The Biobjective Capacitated *m*-Ring Star Problem

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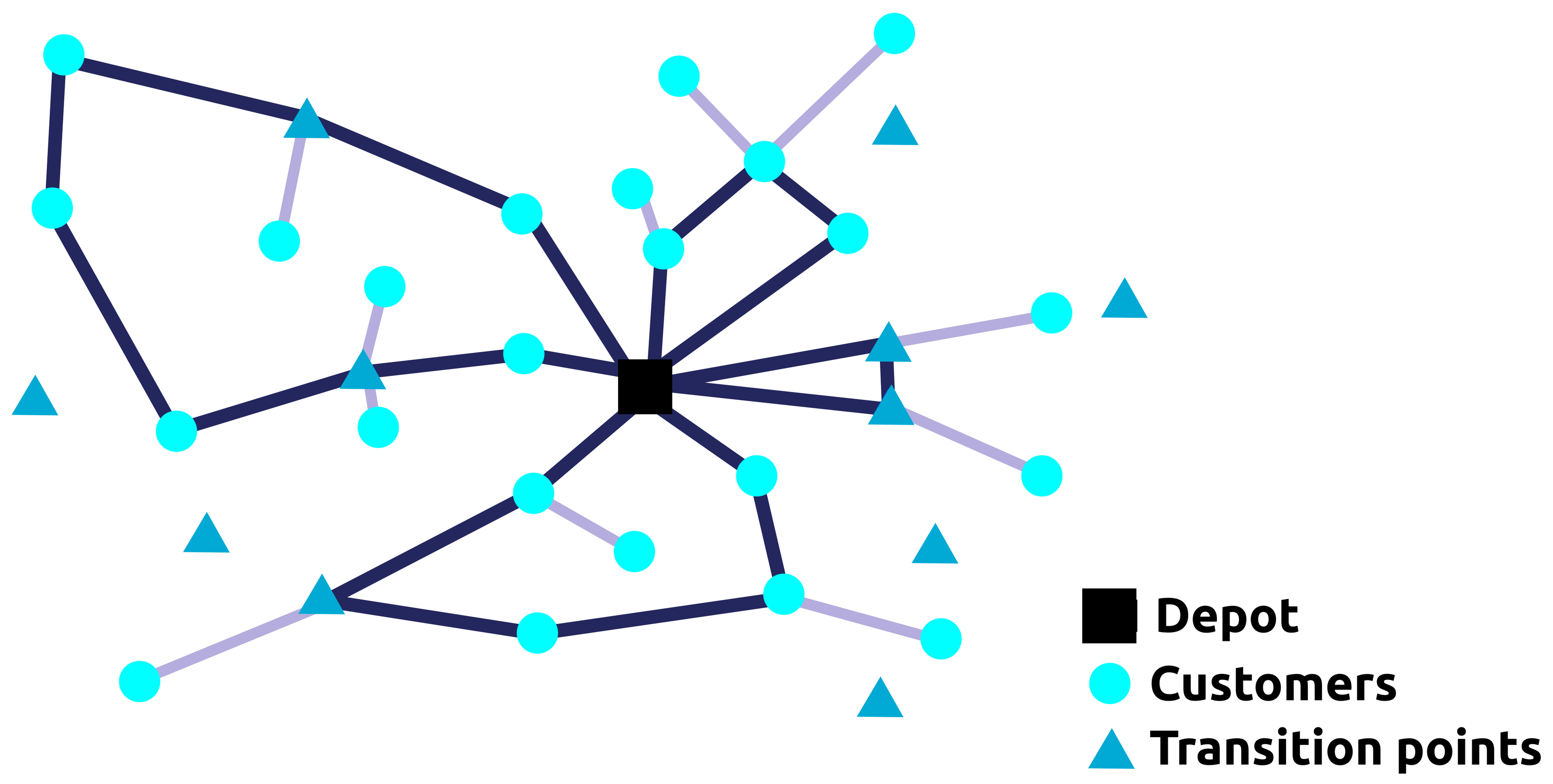
THE BIOBJECTIVE CAPACITATED *m*-RING STAR PROBLEM

Let G be a network wich consists of a depot, some customers and some transition points. An m-ring star is a set of m simple cycles (rings) through a subset of nodes of G satisfying:

- Each ring contains the depot.
- The rings must be node-disjoint (except for the depot).
- The customers not in any ring are directly allocated to nodes in the rings.
- The number of customers visited or allocated to a ring is limited by a given Q.



