INTRODUCTION TO DISTRICTING

Districting in General

Planning task

Group small geographic units (basic areas) into larger cluster (districts or territories) such that some relevant planning criteria are satisfied.

Typical planning criteria:
- Compactness
- Contiguity
- Balance

Classification of Districting Literature

Political districting
- Has attracted the attention of many researchers since the 1960s
- Goals:
  - Prevent gerrymandering
  - Ensure that each vote has the same power

Design of service territories
- Services at fixed locations
  - „Customer comes to the service.“
  - Examples: School districts, districts for social facilities
  - Goals:
    - Short distances, good accessibility
    - Same population or racial balance
- On-site services
  - „Service comes to the customer.“
  - Examples: Sales territories, districts for pickup/delivery operations
  - Goals:
    - Little travel time
    - Same workload or earning opportunities
Motivation

Companies with a field service workforce
- Provide recurring services at customers’ locations
- Examples:
  - Sales force of manufacturers and wholesalers of consumer goods (Fleischmann and Paraschis, 1988; Polacek et al., 2007)
  - Field service technicians of engineering companies (Blakeley et al., 2003)

Importance of service consistency
- Personal consistency: long-term personal relations with customers (Zoltners and Sinha, 2005)
- Temporal consistency: regularity of service visits (cf. Groër et al., 2009)

**DESCRIPTION OF THE MPSTDP-S**

**Customer-specific visiting requirements**

**Week patterns**
- Feasible combinations of visiting weeks
- Rigid week rhythm

**Weekday patterns**
- Feasible combinations of weekdays within visiting weeks

<table>
<thead>
<tr>
<th>Week</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
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</table>

Example: Two service visits per week, but not on consecutive days

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<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week pattern</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

Example: Week rhythm = 4

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**Weekday regularities**
- Strict: same weekday pattern in each visiting week

<table>
<thead>
<tr>
<th>Weekday</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
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<tbody>
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<td>4</td>
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</table>

Example: Visits always on Tuesday and Friday
Customer-specific visiting requirements

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed

Example: Visits always on Tuesday and Friday except for the third visiting week

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed
- No regularity requirements: No restrictions

Example: Different weekday pattern in each visiting week

Week patterns
- Feasible combinations of visiting weeks
- Rigid week rhythm

Weekday patterns
- Feasible combinations of weekdays within visiting weeks

Weekday regularities
- Strict: same weekday pattern in each visiting week
- Partial: Pre-specified number of deviations allowed
- No regularity requirements: No restrictions

Example: Four visits with different service times

Planning criteria

Geographic compactness
- Compact day clusters
Planning criteria

Geographic compactness
- Compact day clusters
- Compact week clusters

Balance
- Service time evenly distributed across days
- Service time evenly distributed across weeks

Feasibility
- Feasible schedule with respect to all customer-specific visiting requirements

Related Problems

Other multi-period districting problems
- Districting in a setting with a dynamically varying customer base
- Only two papers: Lei et al., 2015, 2016

No consideration of week or weekday patterns

Extensions of the vehicle routing problem
- Route planning across several time periods
- Examples: IRP (Imich et al., 2014), PVRP (Coelho et al., 2014)

Optimization of routing cost instead of compactness

Multi-period scheduling problems
- Scheduling of tasks according to strict rhythms
- Examples: Machine maintenance (Wei and Liu, 1983), logistics (Campbell and Hardin, 2005)

No consideration of geographical aspects


MATHEMATICAL FORMULATION OF THE MPSTDP-S

MIP Formulation of the MPSTDP-S

Decision variables

- Compactness
- Balance
- Feasibility

\[ g_{i\beta} = \begin{cases} 1 & \text{if week pattern } i \in P(b) \\ 0 & \text{otherwise} \end{cases} \]

\[ b_{\beta\gamma}^{w} = \begin{cases} 1 & \text{if weekday pattern } \beta \in Q(b) \\ 0 & \text{otherwise} \end{cases} \]

Auxiliary variables

- Compactness measurement

\[ w_{i\beta} = \begin{cases} 1 & \text{if customer } b \in B \text{ is assigned to week center } i \in B \\ 0 & \text{in week } w \in W \\ \text{otherwise} \end{cases} \]

\[ r_{i\beta} = \begin{cases} 1 & \text{if customer } b \in B \text{ is assigned to day center } i \in B \\ 0 & \text{on day } d \in D \\ \text{otherwise} \end{cases} \]

\[ x_{i\beta} = \begin{cases} 1 & \text{if customer } b \in B \text{ is selected as the week center} \\ 0 & \text{in week } w \in W \\ \text{otherwise} \end{cases} \]

\[ y_{i\beta} = \begin{cases} 1 & \text{if customer } b \in B \text{ is selected as the day center} \\ 0 & \text{on day } d \in D \\ \text{otherwise} \end{cases} \]
**MIP Formulation of the MPSTDP-S**

- Week Day: Link week pattern and weekday pattern selection
- Day: Assignment of customers to day centers
- Day: Day center selection
- Day: Daily balance
- Week Day: + Domain constraints

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**Symmetry**

**Symmetrical solutions**
- Feasible permutations of clusters form symmetrical solutions
- Symmetry exists on the level of day and week clusters

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**Symmetry (cont.)**

**Feasible permutations of week clusters**
- The set of feasible permutations depends on the planning horizon and week rhythms
- A set of feasible permutations may be determined which is valid for all instances of a given horizon
- 4 weeks planning horizon: Symmetry only constrained by biweekly customers

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**SOLUTION APPROACHES FOR THE MPSTDP-S**
Two new solution approaches

Location-allocation
- Fast heuristic
- Covers all planning requirements of the MPSTDP-S
- Is based on model $SCHEDULE_{MIP}$ with variable fixations
- Extends the decomposition idea of Hess et al. (1965) to a multi-period setting

