A Stochastic Mixed 0-1 Model for the Capacitated Bank Restructuring Problem with Redundant Branches

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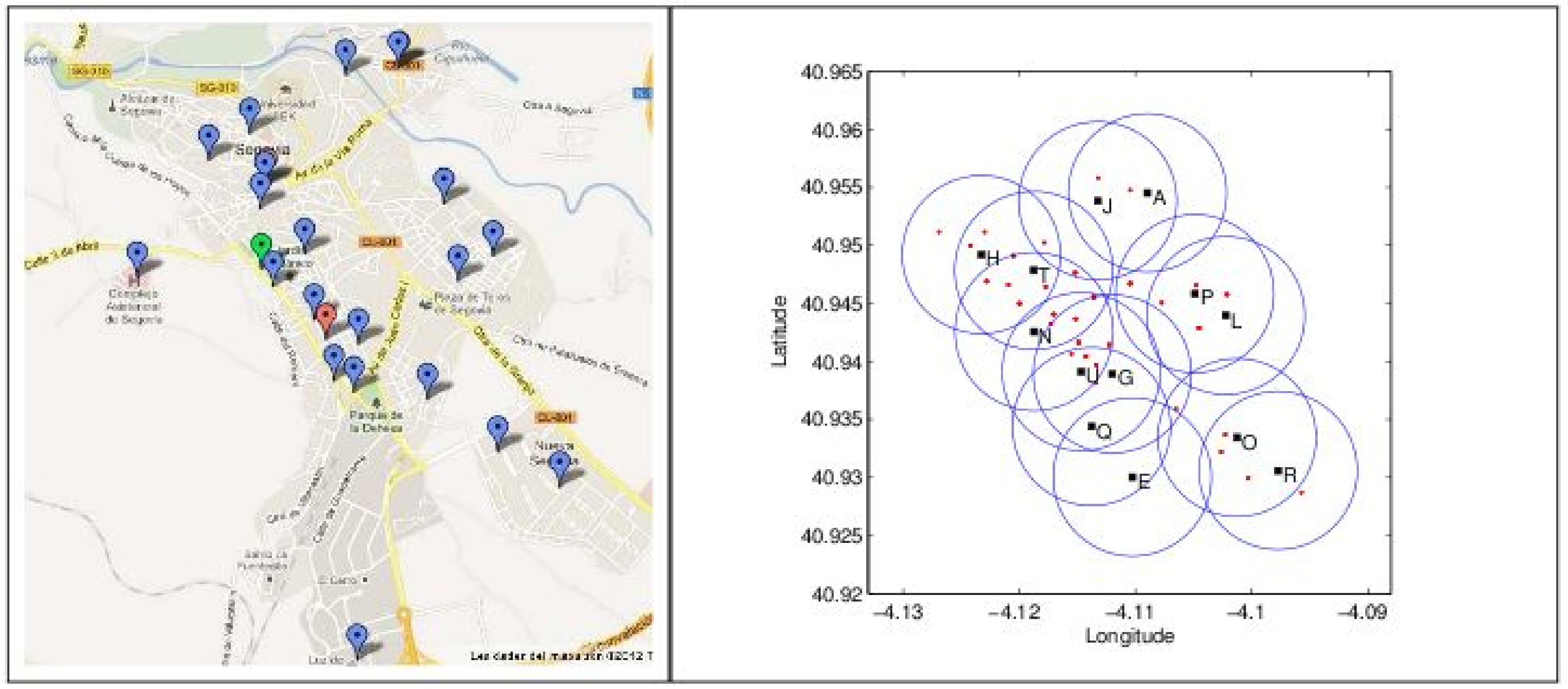
Background

- Merger of the Spanish saving banks opens questions regarding management of the resulting institutions.
- One of the most challenging: closing and resizing of redundant branches in neighbouring areas.
- In the past saving banks open a large number of branches:
  - To capture market share.
  - Market overlaping strategy.
  - Multimarket strategy.
- The merger of two or three small saving banks into one larger institution results in redundant branches.
- Necessary to reduce the number of redundant branches and to resize the remaining ones.

Application: Bankia Network, Segovia, Spain



Problem: find an optimal subset of offices to be kept open out of a collection of possible redundant branches.

Consider

- Minimal distance between branches (*redundancy*);
- Maximal distance from customers (*accessibility*);
- Distance to competitors' branches (*capture*);
- Potential demand (customers, clients);
- Branches' capacity (physical size, virtual service apacity);
- Operation, closing-down and resizing costs;

Methodology

- Design a suitable banking network based on the well know Swain network.
- Develop two alternative versions of the *Capacitated Branch Restructuring Model*: with and without ceding customers.
- Construct two alternative two-stage stochastic mixed integer optimization models.
  1. Closing and restructuring are performed during the first stage. Clientèle reaction is captured in the second stage.
  2. Branch closing is performed during the first stage. Clientèle reacts and reburbishment is performed in the second stage.
- Formulate and solve the *Deterministic Expected Value Version*
- Formulate and solve the *Risk Neutral Strategy*
- (If necessary) formulate and solve *Risk Averse (stochastic dominance constraints) Strategies.*

Model Formulation: Constraints and Variables

Accessibility Constraints

At least one open branch accessible to any demand node,

$$\sum_{i \in \mathcal{I}_j^a} W_i \geq 1, \qquad j \in \mathcal{J}.$$

where

$$\mathcal{I}_j^a = \{i \in \mathcal{I} : d_{ij} \leq d_j\} \quad \forall j \in \mathcal{J}$$

