The Probabilistic Pickup and Delivery Problem

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Contents

PPDTSP

n customer requests

For each request i an item must be picked up at a location and delivered at another location

Probabilistic case:

Each request has a probability p to exist. The probabilities are

- Homogeneous
- Independent

Vehicles:

- Only one vehicle
- Enough capacity

Find a route for the vehicle that service all the requests with minimum expected length

- The route must start and end at the depot and satisfy the precedence constraint between the origin and destination of each request. If a request does not exist the corresponding locations are skipped

PPDTSP (2)

We assume, for each request i: pickup node i delivery node $n+i$

$G=(V,A)$ complete directed graph

$V=P\cup D\cup\{0\}$ set of nodes

0 depot node
$P=\{1,\dots,n\}$ pickup nodes
$D=\{n+1,\dots,2n\}$ delivery nodes

p probability that a given request really exist

c_{ij} cost (length) of traveling from i to j, $i,j\in V$ (satisfying triangular inequality)

We also denote by $c(i)$ to the *companion* of node i, that is

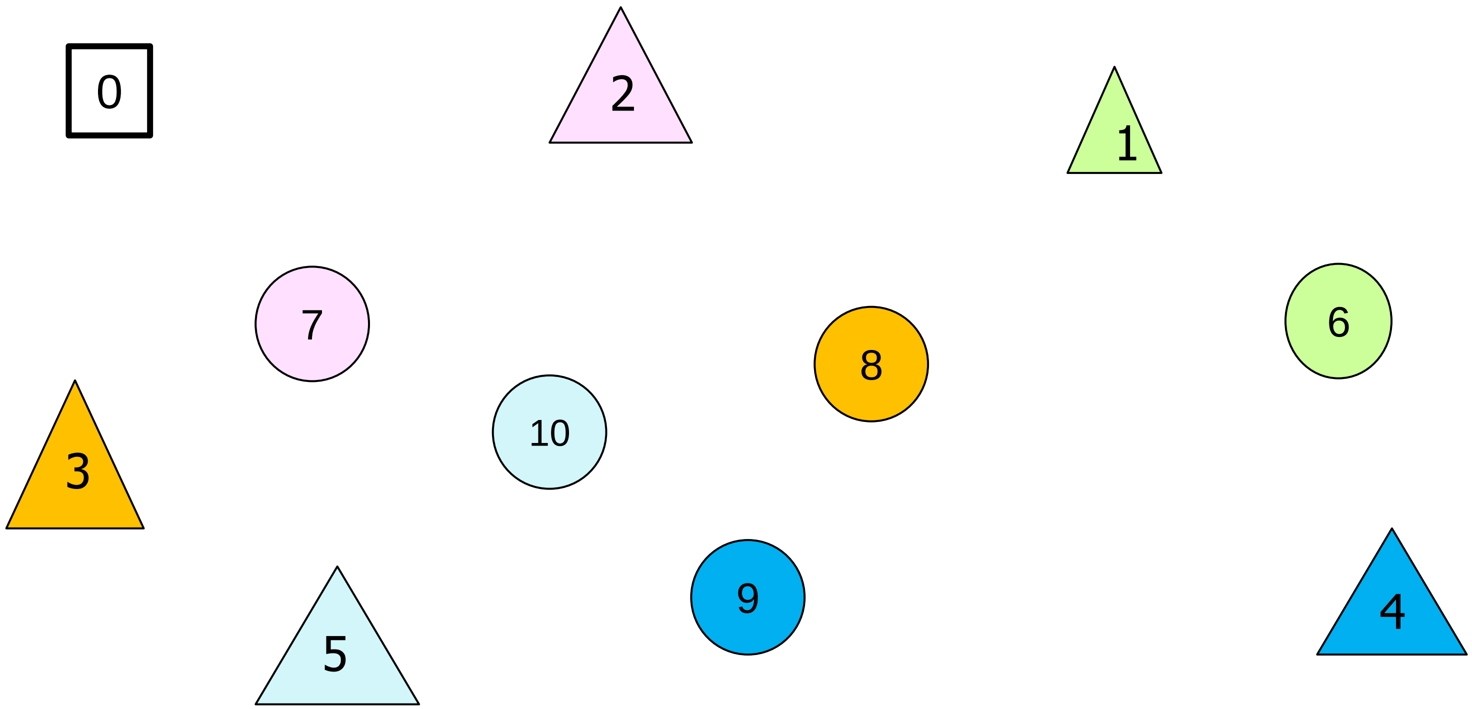
- if $i\in P$, $c(i)=i+n$
- if $i\in D$, $c(i)=i-n$

