{operating cost in location } i,\\
a_{ij} &= \text{binary parameter indicating whether } i \text{ covers } j \text{ or not,}\\
e_i &= \text{maximum number of facilities that can be operating in } i \in I,\\
b_j &= \text{coverage threshold of } j,\\
g_{jk} &= \text{ negative cost for } j \text{ if it is served by at least } b_j + k \text{ facilities,}\\
& \quad (g_{j1} \leq g_{j2} \leq \ldots \leq g_{j,|K|})
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$$

$$\underbrace{\underbrace{\blacksquare \quad \blacksquare \cdots \blacksquare}_{b_j} \quad \blacksquare \quad \blacksquare}_{b_j + 2 \to g_{j1} + g_{j2}} \cdots \square \quad \square$$

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A general covering location model

$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_i y_i + \sum_{j \in J} \sum_{k \in K} g_{jk} w_{jk}, & \\
\text{s.t.} \quad & \sum_{i \in I} y_i \leq p, & \\
& \sum_{i \in I} a_{ij} y_i = b_j + \sum_{k \in K} w_{jk}, & j \in J, \\
& y_i \in \{0, \ldots, e_i\}, & i \in I, \\
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\end{aligned}
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A general covering location model

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\begin{aligned}
\min \quad & \sum_{i \in I} 1 y_i + \sum_{j \in J} \sum_{k \in K} 0 w_{jk}, & \\
\text{s.t.} \quad & \sum_{i \in I} y_i \leq |I|, & \\
& \sum_{i \in I} a_{ij} y_i = 1 + \sum_{k \in K} w_{jk}, & j \in J, \\
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\min \quad & \sum_{i \in I} 0 y_i + \sum_{j \in J} (-1) w_{j1} + \sum_{j \in J} \sum_{k=2}^{|K|} 0 w_{jk}, & \\
\text{s.t.} \quad & \sum_{i \in I} y_i \leq p, & \\
& \sum_{i \in I} a_{ij} y_i = 0 + \sum_{k \in K} w_{jk}, & j \in J, \\
& y_i \in \{0, 1\}, & i \in I, \\
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The proposed model

Two relevant features:

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- Multiperiod problem in a finite planning horizon

G. Gunawardane.
Dynamic versions of set covering type public facility location problems.
European Journal of Operational Research, 10(2):190–195, 1982.

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- Multiperiod problem in a finite planning horizon

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  Dynamic versions of set covering type public facility location problems.
  European Journal of Operational Research, 10(2):190–195, 1982.

- Uncertainty

  A. K. F. Vatsa and S. Jayaswal.
  A new formulation and Benders decomposition for the multi-period maximal covering facility location problem with server uncertainty.
  European Journal of Operational Research, 251:404–418, 2016.

Notation

Sets:

$\mathcal{T} = \{1, \ldots, T\}$,	set of periods in the planning horizon.
$I = \{1, \ldots, m\}$,	set of potential location for the facilities.
$J = \{1, \ldots, n\}$,	set of demand points.
$\mathcal{S} = \{1, \ldots, S\}$,	set of scenarios.

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$\mathcal{S} = \{1, \ldots, S\}$,	set of scenarios.

Parameters:

o_{it},	cost of opening a facility at $i \in I$ at the beginning of $t \in \mathcal{T}$.
c_{it},	cost of closing a facility at $i \in I$ at the end of $t \in \mathcal{T} \setminus \{T\}$.
f_{it},	cost of operating a facility at $i \in I$ during $t \in \mathcal{T}$.
e_i,	maximum number of facilities that can be operating in $i \in I$.
p_t,	maximum number of facilities that can be operating in $t \in \mathcal{T}$.
$\bar{y}_{i0}$,	number of facilities that are open at $i \in I$ before the beginning of the planning horizon.

