The effect of product's short life-cycle on network design

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Outline

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- Trade-off between over and under-capacity affects location.

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- For a given facility: capacity is a variation of the so-called newsboy model;
- For the multi-facility problem, a critical-ratio based heuristic is proposed.

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Decisions are taken in two stages:

1st Facilities and capacities are determined to $\max E\left[Z^{\pi}(S, C, a)\right]$

2nd Once demand is revealed, chose allocation policy a^d such that $\max Z^{\pi}\left(a^d | S, C\right)$

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$$a^d = \operatorname*{argmax}_a \left\{ Z^{\pi}\left(a^d | S, C\right)\right\}, \quad \forall d \in \mathcal{D}$$

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- At the upper levels we have a p-median location and a newsboy capacity problems;
- Capacity costs are convex increasing;
- There always exists C such that $E\left[Z^{\pi}\left(S, C, a\right)\right]$ is maximised.

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- For each candidate facility, solve a newsboy problem and compute expected profit as follows:

Newsboy Algorithm

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1. For each step k, take the node, i, closest to the candidate location, j, and compute the critical ratio:

$$Cr_j(k) = \frac{m_{ij} - m_0}{m_{ij} - m_0 + beta_j(k-1)}$$

where

$$\beta_j(k) = \psi(\delta_{j,k}) - \psi(\delta_{j,k-1})$$

is the marginal capacity cost increase from including i; and

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3. Stop when $Z(c)$ starts decreasing or Cr_j becomes negative.

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- Greedy approach;
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- Open the facility in the location yielding maximal profit.

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7. Repeat steps 1 to 6 until all candidates are tested;
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10. Repeat until all wanted facilities are located.

Illustrative Example

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- Successive experiments locating 1 to 8 facilities;
- Three alternative methods compared:
  - p-Median solution with implied capacities;
  - p-Median solution with ex-post newsboy capacities;
  - Simultaneous location/newsboy capacity solution;

Numerical results: 5 to 8 facilities

Facilities	Method	Total Capacity	Expected Revenue	Capacity Cost	Expected Profit	Sourced Profit	Tot.Exp. Profit
5	p-Median	5359	18500	6744	11756	0	11756
	p-Median+NB	4809	18355	5262	13092	0	13092
	NB	4631	17441	4631	12810	515	13325
6	p-Median	5360	18762	6308	12454	0	12454
	p-Median+NB	5017	18704	5184	13519	0	13519
	NB	4944	18400	4873	13528	0	13528
7	p-Median	5360	18996	6294	12702	0	12702
	p-Median+NB	5398	18931	5464	13467	0	13467
	NB	5217	18779	5109	13670	0	13670
8	p-Median	5360	19206	5640	13565	0	13565
	p-Median+NB	5429	19169	5387	13782	0	13782
	NB	5425	18994	5210	13783	0	13783

Table: Illustrative Example: Summary Results

Graphical representation: 5 facilities case (pMedian)

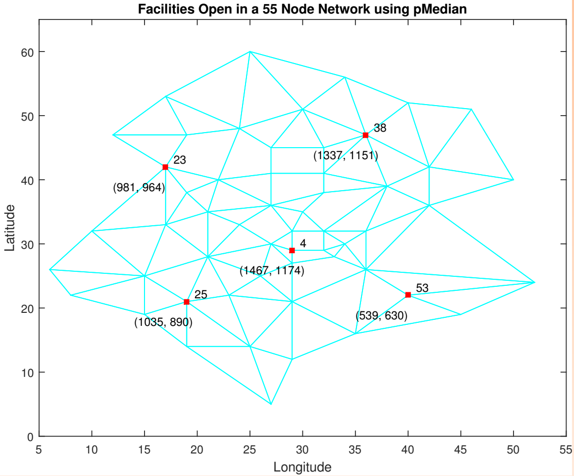

Figure: Open Facilities (Implied capacities, NB capacities)

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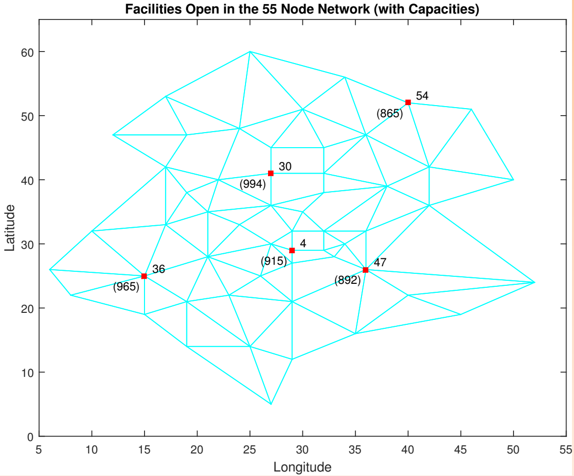
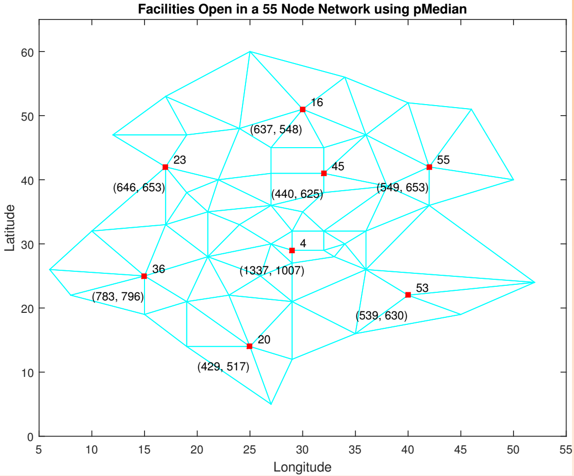