PPDTSP Example

5 requests

pickup nodes $1, \dots, 5$
delivery nodes $6, \dots, 10$

pickup i
delivery $5 + i$



PPDTSP Example

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pickup nodes $1, \dots, 5$
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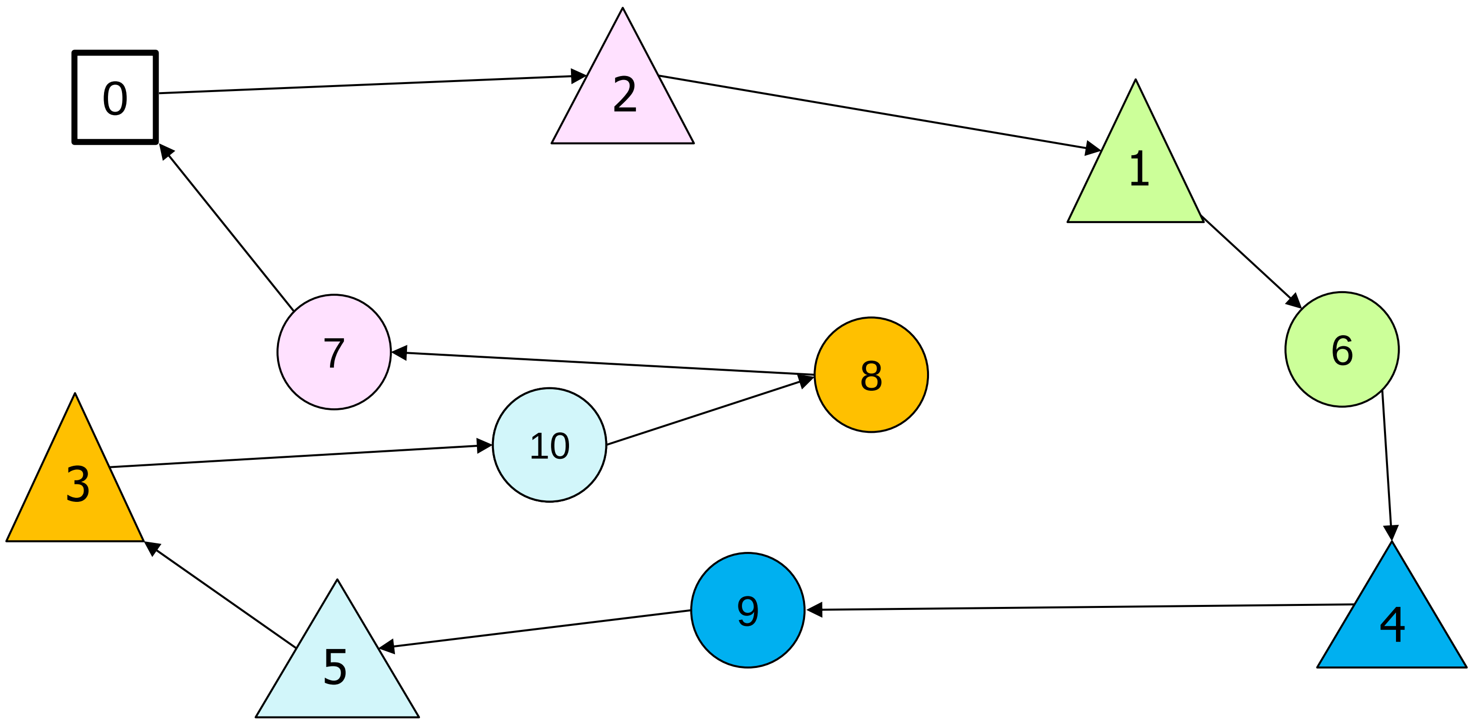
pickup i
delivery $i + 5$