Notation

Parameters depending on the scenario:

π_s, probability that scenario $s \in \mathcal{S}$ occurs. ($\pi_s > 0$ for $s \in \mathcal{S}$ and $\sum_{s\in\mathcal{S}} \pi_s = 1$).

a^s_{ijt}, (binary) parameter indicating whether $i \in I$ can cover $j \in J$ in $t \in \mathcal{T}$ under $s \in \mathcal{S}$.

b^s_{jt}, minimum number of facilities requested to cover $j \in J$ in $t \in \mathcal{T}$ under $s \in \mathcal{S}$.

g^s_{jkt}, negative cost for covering $j \in J$ with a surplus of at least k facilities in $t \in \mathcal{T}$ under $s \in \mathcal{S}$. ($g^s_{j1t} \leq g^s_{j2t} \leq \ldots \leq g^s_{j,|K^s_{jt}|t}$)

h^s_{jkt}, penalty for a shortage of at least k facilities in the coverage of $j \in J$ in $t \in \mathcal{T}$ under $s \in \mathcal{S}$. ($h^s_{j1t} \leq h^s_{j2t} \leq \ldots \leq h^s_{j,|K^{s\prime}_{jt}|t}$)

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$$\underbrace{\underbrace{\blacksquare \cdots \blacksquare \cdots \blacksquare \quad \blacksquare \quad \blacksquare}_{b^s_{jt}} \cdots \blacksquare}_{b^s_{jt} + k \to \sum_{k'=1}^{k} g^s_{jk't}} \qquad \square$$

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$$\underbrace{\underbrace{\blacksquare \cdots \blacksquare \cdots \blacksquare \quad \blacksquare}_{b^s_{jt}-1 \to h^s_{j1t}} \quad \square}_{b^s_{jt}} \cdots \square \quad \square$$

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$$\underbrace{\underbrace{\blacksquare \cdots \blacksquare}_{} \cdots \square \quad \square \quad \square}_{b^s_{jt}} \cdots \square \quad \square$$

$$b^s_{jt} - k \rightarrow \sum_{k'=1}^{k} h^s_{jk't}$$

Variables

$$z_{it} = \text{N. of facilities opened in } i \in I \text{ at the beginning of } t \in \mathcal{T}.$$

$$z'_{it} = \text{N. of facilities closed in } i \in I \text{ at the end of } t \in \mathcal{T} \setminus \{T\}.$$

$$y_{it} = \text{N. of facilities operating in } i \in I \text{ and } t \in \mathcal{T}.$$

$$w^s_{jkt} = \begin{cases} 1, & \text{if demand point } j \text{ is covered by at least } b^s_{jt} + k \\ & \text{facilities in period } t \text{ and scenario } s, \\ 0, & \text{otherwise,} \end{cases}$$

with $j \in J,\ k \in K^s_{jt},\ t \in \mathcal{T},\ s \in \mathcal{S}$.

$$v^s_{jkt} = \begin{cases} 1, & \text{if demand point } j \text{ is covered by at most } b^s_{jt} - k \\ & \text{facilities in period } t \text{ and scenario } s, \\ 0, & \text{otherwise,} \end{cases}$$

with $j \in J,\ k \in K'^s_{jt},\ t \in \mathcal{T},\ s \in \mathcal{S}$.

Where $K^s_{jt} = \{1, \ldots, p_t - b^s_{jt}\}$ and $K'^s_{jt} = \{1, \ldots, b^s_{jt}\}$.

Formulation

Objective function:

$$
\begin{aligned}
\text{O.F.(GSMC)} \quad = \quad & \sum_{i \in I} \sum_{t \in \mathcal{T}} o_{it} z_{it} + \sum_{i \in I} \sum_{t \in \mathcal{T} \setminus \{T\}} c_{it} z'_{it} + \sum_{i \in I} \sum_{t \in \mathcal{T}} f_{it} y_{it} \\
& + \sum_{s \in \mathcal{S}} \pi_s \left[\sum_{t \in \mathcal{T}} \sum_{j \in J} \sum_{k \in K^s_{jt}} g^s_{jkt} w^s_{jkt} + \sum_{t \in \mathcal{T}} \sum_{j \in J} \sum_{k \in K'^s_{jt}} h^s_{jkt} v^s_{jkt} \right]
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & \text{O.F.(GSMC)} && \\
\text{s.t.} \quad & \sum_{i \in I} y_{it} \leq p_t, && t \in \mathcal{T}, && (1)\\
& y_{i1} = z_{i1} + \bar{y}_{i0}, && i \in I, && (2)\\
& y_{it} = y_{i,t-1} + z_{it} - z'_{i,t-1}, && i \in I, t \in \mathcal{T} \setminus \{1\}, && (3)\\
& \sum_{i \in I} a^s_{ijt} y_{it} = b^s_{jt} + \sum_{k \in K^s_{jt}} w^s_{jkt} - \sum_{k \in K'^s_{jt}} v^s_{jkt}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, && (4)\\
& w^s_{j1t} + v^s_{j1t} \leq 1, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, && (5)\\
& w^s_{jkt} \leq w^s_{j1t}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, k \in K^s_{jt} \setminus \{1\}, && (6)\\
& v^s_{jkt} \leq v^s_{j1t}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \in K'^s_{jt} \setminus \{1\}, && (7)\\
& y_{it} \in \{0, \ldots, e_i\}, && i \in I, t \in \mathcal{T}, && (8)\\
& z_{it} \in \{0, \ldots, e_i\}, && i \in I, t \in \mathcal{T}, && (9)\\
& z'_{it} \in \{0, \ldots, e_i\}, && i \in I, t \in \mathcal{T} \setminus \{T\}, && (10)\\
& w^s_{jkt} \in \{0, 1\}, && j \in J, k \in K^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}, && (11)\\
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& y_{it} \in \{0, \ldots, e_i\}, & i \in I, t \in \mathcal{T}, \quad (8) \\
& z_{it} \in \{0, \ldots, e_i\}, & i \in I, t \in \mathcal{T}, \quad (9) \\
& z'_{it} \in \{0, \ldots, e_i\}, & i \in I, t \in \mathcal{T} \setminus \{T\}, \quad (10) \\
& w^s_{jkt} \in \{0, 1\}, & j \in J, k \in K^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}, \quad (11) \\
& v^s_{jkt} \in \{0, 1\}, & j \in J, k \in K'^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (12)
\end{array}
$$

$$\begin{aligned}
\min \quad & \text{O.F.(GSMC)} \\
\text{s.t.} \quad & \sum_{i \in I} y_{it} \le p_t, && t \in \mathcal{T}, && (1)\\
& y_{i1} = z_{i1} + \bar{y}_{i0}, && i \in I, && (2)\\
& y_{it} = y_{i,t-1} + z_{it} - z'_{i,t-1}, && i \in I, t \in \mathcal{T} \setminus \{1\}, && (3)\\
& \sum_{i \in I} a^s_{ijt} y_{it} = b^s_{jt} + \sum_{k \in K^s_{jt}} w^s_{jkt} - \sum_{k \in K'^s_{jt}} v^s_{jkt}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, && (4)\\
& w^s_{j1t} + v^s_{j1t} \le 1, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, && (5)\\
& w^s_{jkt} \le w^s_{j1t}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, k \in K^s_{jt} \setminus \{1\}, && (6)\\
& v^s_{jkt} \le v^s_{j1t}, && j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \in K'^s_{jt} \setminus \{1\}, && (7)\\
& y_{it} \in \{0, \dots, e_i\}, && i \in I, t \in \mathcal{T}, && (8)\\
& z_{it} \in \{0, \dots, e_i\}, && i \in I, t \in \mathcal{T}, && (9)\\
& z'_{it} \in \{0, \dots, e_i\}, && i \in I, t \in \mathcal{T} \setminus \{T\}, && (10)\\
& w^s_{jkt} \in \{0, 1\}, && j \in J, k \in K^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}, && (11)\\
& v^s_{jkt} \in \{0, 1\}, && j \in J, k \in K'^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}. && (12)
\end{aligned}$$

$$
\begin{array}{lll}
\min & \text{O.F.(GSMC)} & \\
\text{s.t.} & \displaystyle\sum_{i\in I} y_{it} \leq p_t, & t \in \mathcal{T}, \quad (1)\\
& y_{i1} = z_{i1} + \bar{y}_{i0}, & i \in I, \quad (2)\\
& y_{it} = y_{i,t-1} + z_{it} - z'_{i,t-1}, & i \in I, t \in \mathcal{T}\setminus\{1\}, \quad (3)\\
& \displaystyle\sum_{i\in I} a^s_{ijt} y_{it} = b^s_{jt} + \sum_{k\in K^s_{jt}} w^s_{jkt} - \sum_{k\in K'^s_{jt}} v^s_{jkt}, & j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \quad (4)\\
& w^s_{j1t} + v^s_{j1t} \leq 1, & j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \quad (5)\\
& w^s_{jkt} \leq w^s_{j1t}, & j \in J, t \in \mathcal{T}, s \in \mathcal{S}, k \in K^s_{jt}\setminus\{1\}, \quad (6)\\
& v^s_{jkt} \leq v^s_{j1t}, & j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \in K'^s_{jt}\setminus\{1\}, \quad (7)\\
& y_{it} \in \{0,\ldots,e_i\}, & i \in I, t \in \mathcal{T}, \quad (8)\\
& z_{it} \in \{0,\ldots,e_i\}, & i \in I, t \in \mathcal{T}, \quad (9)\\
& z'_{it} \in \{0,\ldots,e_i\}, & i \in I, t \in \mathcal{T}\setminus\{T\}, \quad (10)\\
& w^s_{jkt} \in \{0,1\}, & j \in J, k \in K^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}, \quad (11)\\
& v^s_{jkt} \in \{0,1\}, & j \in J, k \in K'^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}. \quad (12)
\end{array}
$$

