Location of Emergency Units in Collective Transportation Line Networks

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Unión Europea
Fondo Europeo de Desarrollo Regional
Fondo Social Europeo

M DE ECONOMÍA Y COMPETITIVIDAD

VI International Workshop on Locational Analysis and Related Problems,
Barcelona, November 25-27, 2015

Outline

- Rapid Transit Networks
- Graph Partitioning
- Hypergraph Partitioning
- Algorithms and Experimental Results

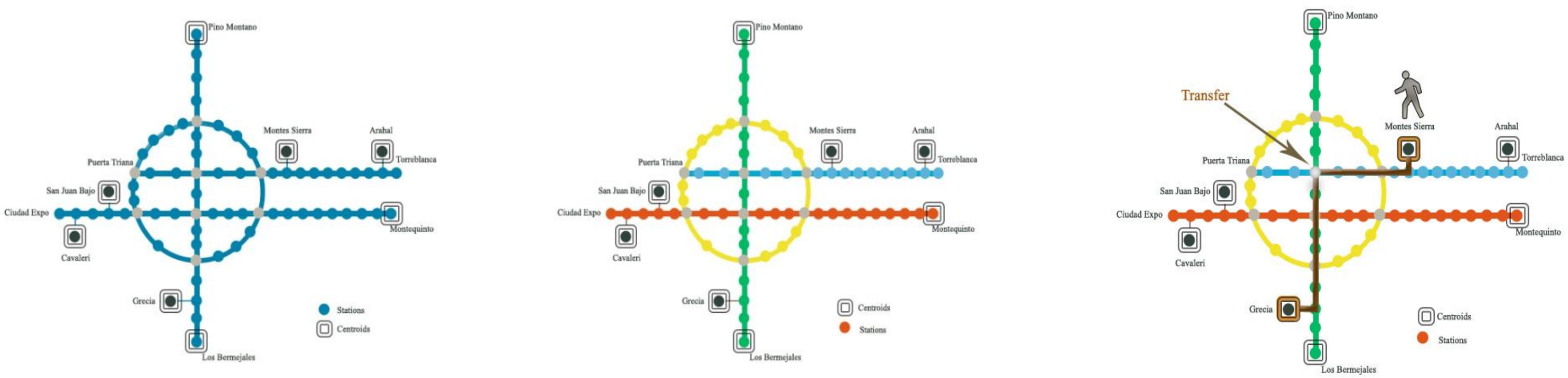
Rapid Transit Networks

- According to UITP (International Association of Public Transport) a metro (underground, subway, U-bahn) is an urban mass passenger transport system operated on his own right of way and segregated from other traffic.
- There are 198 metros in the world (World Metro Database, Mike Rhode, 2015).
- A light rail contains non-segretated sections from road and pedestrian and others with segregated rilight of way.
- The term Rapid Transit System includes metro, light metro, light rail, monorail, commuter trains, and other mass transit systems.

Rapid Transit Networks

Three embedded levels:

- Infrastructure Network.
- Line Routes.
- Passenger Routes.



Rapid Transit Networks

INCIDENTS

Engine breakdowns

Signal and/or telecommunication problems

Catenary problems

Power shortage

Human Causes (error, crew sikness, etc.)

Incident on the track (suicides: 151 people were hit by the train in New York in 2013)

Rapid Transit Systems

- Put emergency units in every station: fast reaction but expensive.
- Put emergency units in every line: line lengths varies (from 3-4 kms. to more than 80 Kms.)
- Partition rapid transit networks in several parts, and assign to each part an emergency unit. Locate the stations to have the emergency units.

Graphs partitioning

- Graph partitioning is a familly of combinatorial problems which consists in to partition the set of vertices of a graph into subsets satisfying size constraints so that an objective function is optimized.
- The graph bisection problem consists in to partition the set of vertices in two equal sets so that the number of edges between vertices in different subset is minimized

Graph partitioning

- K-way partition problem:

  Partition the vertex set into K parts of similar size minimizing the number of edges between different parts. Vertex and edges can be weighted.

Graph partitioning

Applications (Hager, Phan & Zhang, 2009)

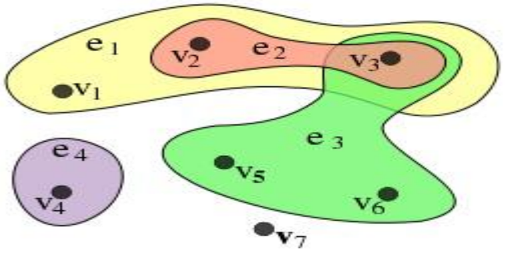
- VLSI design
- Data mining
- Parallel computing
- Sparse matrix factorization

Graph partitioning

- The graph bisection problem is NP-hard except for special graphs (e.g. grids)
- There are exact methods: branch-and-cut, column generation, polyhedral approaches, B&B with QP programming etc.
- There are heuristic methods: spectral, geometrics, multilevel schemes, iteratives (Feduccia-Mattheises), metaheuristics

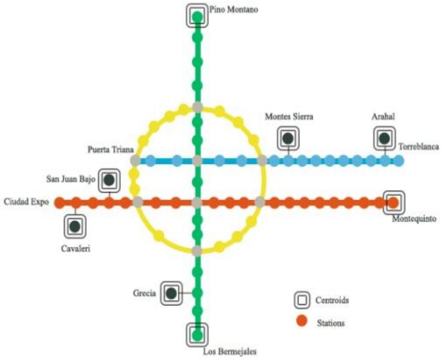
Hypergraph partitioning

- Graphs do not capture all the characteristics of some system since the relationship between elements are not pairwise but among a group of elements.
- A hypergraph is a pair: vertex set, hyperedge set.