The goal of the B-CmRSP is to minimize two costs: the **ring cost** and the **allocation cost**, considered individually.
The CmRSP is used in telecommunications network design, because of the reliability/cost relation.

MATHEMATICAL FORMULATION

- $G = (V, E \cup A)$ is a mixed network, where V is the set of nodes, E is the set of edges and A is the set of arcs.
- Edges in E refer to undirected connections and arcs in A refer to directed allocations.
- Costs: $c_{ij}, \forall [i,j] \in E$; $d_{ij}, \forall (i,j) \in A$.
- Let

$$x_{ij} = \begin{cases} 1, & \text{if edge } [i,j] \text{ is on a cycle} \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if node } i \text{ is allocated to node } j \text{ on a cycle} \\ 0, & \text{otherwise} \end{cases}$$

- Mathematical formulation of the biobjective integer program:

$$\min \left(\sum_{[i,j]\in E} c_{ij} x_{ij} \quad , \quad \sum_{(i,j)\in A} d_{ij} y_{ij} \right)$$
$$\text{s.t.} \quad \{x_{ij}, y_{ij}, y_{jj}\} \text{ defines a feasible solution}$$

- A solution X is called efficient if none is at least as good as X for all objectives and strictly better for at least one. The image in the criterion space of the set of efficient solutions is called Pareto front.

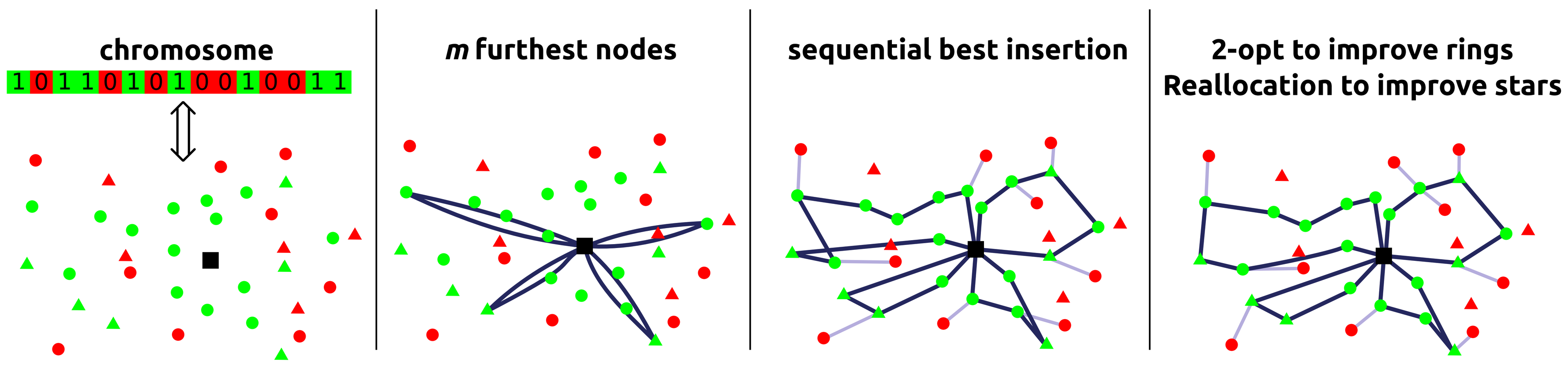
REFERENCES

- R. Baldacci, M. Dell'Amico, J.J. Salazar González. The capacitated m-ring-star problem. Operations Research, 55(6), 1147-1162, 2007.
- E.A. Hoshino, C.C. de Souza. A branch-and-cut-and-price approach for the capacitated m-ring-star problem. Discrete Applied Mathematics, 160(18), 2728-2741, 2012.
- Z. Naji-Azimi, M. Salari, P. Toth. A heuristic procedure for the capacitated m-ring-star problem. European Journal of Operational Research, 207(3), 1227-1234, 2010.
- Z. Naji-Azimi, M. Salari, P. Toth. An integer linear programming based heuristic for the capacitated m-ring-star problem. European Journal of Operational Research, 217(1), 17-25, 2012.