Removing y-variables (GSMC')

$$y_{it} = \bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau}, \qquad i \in I,\ t \in \mathcal{T}.$$

Objective function:

$$\begin{aligned}
\text{O.F.(GSMC')} \quad = \quad & \sum_{i\in I}\sum_{t\in\mathcal{T}} \left(o_{it} + \sum_{\tau=t}^{T} f_{i\tau} \right) z_{it} \\
& + \sum_{i\in I}\sum_{t\in\mathcal{T}\setminus\{T\}} \left(c_{it} - \sum_{\tau=t+1}^{T} f_{i\tau} \right) z'_{it} + \sum_{i\in I}\sum_{t\in\mathcal{T}} f_{it}\bar{y}_{i0} \\
& + \sum_{s\in\mathcal{S}} \pi_s \left(\sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K_{jt}^{s}} g_{jkt}^{s} w_{jkt}^{s} + \sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K_{jt}^{\prime s}} h_{jkt}^{s} v_{jkt}^{s} \right)
\end{aligned}$$

$$\min \quad \text{O.F.(GSMC')}$$

$$\text{s.t.} \quad (5)-(7),(9)-(12),$$

$$\sum_{i\in I}\left(\bar{y}_{i0}+\sum_{\tau=1}^{t} z_{i\tau}-\sum_{\tau=1}^{t-1} z'_{i\tau}\right) \le p_t, \qquad t\in\mathcal{T}, \tag{13}$$

$$\sum_{i\in I} a^s_{ijt}\left(\bar{y}_{i0}+\sum_{\tau=1}^{t} z_{i\tau}-\sum_{\tau=1}^{t-1} z'_{i\tau}\right) = \\ = b^s_{jt}+\sum_{k\in K^s_{jt}} w^s_{jkt}-\sum_{k\in K'^s_{jt}} v^s_{jkt}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \tag{14}$$

$$\bar{y}_{i0}+\sum_{\tau=1}^{t} z_{i\tau}-\sum_{\tau=1}^{t-1} z'_{i\tau} \le e_i, \qquad i\in I,\ t\in\mathcal{T}, \tag{15}$$

$$\bar{y}_{i0}+\sum_{\tau=1}^{t} z_{i\tau}-\sum_{\tau=1}^{t-1} z'_{i\tau} \ge 0, \qquad i\in I,\ t\in\mathcal{T}. \tag{16}$$

Lagrangian relaxation based procedure

Relaxation of constraints:

$$\sum_{i\in I} a_{ijt}^s \left(\bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau}\right) = b_{jt}^s + \sum_{k\in K_{jt}^s} w_{jkt}^s - \sum_{k\in K_{jt}'^s} v_{jkt}^s, \; j\in J, t\in\mathcal{T}, s\in\mathcal{S}.$$

Objective function:

$$\sum_{i\in I}\sum_{t\in\mathcal{T}} \left(o_{it} + \sum_{\tau=t}^{T} f_{i\tau}\right) z_{it} + \sum_{i\in I}\sum_{t\in\mathcal{T}\setminus\{T\}} \left(c_{it} - \sum_{\tau=t+1}^{T} f_{i\tau}\right) z'_{it} + \sum_{i\in I}\sum_{t\in\mathcal{T}} f_{it}\bar{y}_{i0}$$

$$+\sum_{s\in\mathcal{S}} \pi_s \left(\sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K_{jt}^s} g_{jkt}^s w_{jkt}^s + \sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K_{jt}'^s} h_{jkt}^s v_{jkt}^s\right)$$

$$+\sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \alpha_{jt}^s \left(\sum_{i\in I} a_{ijt}^s \left(\bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau}\right)\right)$$

$$+\sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \alpha_{jt}^s \left(-b_{jt}^s - \sum_{k\in K_{jt}^s} w_{jkt}^s + \sum_{k\in K_{jt}'^s} v_{jkt}^s\right)$$