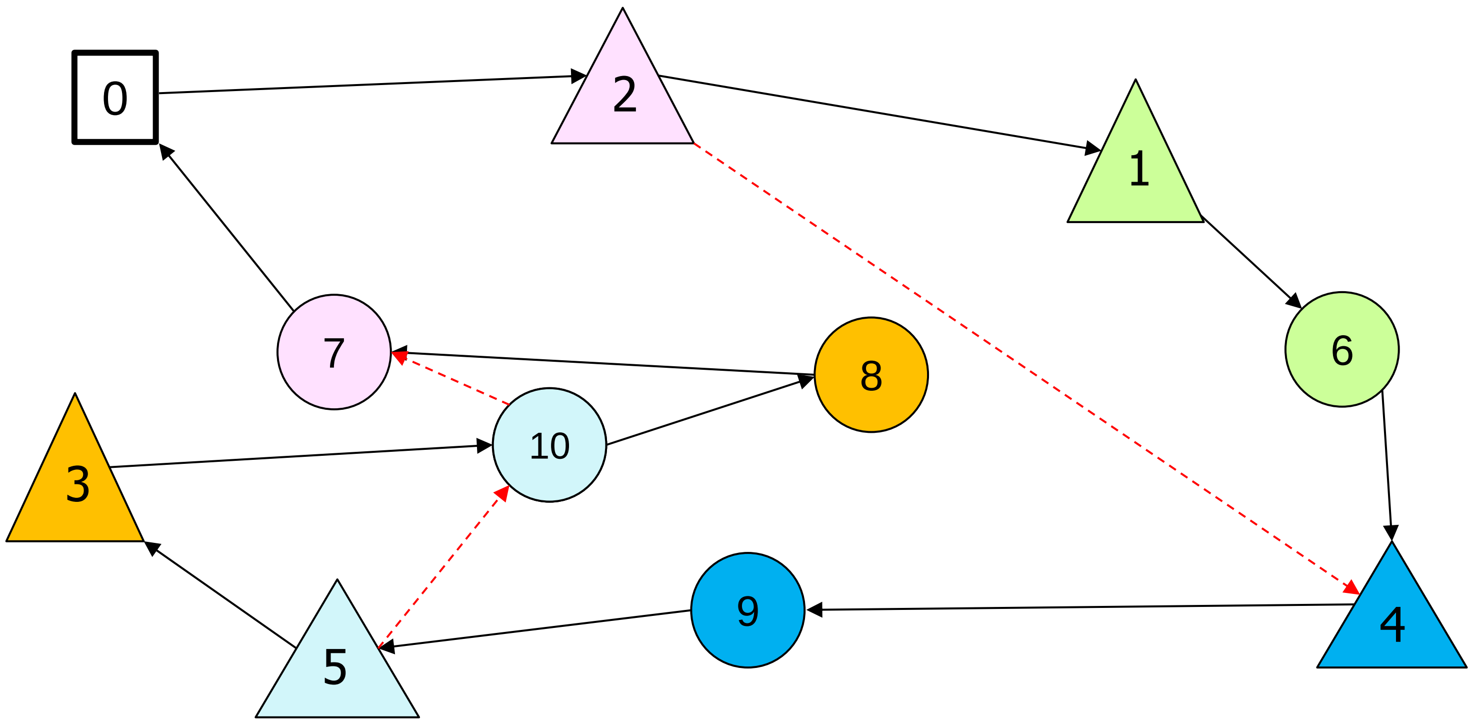
a priori tour

PPDTSP Example

5 requests

pickup nodes $1, \dots, 5$
delivery nodes $6, \dots, 10$

pickup i
delivery $i + 5$



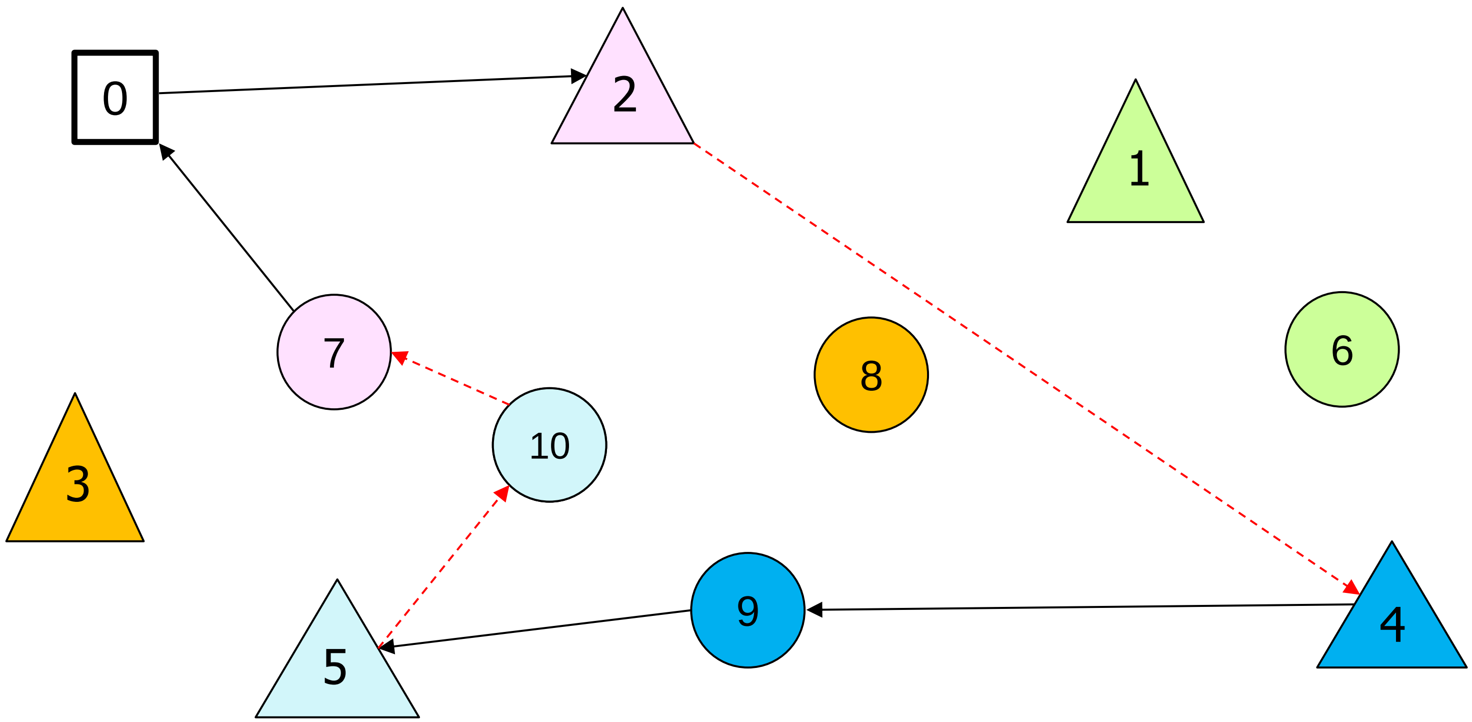
If requests 1 and 3 have been cancelled …