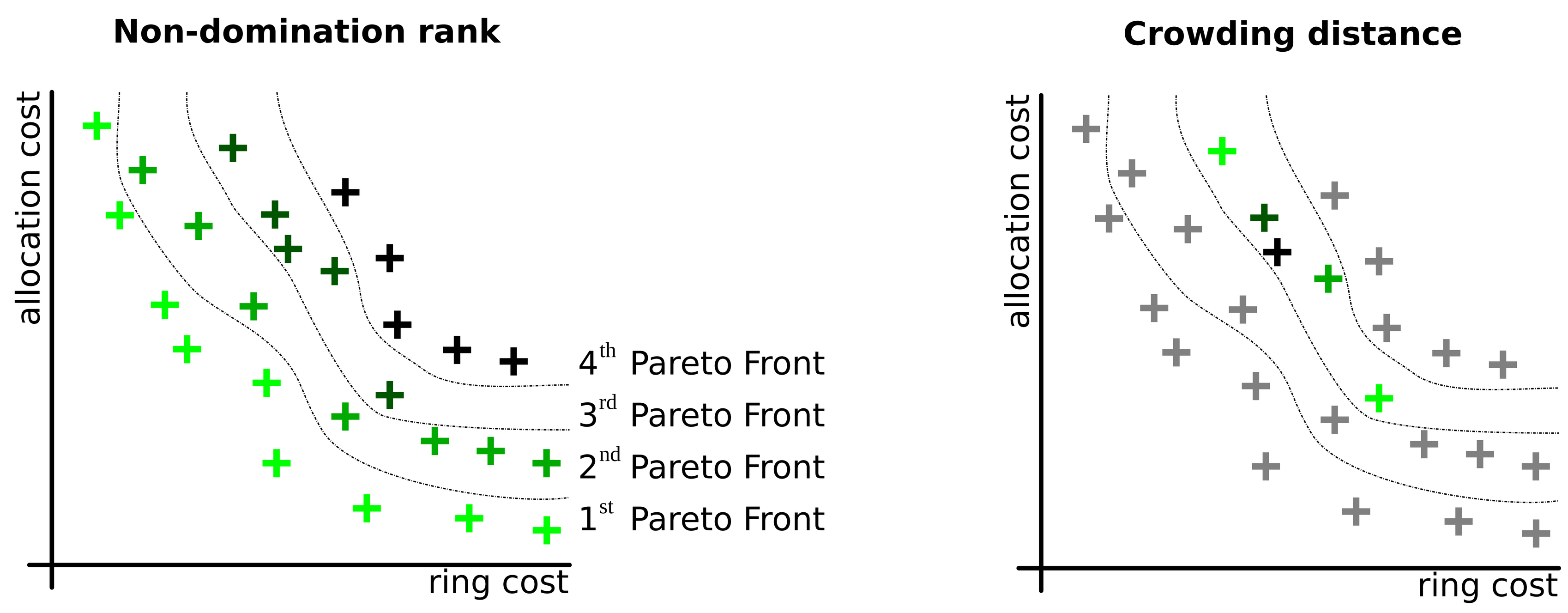
AN EVOLUTIONARY ALGORITHM

An evolutionary algorithm is proposed to aproximate the Pareto front:

- Each chromosome is encoded as a binary $|V|$-string. Each gene indicates if the node either is or is not on a ring.
- For generating each chromosome in the initial population (size N) a random number (at least $Q+1$) of genes selected randomly are set to 1 and the rest of them are set to 0. The gene of node 0 is always 1.
- Crossover: N pairs of chromosomes are randomly selected and uniform crossover is applied. One offspring is generated from each pair.
- Mutation: For each offspring, a random gene is switched with probability 0.5.
- Fitness evaluation: For each chromosome, a feasible solution of the B-RSP is associated:
  - First, a set of m rings is constructed by considering the depot and m ring nodes which are as far as possible one from each other.
  - Second, the remaining nodes randomly browsed are visited or allocated to their best feasible position.
  - Finally, 2-opt is applied to improve the cost of each ring and a reallocation procedure is performed to attempt a better allocation cost, maintaining feasibility.