Constraints:

$$\sum_{i\in I}\left(\bar{y}_{i0}\sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau}\right) \le p_t, \quad t \in \mathcal{T},$$

$$\bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau} \le e_i, \quad i \in I,\ t \in \mathcal{T},$$

$$\bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau} - \sum_{\tau=1}^{t-1} z'_{i\tau} \ge 0, \quad i \in I,\ t \in \mathcal{T}.$$

$$z_{it} \in \{0,\dots,e_i\}, \quad i \in I, t \in \mathcal{T},$$

$$z'_{it} \in \{0,\dots,e_i\}, \quad i \in I, t \in \mathcal{T}\setminus\{T\},$$

$$w^s_{j1t} + v^s_{j1t} \le 1, \quad j \in J, t \in \mathcal{T}, s \in \mathcal{S},$$

$$w^s_{jkt} \le w^s_{j1t}, \quad j \in J, t \in \mathcal{T}, s \in \mathcal{S}, k \in K^s_{jt}\setminus\{1\},$$

$$v^s_{jkt} \le v^s_{j1t}, \quad j \in J, t \in \mathcal{T}, s \in \mathcal{S}, \in K'^s_{jt}\setminus\{1\},$$

$$w^s_{jkt} \in \{0,1\}, \quad j \in J, k \in K^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S},$$

$$v^s_{jkt} \in \{0,1\}, \quad j \in J, k \in K'^s_{jt}, t \in \mathcal{T}, s \in \mathcal{S}.$$

$$
\begin{aligned}
(\mathrm{LR1}_\alpha)\min & \sum_{i\in I}\sum_{t\in\mathcal{T}}\left[o_{it}+\sum_{\tau=t}^{T}(f_{i\tau}+\sum_{j\in J}\sum_{s\in\mathcal{S}}\alpha_{j\tau}^{s}a_{ij\tau}^{s})\right]z_{it}+ \\
& \sum_{i\in I}\sum_{t\in\mathcal{T}\setminus\{T\}}\left[c_{it}-\sum_{\tau=t+1}^{T}(f_{i\tau}+\sum_{j\in J}\sum_{s\in\mathcal{S}}\alpha_{j\tau}^{s}a_{ij\tau}^{s})\right]z'_{it}+ \\
& \sum_{i\in I}\sum_{t\in\mathcal{T}}(f_{it}+\sum_{j\in J}\sum_{s\in\mathcal{S}}\alpha_{jt}^{s}a_{ijt}^{s})\bar{y}_{i0} \\
\text{s.t.} & \sum_{i\in I}\left(\bar{y}_{i0}\sum_{\tau=1}^{t}z_{i\tau}-\sum_{\tau=1}^{t-1}z'_{i\tau}\right)\leq p_t, && t\in\mathcal{T}, \\
& \bar{y}_{i0}+\sum_{\tau=1}^{t}z_{i\tau}-\sum_{\tau=1}^{t-1}z'_{i\tau}\leq e_i, && i\in I,\ t\in\mathcal{T}, \\
& \bar{y}_{i0}+\sum_{\tau=1}^{t}z_{i\tau}-\sum_{\tau=1}^{t-1}z'_{i\tau}\geq 0, && i\in I,\ t\in\mathcal{T}, \\
& z_{it}\in\{0,\ldots,e_i\}, && i\in I, t\in\mathcal{T}, \\
& z'_{it}\in\{0,\ldots,e_i\}, && i\in I, t\in\mathcal{T}\setminus\{T\}.
\end{aligned}
$$