PPDTSP Example

5 requests

pickup nodes $1, \dots, 5$
delivery nodes $6, \dots, 10$

pickup i
delivery $i + 5$



Final tour if requests 1 and 3 have been cancelled

PPDTSP

The length of the tour really operated by the vehicle (final tour) depends on the requests that have not been cancelled

The objective is to find an a priori tour for which the expected length of the final tour is minimum

Applications:

- Pickup and delivery problems when there is uncertainty in the demand
- Design of robust tours for long term operation where not all the costumers request service every day

Literature

Probabilistic TSP

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  A Priori Solution of a Traveling Salesman Problem in Which a Random Subset of the Customers are Visited
  Oper. Res., 1988, 36, 6, 929-936

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  Operations Research/ Computer Science Interfaces Series, 2008, 43, 123-142

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  Comput.Oper.Res., 2007, 34, 10, 2949-2963

- Marinakis,Y; Marinaki, M
  A Hybrid Multi-Swarm Particle Swarm Optimization algorithm for the Probabilistic Traveling Salesman Problem
  Comput. Oper.Res., 2010, 37, 3, 432-442

Literature

To our knowledge, only one paper exists on the PPDTSP:

Probabilistic Pickup and Delivery Traveling Salesman Problem (PPDTSP)

- Beraldi, P.; Ghiani, G.; Laporte, G.; Musmanno, R.
  Efficient neighborhood search for the probabilistic Pickup and Delivery Travelling Salesman Problem
  Networks, 2005, 45, 4, 195-198

- Ho, S.C.; Haugland, D.
  Local search heuristics for the probabilistic dial-a-ride problem
  OR Spectrum, 2011, 33, 4, 961-988

PPDTSP formulation: variables

Classical variables for the TSP (directed)

$$x_{ij}^0 = \begin{cases} 1 & \text{if the vehicle travels directly from } i \text{ to } j \\ 0 & \text{otherwise} \end{cases} \qquad i,j \in V$$

Define the a priori tour

Skip-variables

$$x_{ij}^r = \begin{cases} 1 & \text{if there are nodes of } r \text{ requests between } i \text{ to } j \text{ in the a priori tour} \\ & \text{and none of them correspond to requests } i \text{ or } j \\ 0 & \text{otherwise} \end{cases}$$

$$r \geq 1, i,j \in V$$

PPDTSP formulation: The expected length of the a priori tour

Given an a priori tour, $x_{ij}^{r} = 1$ implies that

$$\Pr(\text{arc } (i,j) \text{ is in the final tour}) = p^2(1-p)^r$$

- i is visited before j and correspond to different requests,
- there are nodes of r requests between i and j, and none of them correspond to the same requests than i or j

$$\Pr(\text{arc } (i,j) \text{ is in the final tour}) = p(1-p)^r$$

- i and j correspond to the same request, or one is the depot
- there are nodes of r requests between i and j, and none of them correspond to the same requests than i or j

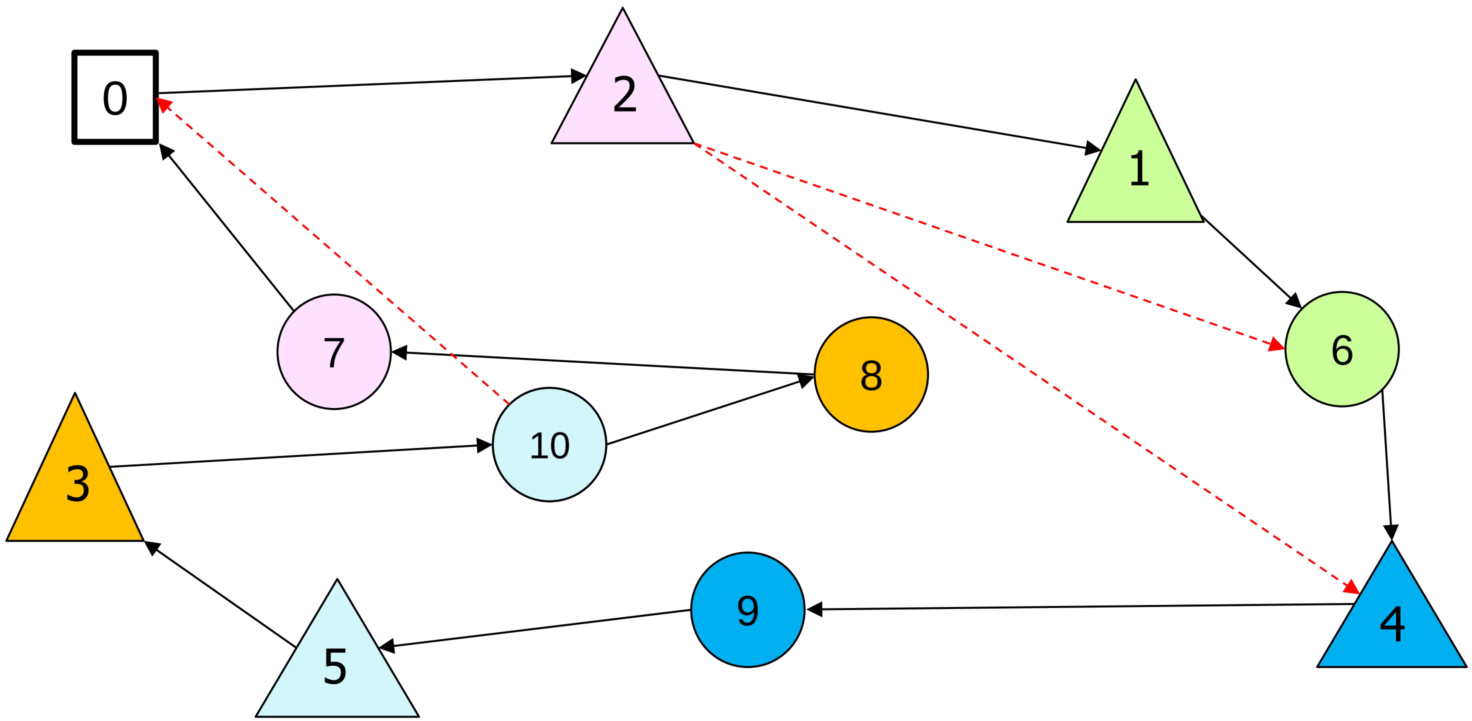
In all the other cases $\Pr(\text{arc } (i,j) \text{ is in the final tour}) = 0$