Redundancy Constraints

At most one branch open in a redundancy area,

$$W_i + \sum_{i' \in \mathcal{I}_i^r} W_{i'} \leq 1, \qquad i \in \mathcal{I}.$$

where

$$\mathcal{I}_i^r = \left\{i' \in \mathcal{I} : d_{ii'} \leq d^i\right\} \quad \forall i \in \mathcal{I}$$

Clients Relocation

Version 1 (first stage):

$$B_{ii} + \sum_{i' \in \mathcal{I}_i^a} B_{i'i} = b_i$$

(second stage):

$$L_i^{cl,\omega} \geq \sum_{i' \in \mathcal{I}_i^a, i \neq i'} \alpha_{i'i}^{\omega} B_{i'i}, \quad \forall \omega \in \Omega^i, i \in \mathcal{I}$$

Version2 (second stage):

$$B_{ii}^{\omega} + \sum_{i' \in \mathcal{I}_i^a} B_{i'i}^{\omega} + L_i^{cl,\omega} = b_i, \quad \forall i \in \mathcal{I}$$

$$L_i^{cl,\omega} \geq \sum_{i' \in \mathcal{I}_i^a, i \neq i'} \alpha_{i'i}^{\omega} B_{i'i}^{\omega}, \quad \forall \omega \in \Omega^i, i \in \mathcal{I}$$

Capture Constraints

At least one open branch in a competitor's vicinity,

$$\sum_{i \in \mathcal{I}_k^c} W_i \geq 1, \qquad k \in \mathcal{K}.$$

where

$$\mathcal{I}_k^c = \{i \in \mathcal{I} : d_{ik} \leq v_k\} \quad \forall k \in \mathcal{K}$$

Service Capacity Constraints

- If the virtual capacity of an open branch is exceeded by the relocated clients an expansion is required,

$$B_{ii} + \sum_{i' \in \mathcal{I}} B_{i'i} \leq \overline{b}_i W_i + m_i G_i, \quad i \in \mathcal{I};$$

$$G_i \leq W_i, \quad i \in \mathcal{I}.$$

- In *Version 2*, these are second stage constraints and we will have B_{ii}^{ω}, $B_{i'i}^{\omega}$, and G_i^{ω}, $\forall \omega \in \Omega^i$, correspondingly.

Customer Allocation

Version 1 (first stage):

$$\sum_{i \in \mathcal{I}_j^a} Z_{ji} + C_j = a_j, \quad \forall j \in \mathcal{J}$$

(second stage):

$$L_j^{cu,\omega} \geq \sum_{i \in \mathcal{I}_j^f} \sigma_j^{\omega} Z_{ji}, \quad \forall \omega \in \Omega^j, j \in \mathcal{J}$$

where

$$\mathcal{I}_j^f = \left\{i \in \mathcal{I}_j^a : d_{ij} > d_j^k\right\}$$

and d_j^k is the distance from $j \in \mathcal{J}$ to the closest competitor.

Physical Capacity Constraints

- If the capacity of the branch is exceeded by relocated customers, an expansion will be required,

$$\sum_{j\in\mathcal{J}:i\in\mathcal{I}_j^a} Z_{ji} \le \bar{a}_i W_i + n\cdot T_i \qquad \forall i\in\mathcal{I},\omega\in\Omega.$$

- Upper limit on the size physical capacity expansion,

$$T_i \le e_i R_i, \quad i\in\mathcal{I};$$

$$R_i \le W_i, \quad i\in\mathcal{I}.$$

- In *Version 2*, these are second stage constraints and we will have Z_{ji}^{ω} $\forall\omega\in\Omega^j$ and R_i^{ω}, T_i^{ω} $\forall\omega\in\Omega^i$, correspondingly.

Risk Neutral Strategy

Objective Function (*Version 1*)

The objective is to minimise the expected (discounted) sum the following:

- Operation costs,

$$\sum_{i\in\mathcal{I}} o_i^{cl}\Big[B_{ii} + \sum_{i'\in\mathcal{I},i'\neq i} B_{i'i}\Big] + \sum_{i\in\mathcal{I}} o_i^{cu}\Big[\bar{a}_i + nT_i\Big];$$

- Closing down costs

$$\sum_{i\in\mathcal{I}}\Big[c_i\left(1-W_i\right)\Big];$$

- Capacity expansion and refurbishment costs,

$$\sum_{i\in\mathcal{I}}\Big[g_i G_i + f_i R_i + q_i T_i\Big];$$

- Client relocation cost and customer displacement penalties,

$$\sum_{i\in\mathcal{I}}\sum_{i'\in\mathcal{I}} r_{i'i}B_{i'i} + \sum_{j\in\mathcal{J}}\sum_{i\in\mathcal{I}_j^a} \rho\cdot d_{ji}Z_{ji};$$

- Penalty for ceding customers,

$$\sum_{j\in\mathcal{J}} \ell_j^{cd} C_j;$$

- Expected cost of losing clients and customers,

$$\sum_{i\in\mathcal{I}}\sum_{\omega\in\Omega^i} w^{\omega}\ell_i^{cl}L_i^{cl,\omega} + \sum_{j\in\mathcal{J}}\sum_{\omega\in\Omega^j} w^{\omega}\ell_j^{cu}L_j^{cu,\omega}.$$