$$
\begin{aligned}
(\mathrm{LR2}_\alpha)\ \min & \sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\sum_{k\in K^s_{jt}} (\pi_s g^s_{jkt} - \alpha^s_{jt}) w^s_{jkt} + \\
& \sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}}\sum_{k\in K'^s_{jt}} (\pi_s h^s_{jkt} + \alpha^s_{jt}) v^s_{jkt} \\
& - \sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \alpha^s_{jt} b^s_{jt} \\
\text{s.t } & w^s_{j1t} + v^s_{j1t} \leq 1, && j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \\
& w^s_{jkt} \leq w^s_{j1t}, && j\in J, t\in\mathcal{T}, s\in\mathcal{S}, k\in K^s_{jt}\setminus\{1\}, \\
& v^s_{jkt} \leq v^s_{j1t}, && j\in J, t\in\mathcal{T}, s\in\mathcal{S}, \in K'^s_{jt}\setminus\{1\}, \\
& w^s_{jkt} \in \{0,1\}, && j\in J, k\in K^s_{jt}, t\in\mathcal{T}, s\in\mathcal{S}, \\
& v^s_{jkt} \in \{0,1\}, && j\in J, k\in K'^s_{jt}, t\in\mathcal{T}, s\in\mathcal{S}.
\end{aligned}
$$

Proposition

Subproblems $LR1_\alpha$ and $LR2_\alpha$ have the integrality property.

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- Lagrangian relaxation of the linear relaxation of GSMC'.

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Subproblems $LR1_\alpha$ and $LR2_\alpha$ have the integrality property.

- Lagrangian relaxation of the linear relaxation of GSMC'.
- Using the solutions of $LR1_\alpha$ allows to obtain a feasible solution for the model.

Deriving feasible solutions

Let $\{\mathbf{z}^*, \mathbf{z}'^*\}$ be an optimal solution to $(\mathrm{LR1}_\alpha)$ and (UB_α):

$$\min \sum_{i\in I}\sum_{t\in\mathcal{T}}\left(o_{it}+\sum_{\tau=t}^{T} f_{i\tau}\right) z^*_{it} + \sum_{i\in I}\sum_{t\in\mathcal{T}\setminus\{T\}}\left(c_{it}-\sum_{\tau=t+1}^{T} f_{i\tau}\right) z'^*_{it}$$

$$+\sum_{i\in I}\sum_{t\in\mathcal{T}} f_{it}\bar{y}_{i0} + \sum_{s\in\mathcal{S}}\pi_s\left(\sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K^s_{jt}} g^s_{jkt}w^s_{jkt} + \sum_{t\in\mathcal{T}}\sum_{j\in J}\sum_{k\in K'^s_{jt}} h^s_{jkt}v^s_{jkt}\right)$$

$$\text{s.t.} \sum_{k\in K^s_{jt}} w^s_{jkt} - \sum_{k\in K'^s_{jt}} v^s_{jkt} = \sum_{i\in I} a^s_{ijt}\left(\bar{y}_{i0}+\sum_{\tau=1}^{t} z^*_{i\tau}-\sum_{\tau=1}^{t-1} z'^*_{i\tau}\right) - b^s_{jt}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S},$$

$$w^s_{j1t}+v^s_{j1t}\le 1,$$

$$w^s_{jkt}\le w^s_{j1t}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S}, k\in K^s_{jt},$$

$$v^s_{jkt}\le v^s_{j1t}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S}, k\in K'^s_{jt},$$

$$w^s_{jkt}\in\{0,1\}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S}, k\in K^s_{jt},$$

$$v^s_{jkt}\in\{0,1\}, \qquad j\in J, t\in\mathcal{T}, s\in\mathcal{S}, k\in K'^s_{jt}.$$