Branch-and-price
- Exact method
- For a special planning scenario of the MPSTDP-S
- Is based on a formulation with a huge number of variables
- Contains new, specially-tailored acceleration techniques


Location-allocation: Flowchart

Start

Initialize centers

Update centers

Solve resulting IP

Terminate?

no

yes

Stop

The Multi-Period Service Territory Design Problem
Institute of Operations Research (IOR)
Discrete Optimization and Logistics
**Integer Program with Fixed Centers**

Attach distances to pattern variables

\[
\min \sum_{i,j} c_{ij} x_{ij} + \sum (1 - \lambda_{ij}) \sum \sum \sum \sum a_{ijkl} y_{ijkl}^{w}
\]

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**Location-allocation: Flowchart**

Start

- Initialize centers

- Update centers

- Solve resulting IP

- Terminate?
  - no
  - yes

Stop

**Branch-and-price: Introductory remarks**

**Considered planning scenario**
- At most one service visit per customer and week
- No customer-specific restrictions of the feasible visiting days
- Identical service times for each service visit of the same customer

→ Highly relevant scenario in practice

**Basic idea of the approach**
- Do not consider individual customers, but feasible week and day clusters as variables/columns in the model.
- Select optimal combination of week and day clusters.
- Work with restricted master problem (RMP), which contains only a subset of all clusters, and generate new clusters only when needed.

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**The Multi-Period Service Territory Design Problem**

- Week pattern selection
- Weekly balance
- Link week pattern and weekday pattern selection
- Daily balance
- Week and weekday pattern variables

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**Locational-allocation: Flowchart**

Start

- Initialize centers

- Update centers

- Solve resulting IP

- Terminate?
  - no
  - yes

Stop

---

**Locational-allocation: Flowchart**

Start

- Initialize centers

- Update centers

- Solve resulting IP

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  - no
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Stop

---

**Locational-allocation: Flowchart**

Start

- Initialize centers

- Update centers

- Solve resulting IP

- Terminate?
  - no
  - yes

Stop
Branch-and-Price: Flowchart

1. Initialization
   - Generate initial set of columns using the location-allocation heuristic
   - Add root node to the set of active nodes

2. Node selection
   - Best-first strategy

3. Select node
   - Solve LP-relaxation of RMP
   - Solve pricing problems

4. More columns?
   - Yes
   - Search for violated cuts
   - Cuts found?
     - Yes
     - Add cuts to RMP
     - More nodes?
       - Yes
       - Branch/update incumbent
       - More nodes?
         - Yes
         - Stop
       - No
       - No
     - No
     - No
   - No
   - No

5. Add columns to RMP

6. Add cuts to RMP

7. Stop
The Multi-Period Service Territory Design Problem

Cutting

- Subset-row inequalities (Jepsen et al., 2008)


Handling week symmetry

Fixed
Prohibited

Add columns to RMP

Add cuts to RMP

More columns?

Search for violated cuts

Cuts found?

Branch/update incumbent

More nodes?

Stop

Add columns to RMP

Add cuts to RMP

More columns?

Search for violated cuts

Cuts found?

Branch/update incumbent

More nodes?

Stop

Check termination

- Optimal solution found if set of active nodes empty
**COMPUTATIONAL RESULTS**

**Evaluation of Location-Allocation Heuristic**

Comparison with PTV xCluster Server version 1.18

- 480 real-world test instances and test instances derived from real-world data
  - On average 115 customers per test instance
  - Planning horizon consists of 16 or 48 weeks with 5 days per week
- Negative values correspond to improvements compared to xCluster.

→ Significant improvement in all relevant measures

**Evaluation of Branch&Price Algorithm**

Symmetry breaking

- 9 real-world test instances
  - Between 25 and 35 customers per test instance
  - Planning horizon consists of 4 weeks with 5 days per week
- 3 tested settings
  - No symmetry reduction (NONE)
  - Fixing a reference customer (FRC)
  - Full symmetry reduction (FULL)
- Time limit of 10 h

→ Running time reduction of over 97% by full symmetry reduction

**Evaluation of Branch&Price Algorithm**

Comparison to MIP solver Gurobi

- 9 real-world test instances
  - Between 25 and 35 customers per test instance
  - Planning horizon consists of 4 weeks with 5 days per week
  - Compact formulation
  - Includes symmetry-breaking constraints
  - Solved with MIP solver Gurobi
  - Time limit of 10 hours

→ Running time reduction of over 98%
Comparison Location-Allocation and Branch&Price

Solution quality
- 16 real-world test instances
- Contain 25 – 55 customers
- Planning horizon consists of 4 weeks with 5 days per week
- Percentage values correspond to the deviation of the location-allocation solution from the optimal solution

<table>
<thead>
<tr>
<th>Deviation of Location-Allocation solution from optimum</th>
<th>Objective function value</th>
<th>Compactness of day clusters</th>
<th>Compactness of week clusters</th>
<th>Travel time within day clusters</th>
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</thead>
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Exemplary day clusters: Mon – Wed

- Monday
- Tuesday
- Wednesday

Exemplary day clusters: Thu – Fri

- Thursday
- Friday

Exemplary week cluster
CONCLUSION

Summary and Outlook

Summary

- We have introduced a highly relevant new problem.
- We have proposed two solution approaches and evaluated their performance:
  - The location-allocation heuristic clearly beats the software product PTV xCluster.
  - The branch-and-price algorithm outperforms the MIP solver Gurobi.
- With the release in December 2016, PTV Group has replaced the previous algorithm in their xCluster Server with an algorithm based on our location-allocation approach.

Outlook

- Integration of additional planning criteria, e.g.
  - Planning of overnight stays
  - Incorporation of travel time approximations

Literature