Figure: Open Facilities (Capacities)

Graphical representation: 8 facilities case (pMedian)



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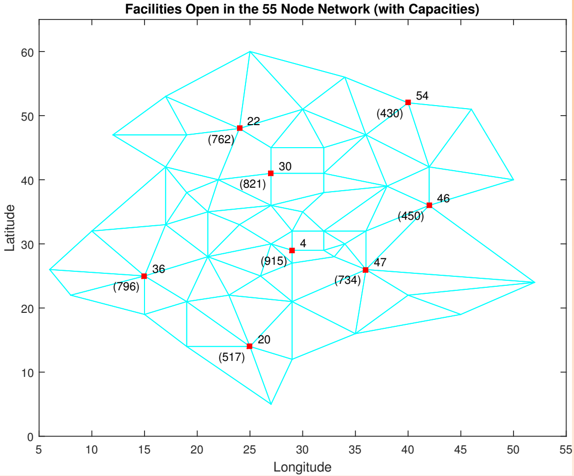

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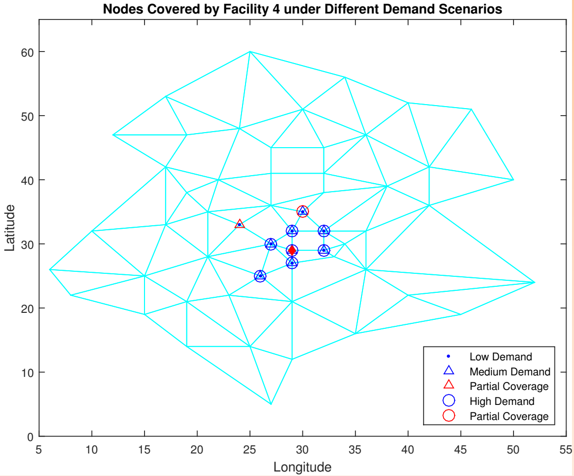

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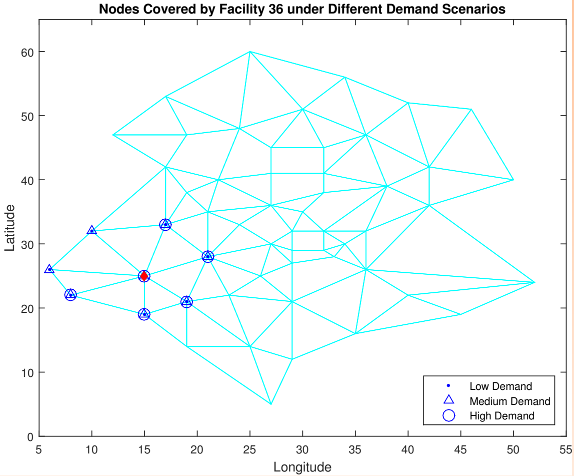

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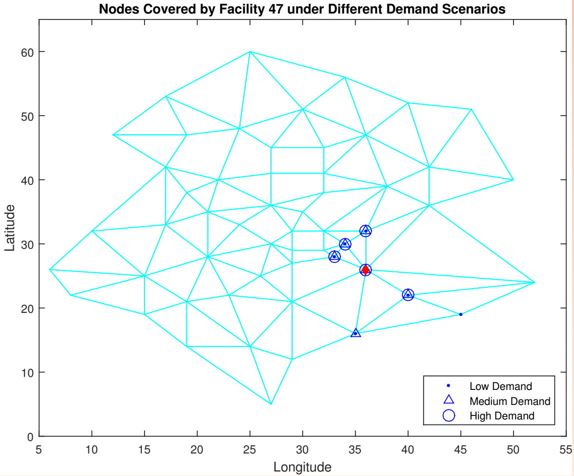

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- The proposed heuristic, by exploiting certain particular features of the problem, is efficient and intuitive;
- To-dos: test in larger (real) networks, analyse different capacity cost configurations (returns to scale).

Thank You

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