$$
\begin{aligned}
(\mathrm{UB}_{\alpha jts})\ \min\ & \sum_{k \in K_{jt}^{s}} \pi_s g_{jkt}^{s} w_{jkt}^{s} + \sum_{k \in K_{jt}^{\prime s}} \pi_s h_{jkt}^{s} v_{jkt}^{s} & \\
\text{s.t. } & \sum_{k \in K_{jt}^{s}} w_{jkt}^{s} - \sum_{k \in K_{jt}^{\prime s}} v_{jkt}^{s} = \sum_{i \in I} a_{ijt}^{s} \left(\bar{y}_{i0} + \sum_{\tau=1}^{t} z_{i\tau}^{*} - \sum_{\tau=1}^{t-1} z_{i\tau}^{\prime *} \right) - b_{jt}^{s}, & \\
& w_{j1t}^{s} + v_{j1t}^{s} \leq 1, & \\
& w_{jkt}^{s} \leq w_{j1t}^{s}, & k \in K_{jt}^{s}, \\
& v_{jkt}^{s} \leq v_{j1t}^{s}, & k \in K_{jt}^{\prime s}, \\
& w_{jkt}^{s} \in \{0,1\}, & k \in K_{jt}^{s}, \\
& v_{jkt}^{s} \in \{0,1\}, & k \in K_{jt}^{\prime s}.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{V}(\mathrm{UB}_\alpha) &= \sum_{i\in I}\sum_{t\in\mathcal{T}}\left(o_{it}+\sum_{\tau=t}^{T} f_{i\tau}\right) z^*_{it} \\
&+ \sum_{i\in I}\sum_{t\in\mathcal{T}\setminus\{T\}}\left(c_{it}-\sum_{\tau=t+1}^{T} f_{i\tau}\right) z'^*_{it} \\
&+ \sum_{i\in I}\sum_{t\in\mathcal{T}} f_{it}\bar{y}_{i0} + \sum_{j\in J}\sum_{t\in\mathcal{T}}\sum_{s\in\mathcal{S}} \mathcal{V}(\mathrm{UB}_{\alpha jts})
\end{aligned}
$$

Lagrangian relaxation based procedure

Initialization: $\alpha[0]_{jts} = 0$ for $j \in J, t \in \mathcal{T}, s \in \mathcal{S}$.

$\mathrm{LB} = \mathcal{V}(\mathrm{LR1}_{\alpha[0]}) + \mathcal{V}(\mathrm{LR2}_{\alpha[0]}),\ \mathrm{UB} = \mathcal{V}(\mathrm{UB}_{\alpha[0]})$.

Update $\alpha[0]$ to $\alpha[1]$.

Lagrangian relaxation based procedure

Initialization: $\alpha[0]_{jts} = 0$ for $j \in J, t \in \mathcal{T}, s \in \mathcal{S}$.

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Update $\alpha[0]$ to $\alpha[1]$.

General Step: $\mathrm{LB}_\kappa = \mathcal{V}(\mathrm{LR1}_{\alpha[\kappa]}) + \mathcal{V}(\mathrm{LR2}_{\alpha[\kappa]})$.

If $\mathrm{LB}_\kappa > \mathrm{LB}$, then $\mathrm{LB} := \mathrm{LB}_\kappa$.

$\mathrm{UB}_\kappa = \mathcal{V}(\mathrm{UB}_{\alpha[\kappa]})$. If $\mathrm{UB}_\kappa < \mathrm{UB}$, $\mathrm{UB} := \mathrm{UB}_\kappa$.

Update $\alpha[\kappa]$ to $\alpha[\kappa + 1]$.

Lagrangian relaxation based procedure

Initialization:	$\alpha[0]_{jts} = 0$ for $j \in J, t \in \mathcal{T}, s \in \mathcal{S}$.
	$\mathrm{LB} = \mathcal{V}(\mathrm{LR1}_{\alpha[0]}) + \mathcal{V}(\mathrm{LR2}_{\alpha[0]}),\ \mathrm{UB} = \mathcal{V}(\mathrm{UB}_{\alpha[0]})$.
	Update $\alpha[0]$ to $\alpha[1]$.
General Step:	$\mathrm{LB}_{\kappa} = \mathcal{V}(\mathrm{LR1}_{\alpha[\kappa]}) + \mathcal{V}(\mathrm{LR2}_{\alpha[\kappa]})$.
	If $\mathrm{LB}_{\kappa} > \mathrm{LB}$, then $\mathrm{LB} := \mathrm{LB}_{\kappa}$.
	$\mathrm{UB}_{\kappa} = \mathcal{V}(\mathrm{UB}_{\alpha[\kappa]})$. If $\mathrm{UB}_{\kappa} < \mathrm{UB}$, $\mathrm{UB} := \mathrm{UB}_{\kappa}$.
	Update $\alpha[\kappa]$ to $\alpha[\kappa + 1]$.
Stop criterion:	• Small LB-UB gap (0.01%).
	• Number of iterations (500).
	• Small value of $\epsilon[\kappa]$ ($\epsilon[\kappa] < 0.005$).