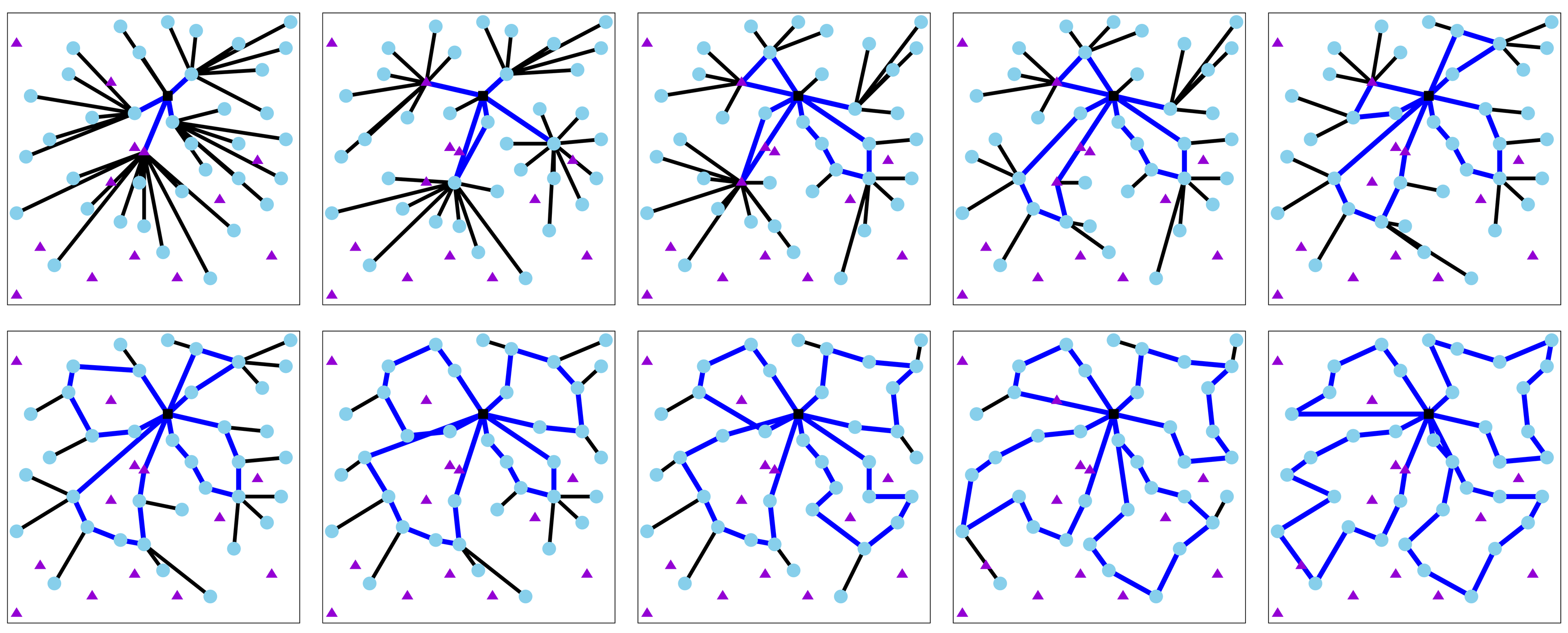


- Survival selection: An elitism selection is performed using general ideas of NSGA-II. The new population is generated by selecting the best N individuals in accordance with the non-domination rank or, in case of a tie, in accordance with the crowding distance.



COMPUTATIONAL RESULTS

- A computational experiment is being carried out on TSPLIB benchmark instances which involve between 26 and 101 nodes. As an example, these are some efficient solutions of the Pareto front approximation for the problem *eil*51 computed with the proposed evolutionary algorithm:



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