PPDTSP Example

5 requests

pickup nodes $1, \dots, 5$
delivery nodes $6, \dots, 10$

pickup i
delivery $i + 5$



$\Pr(2,4) = p^2(1-p)$ $\Pr(10,0) = p\,(1-p)^2$ $\Pr(2,6) = P(4,2) = 0 \dots.$

PPDTSP formulation

Objective function

$$\text{Min} \sum_{r\in R}\left(\sum_{(i,j)\in A_1} c_{ij}p(1-p)^r x_{ij}^r + \sum_{(i,j)\in A_2} c_{ij}p^2(1-p)^r x_{ij}^r\right) \qquad (0)$$

$R = \{0, \dots, n-1\}$

$A_1 = \{(i,j) \in A: j = i+n, \text{or } i = 0, \text{or } j = 0\}$ $\qquad A_2 = A \backslash A_1$

Compact formulation

Pickup and Delivery constraints (a priori tour)

$$\sum_{i \in V\, i \neq j} x_{ij}^{0} = 1 \qquad j \in P \cup D \qquad (1)$$

$$\sum_{i \in V\, i \neq j} x_{ji}^{0} = 1 \qquad j \in P \cup D \qquad (2)$$

$$\sum_{j \in P} x_{0j}^{0} = 1 \qquad \sum_{i \in D} x_{i0}^{0} = 1 \qquad (3)$$

Compact formulation

Pickup and Delivery constraints (a priori tour)

$$x_{ij}^0 \leq w_{ij} \qquad \forall i, j \in V : i \neq j \qquad (4)$$

$$w_{ij} + w_{ji} = 1 \qquad \forall i, j \in V : i \neq j \qquad (5)$$

$$w_{ij} + w_{jk} + w_{ki} \leq 2 \qquad \forall i, j, k \in V : i \neq j, k; j \neq k \qquad (6)$$

$$w_{i,i+n} = 1 \qquad \forall i \in P \qquad (7)$$

Precedence variables

$$w_{ij} = \begin{cases} 1 & \text{if node } i \text{ is visited before node } j \text{ in the a priori tour} \\ 0 & \text{otherwise} \end{cases} \qquad \forall i, j \in V$$

Compact formulation

Constraints forcing the skip-variables x_{ij}^r

$$\sum_{r \in R} x_{ij}^r + \sum_{r \in R} x_{ji}^r \leq 1 \qquad \forall i, j \in V : i \neq j \qquad (8)$$

$$\sum_{r \in R} x_{0i}^r = 1 \qquad \sum_{r \in R} x_{i0}^r = 0 \qquad \forall i \in P \qquad (9)$$

$$\sum_{r \in R} x_{0j}^r = 0 \qquad \sum_{r \in R} x_{j0}^r = 1 \qquad \forall j \in D \qquad (10)$$

$$\sum_{r \in R} x_{i,i+n}^r = 1 \qquad \sum_{r \in R} x_{i+n,i}^r = 0 \qquad \forall i \in P \qquad (11)$$

Compact formulation

Constraints forcing the skip-variables x_{ij}^r

$\sum_{r\in R} x_{ij}^r = 1$ iff	i is visited before j and there is no node related to the requests of i and j between them in the a priori tour

$$\sum_{r\in R} x_{ij}^r \leq w_{ij} \qquad \forall i,j \in V\backslash\{0\} \qquad \sum_{r\in R} x_{ij}^r = w_{ij} \qquad \forall i \in D, j \in P \qquad (12)$$

$$\sum_{r\in R} x_{ij}^r \geq w_{ij} - w_{i+n,j} \qquad \forall i,j \in P \qquad (13)$$

$$\sum_{r\in R} x_{ij}^r \geq w_{ij} - w_{i,j-n} \qquad \forall i,j \in D \qquad (14)$$

$$\sum_{r\in R} x_{ij}^r \geq w_{ij} - w_{i,j-n} - w_{i+n,j} \qquad \forall i \in P, j \in D \qquad (15)$$