Computational results

- Intel(R) Core(TM) i7-4790K CPU 32 GB RAM
- C++ using ILOG Concert Technology CPLEX 7.0.
- Generated instances:
  - Demand points and potential facilities in $[0,10] \times [0,50]$.
  - $m, n \in \{30, 50, 100\}$, $T \in \{3, 5, 10\}$, $S \in \{3, 5, 10\}$.
  - Opening/closing/operating costs in $[1, 10]$.
  - Surplus benefits/ shortage costs in $[1, 10]$.
  - Covering capability based on the distance between points, decreasing in the planning horizon and different depending on the scenario.
  - $e_i = 2$, $i \in I$, $b^s_{jt} =$round$(0.3 \times \sum_{i \in I} a^s_{ijt})$, random $p_t \in U\{\max\{1, 0.1|J|\}, 0.3|J|\}$,
  - π_s are random generated.

Formulation results

m/n	T=3/S=3		T=3/S=5		T=5/S=10	
	LP-/BB-Gap	OPT-Time	LP-/BB-Gap	OPT-Time	LP-/BB-Gap	OPT-Time
30	9.15	3.33	7.22	2.66	20.76/0.01	943.88
	13.92	1.30	12.89	14.33	19.17/2.22	>10800
	9.56	1.57	14.74	1.84	12.45/0.01	1242.70
	7.98	3.54	19.50	25.74	37.31/3.7	>10800
	13.99	1.42	14.64	8.38	13.02/1.08	>10800
50	10.67	1320.88	20.19	573.93	11.26/5.6	>10800
	5.06	3.62	4.00	12.07	21.02/11.35	>10800
	4.98	1.79	10.23	1810.31	11.94/6	>10800
	13.36	135.08	8.03	9295.68	4.76/0.01	3188.16
	10.97	1716.52	8.18	372.20	10.62/3.51	>10800
100	5.16/0.19	>10800	19.26/4.15	>10800	23.27/16.49	>10800
	18.26/6.34	>10800	11.54/5.38	>10800	12.17/8.8	>10800
	9.08/3.96	>10800	11.57/6.38	>10800	21.39/13.59	>10800
	11.38/2.04	>10800	11.02/4.06	>10800	13.49/9.37	>10800
	17.83/5.23	>10800	12.00/4.24	>10800	15.54/10.86	>10800

Lagrangian relaxation based procedure

m/n	T=3/S=3			T=3/S=5			T=5/S=10		
	Gap	LAG-T	OPT-T	Gap	LAG-T	OPT-T	Gap	LAG-T	OPT-T
30	0.22	2.40	3.33	0.56	4.27	2.66	0.00	17.00	943.88
	2.01	2.71	1.30	0.00	4.06	14.33	1.57	18.85	>10800
	0.51	2.90	1.57	0.00	4.29	1.84	0.00	16.71	1242.70
	0.43	2.68	3.54	0.37	4.17	25.74	0.25	16.24	>10800
	0.89	2.57	1.42	0.25	4.69	8.38	4.09	15.87	>10800
50	0.92	5.84	1320.88	0.43	9.69	573.93	-0.79	45.71	>10800
	0.63	7.68	3.62	0.00	13.27	12.07	-0.92	46.79	>10800
	0.00	7.57	1.79	0.64	9.11	1810.31	-1.37	54.46	>10800
	0.25	8.41	135.08	1.12	9.82	9295.68	0.19	48.25	3188.16
	0.50	7.48	1716.52	0.32	10.83	372.20	0.66	50.64	>10800
100	0.91	22.35	>10800	0.98	44.95	>10800	-8.71	208.37	>10800
	-0.04	19.33	>10800	-2.02	48.72	>10800	-4.38	249.65	>10800
	-0.71	27.20	>10800	-1.52	53.27	>10800	-2.94	189.48	>10800
	0.04	24.85	>10800	-0.02	45.52	>10800	-4.06	219.87	>10800
	-0.80	24.87	>10800	-0.74	38.27	>10800	-5.28	212.49	>10800

Conclusions and future research

Conclusions

- General covering model
  - Uncertainty
  - muti-periods
- Development of a formulation
- Lagrangian relaxation based procedure

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Future research

- Valid inequalities for the formulation.
- Particularizations of the model.
- Development of alternative formulations.

Thank you for your attention!