Compact formulation

Constraints forcing the skip-variables x_{ij}^r

$$\sum_{j \in V} x_{ij}^r \leq 1 \qquad \forall i \in V, r \in R \qquad (16)$$

$$\sum_{j \in V} x_{ij}^{r+1} \leq \sum_{j \in V} x_{ij}^r \qquad \forall i \in V, r \in \{0, \dots, n-2\} \qquad (17)$$

$$x_{ij}^r + w_{jt} + \sum_{s=0}^{r} x_{it}^s \leq 2 \qquad \forall i, t \in V, j \in V \setminus \{0\}, r < n-1 \qquad (18)$$

$$x_{i0}^r + \sum_{s=r}^{n-1} x_{ij}^s \leq 1 \qquad \forall i \in D, j \in P, r < n-1 \qquad (19)$$

Compact formulation

$$\text{Min} \quad \sum_{r\in R}\left(\sum_{(i,j)\in A_1} c_{ij}p(1-p)^r x_{ij}^r + \sum_{(i,j)\in A_2} c_{ij}p^2(1-p)^r\, x_{ij}^r\right) \qquad (0)$$

s.t. (1) ... (19)

$$x_{ij}^r \in \{0,1\} \qquad \forall (i,j) \in A, r \in R$$
$$w_{ij} \in \{0,1\} \qquad \forall (i,j) \in A$$

Non-compact formulation

This formulation uses only the skip-variables, $\forall i, j \in V, r \in R$

$$x_{ij}^{r} = \begin{cases} 1 & \text{if there are nodes of } r \text{ requests between } i \text{ to } j \text{ in the a priori tour} \\ & \quad \text{and none of them correspond to requests } i \text{ or } j \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Min} \quad \sum_{r \in R} \left(\sum_{(i,j) \in A_1} c_{ij} p (1-p)^r x_{ij}^r + \sum_{(i,j) \in A_2} c_{ij} p^2 (1-p)^r \, x_{ij}^r \right)$$

$$\sum_{i \in \mathrm{V}\; i \neq j} x_{ij}^0 = 1 \qquad \sum_{i \in \mathrm{V}\; i \neq j} x_{ji}^0 = 1 \qquad j \in P \cup D$$

$$\sum_{j \in P} x_{0j}^0 = 1 \qquad \sum_{i \in D} x_{i0}^0 = 1$$

Non-compact formulation (2)

Notation $S, S' \subseteq V$ $(S\!:\!S') = \{(i,j) \in A, i \in S, j \in S'\}$

$$\gamma(S) = (S\!:\!S)$$

$$x^r(F) = \sum_{(i,j)\in F} x^r_{ij} \qquad F \subseteq A$$

In order to simplify the notation

$$x^r(\{i\}\!:\!S) = x^r(i\!:\!S)$$

$$x^r(S\!:\!\{i\}) = x^r(S\!:\!i)$$

$$x^r\big((S\!:\!S')\big) = x^r(S\!:\!S')$$

Non-compact formulation (3)

Pickup and Delivery constraints (a priori tour)

Balas, Fischetti, Pulleyblank(1995)
The precedence-constrained asymmetric traveling salesman polytope

$$\pi(S) = \{i \in P: i + n \in S\}$$

$$x^0\big(S\backslash\pi(S): \bar{S}\backslash\pi(S)\big) \geq 1 \qquad S \subseteq V\backslash\{0\} \qquad \pi - inequality$$

$$\pi, \sigma - inequalities$$

$$x^0(0: S) + x^0\big(\gamma(S)\big) \leq |S| - 1 \qquad \forall S \subseteq V\backslash\{0\}: \exists i + n \in S \cap D, i \notin S$$

$$x^0\big(\gamma(S)\big) + x^0(S: 0) \leq |S| - 1 \qquad \forall S \subseteq V\backslash\{0\}: \exists i \in S \cap P, i + n \notin S$$

Non-compact formulation (4)

Skip-constraints

Let $i \in V, j \in V, i \neq j$, and $S \subseteq V \backslash \{0, i, j. c(i), c(j)\}$ containing nodes of exactly r requests

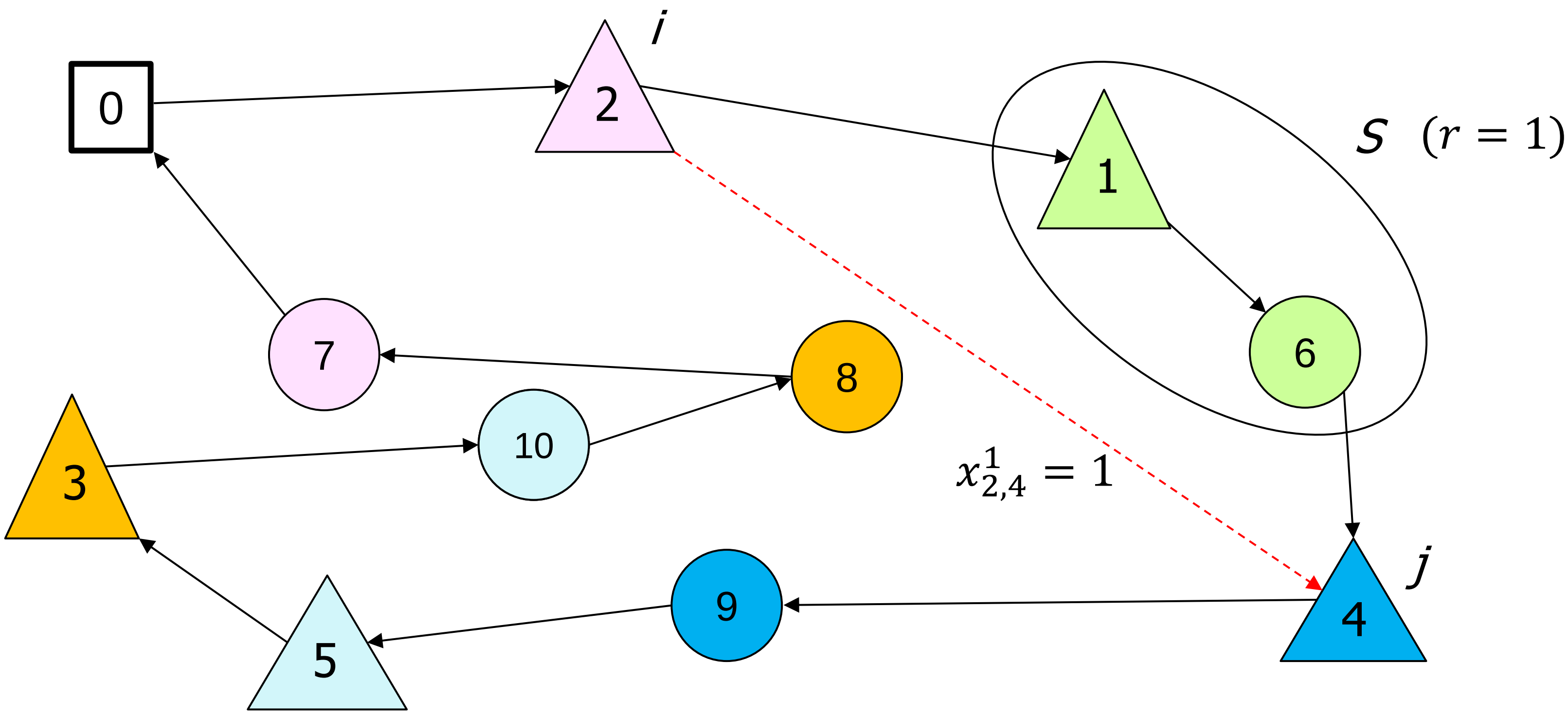
$$x_{ij}^{r} \geq x^{0}(i{:}S) + x^{0}(S{:}j) + x^{0}\big(\gamma(S)\big) - |S|$$

$x^{0}(i{:}S) \leq 1, x^{0}(S{:}j) \leq 1, x^{0}\big(\gamma(S)\big) \leq |S| - 1,$

If $x^{0}(i{:}S) = 1, x^{0}(S{:}j) = 1, x^{0}\big(\gamma(S)\big) = |S| - 1$, then there is a path between i and j with r requests between them in the a priori tour

Skip-constraint example

$$x^1_{2,4} \geq x^0(2{:}\,S) + x^0(S{:}\,4) + x^0\big(\gamma(S)\big) - |S| = 1 + 1 + 1 - 2 = 1$$



pickup i
delivery $i + 5$

Non-compact formulation (5)

Weak skip-constraints

$$\sum_{r \in R} x_{ij}^r \geq x^0(i:S) + x^0(S:j) + x^0\big(\gamma(S)\big) - |S|$$

$$i \in V, j \in V, i \neq j, \text{and } S \subseteq V \setminus \{0, i, j, c(i), c(j)\}$$

Weak skip-constraints can be separated in polynomial time:

- for each pair i, j compute a max-flow in a transformed graph to obtain the subset S that maximizes the RHS

Once the subset S has been obtained, the stronger skip-constraint can be checked for possible violation

Non-compact formulation
Branch-and-Cut algorithm

Initial LP:

$$\text{Min} \sum_{r\in R}\left(\sum_{(i,j)\in A_1} c_{ij}p(1-p)^r x_{ij}^r + \sum_{(i,j)\in A_2} c_{ij}p^2(1-p)^r x_{ij}^r\right)$$

$$\sum_{i\in V\, i\neq j} x_{ij}^0 = 1 \qquad j\in V$$

$$\sum_{i\in V\, i\neq j} x_{ij}^0 = 1 \qquad j\in V$$

$$\sum_{r\in R} x_{ij}^r + \sum_{r\in R} x_{ji}^r \leq 1 \qquad \forall i,j\in V: i\neq j$$

$$\sum_{r\in R} x_{0i}^r = 1 \qquad \sum_{r\in R} x_{i0}^r = 0 \qquad \forall i\in P$$

$$\sum_{r\in R} x_{0j}^r = 0 \qquad \sum_{r\in R} x_{j0}^r = 1 \qquad \forall j\in D$$

$$\sum_{r\in R} x_{i,i+n}^r = 1 \qquad \sum_{r\in R} x_{i+n,i}^r = 0 \qquad \forall i\in P$$

$$1\geq x_{ij}^r \geq 0 \qquad i,j\in V, i\neq j, r\in R$$

Non-compact formulation Branch-and-Cut algorithm

Separation algorithms

- **Generate "promising" sets of nodes S from the support graph $G(\overline{x^0})$**
  - Connected components, shrinking heuristic, sequences of nested subsets
- **For each subset S**
  - Check $\pi -$ and $\pi, \sigma -$constraints for a priori variables x_{ij}^0
  - Generate appropriate nodes $i, j \in V\backslash S$ and check skip-constraints for skip-variables x_{ij}^r with $r \geq 1$
- **Checking the solution**
  - For all nodes $i \in V$

    If arcs with $\overline{x^0}$-integer values form a path from node i to a node j, check the π-constraint and the skip-constraint for all sub-paths starting at i

Exact separation for weak skip constraints has not been implemented

Computational experience

- Branch-and-cut Implemented in C++ using CPLEX 9.0 to solve the LP relaxations
- Computer with an Intel(R) Core(TM) 2 Quad Q6700 with 2 GB of RAM running at 2.66 GHz under Windows XP
- CPU time limit: 2 hours
- 15 PDPLT small instances of the PDTSP taken from the ones used by

  Dumitrescu, Ropke, Cordeau & Laporte (2010) The Traveling Salesman Problem with Pickup and Delivery: Polyhedral Results and a Branch-and-Cut Algorithm

  - 11, 21, 31 nodes of Dataset 1(5, 10, 15 requests)
  - several values for the probability

Computational results

COMPACT FORMULATION

small instances

		p=1			p=0,9			p=0,8		
Instance	size	UB	gap	time	UB	gap	time	UB	gap	time
prob5a	11	3585	0.0	4.0	3400.0	0.0	7.0	3202.6	0.0	13.5
prob5b	11	2565	0.0	0.5	2487.1	0.0	4.3	2403.0	0.0	2.2
prob5c	11	3787	0.0	2.2	3657.3	0.0	4.6	3505.5	0.0	4.4
prob5d	11	3128	0.0	0.4	2976.8	0.0	1.5	2810.2	0.0	3.8
prob5e	11	3123	0.0	3.5	2989.8	0.0	4.8	2832.5	0.0	10.6
prob10a	21	5448	19.8	7200	4761.0	20.4	7200	5114.9	35.6	7200
prob10b	21	4690	14.7	7200	4405.6	22.1	7200	4742.5	37.6	7200
prob10c	21	4070	0.0	5668	4045.9	14.1	7200	2889.9	35.3	7200
prob10d	21	4551	0.0	7197	4431.0	13.7	7200	4255.2	20.7	7200
prob10e	21	4874	0.0	2157	4843.9	12.8	7200	5800	36.5	7200

Computational results

NON-COMPACT FORMULATION

small instances

		p=1			p=0,9			p=0,8		
Instance	size	UB	gap	time	UB	gap	time	UB	gap	time
prob5a	11	3585	0,0	0,3	3400,0	0,0	1,6	3202,6	0,0	16,3
prob5b	11	2565	0,0	0,2	2487,1	0,0	0,6	2403,0	0,0	3,9
prob5c	11	3787	0,0	0,2	3657,3	0,0	1,2	3505,5	0,0	8,2
prob5d	11	3128	0,0	0,2	2976,8	0,0	1,0	2810,2	0,0	5,6
prob5e	11	3123	0,0	0,6	2989,8	0,0	4,0	2832,5	0,0	18,4
prob10a	21	4896	0,0	23,7	4704,2	0,0	1503,0	4494,4	9,8	7200,7
prob10b	21	4490	0,0	33,1	4346,0*	0,0	5489,9	4177,1	11,3	7200,7
prob10c	21	4070	0,0	2,3	3981,0	0,0	149,0	3872,4	5,6	7200,6
prob10d	21	4551	0,0	3,3	4384,4*	0,0	68,6	4204,7	5,3	7200,7
prob10e	21	4874	0,0	4,5	4668,7	0,0	394,2	4449,7	3,9	7200,6

Computational results

NON-COMPACT FORMULATION

medium size instances

Instance	size	p=1			p=0,9		
		UB	gap	time	UB	gap	time
prob15a	31	5150	0,0	2907,2	4948,0	6,5	7201,7
prob15b	31	***	***	7202,4	***	***	7201,5
prob15c	31	5008	0,0	15,1	4840,1	1,5	7201,4
prob15d	31	5566	0,0	4985,1	5367,3	9,3	7201,7
prob15e	31	5229	0,0	10,8	5098,0	3,8	7201,6

Conclusions

- Two mixed integer formulations for the PPDTSP were proposed
- Branch-and-cut based on non-compact formulation is able to solve small instances
- The difficulty of the problem increases as p decreases

Future research

A lot of work remains to be done

- Improve the compact formulation
- Add new families of constraints from the Pickup and Delivery TSP polytope and improve their separation
- Try to derive new valid constraints involving the skip-variables
- Generalize the problem by allowing requests with probability 1 as in the Probabilistic TSP (black nodes)

Thanks you for your attention

Questions?

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