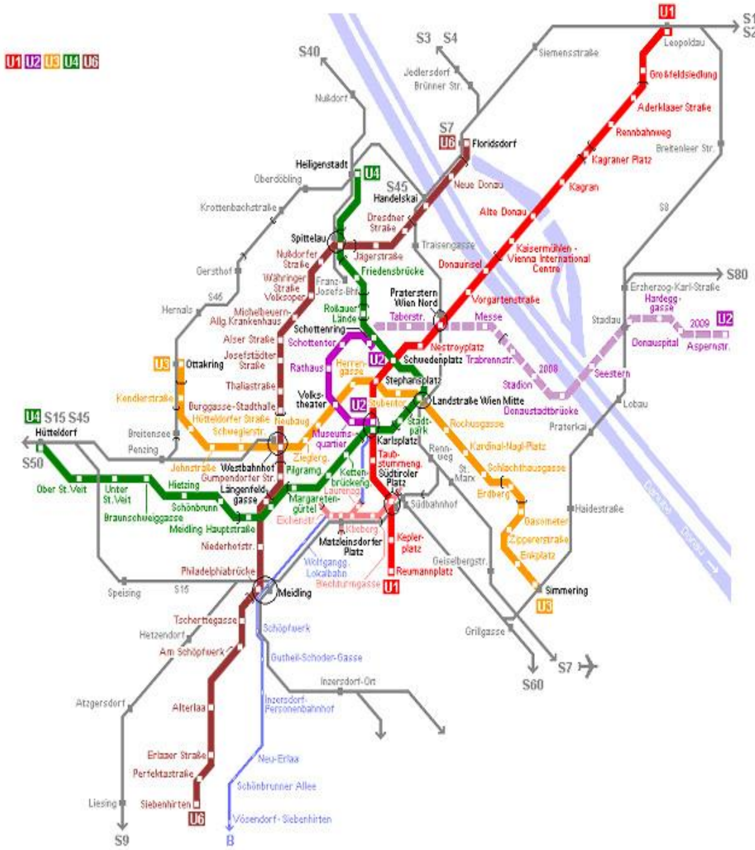


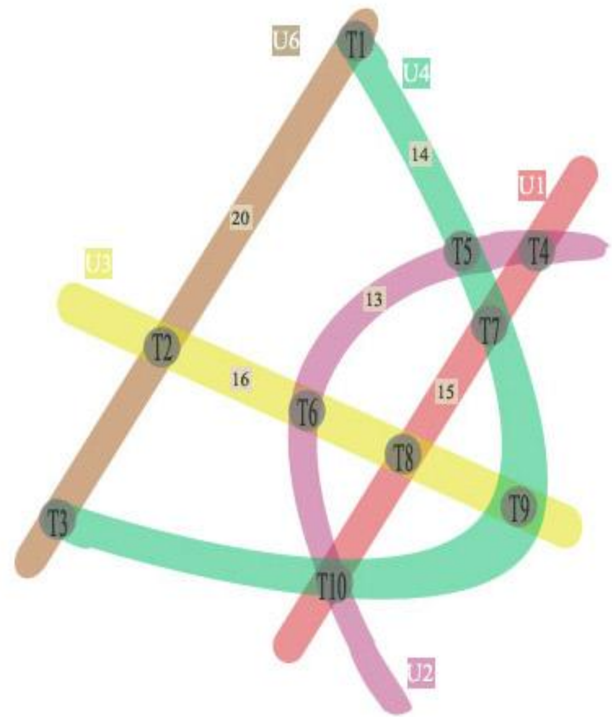
Hypergraph partitioning

- Rapid Transit Networks can be represented by hypergraphs since each line can be considered as a hyperedge.



Hypergraph partitioning





Hypergraph partitioning

- However, the hypergraph structure it is not sufficient due to the lack of connections between adjacent stations in the same line. Thus, does not distinguish between closed stations and stations far away.
- Hyperstructure: $S = (X, E, H)$

  It is a triple formed by the vertex set X ,the edge set E, and the hyperedge set H

Hypergraph partition

Let $S = (N, E, H)$ be a hyperstructure, where $N = \{1, 2, \ldots, n\}$ is the vertex set,

$E = \{e_1, e_2, \ldots, e_m\}$ is the edge set, and

$H = \{h_1, h_2, \ldots, h_L\}$ is the hyperedge set.

$A = (a_{i,j})$ is the adjacent matrix of the graph.

$D = (d_{k,l})$ is the incident matrix of the hypergraph.

Hypergraph partitioning

- Let $P = \{N_1, N_2, \ldots, N_K\}$ be a partition of the set N.
- Each partition determines an asignment of vertices to part: $X = (x_{i,k})$, where $x_{i,k} = 1$ indicates that vertex i is assigned to the part N_k.

Let A_k the adjacent submatrix of part N_k

Hypergraph partitioning

If $N_k = \{k_1, k_2, \ldots, k_{|N_k|}\}$ then

$A_k = I_k \times A \times I_k^T$ where

$i_{i,j}^{(k)} = 1$ if $j \in N_k$ and $j = k_i$

For example, if n=5 and $N_k = \{1, 3\}$ then

$$I_k = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad A_k = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$$

Hypergraph partitioning

- Relationship between matrices X and A_k:

$$x_{i,k} = \sum_{j=1}^{|N_k|} i_{j,i}^{(k)}.$$

- The load of a part is the number of different hyperedges associated with the nodes assigned to the part

$$L_k(x) = \sum_{l=1}^{L} \min \left\{ \sum_{i \in N} d_{i,l} x_{i,k}, 1 \right\}$$

Hypergraph partitioning

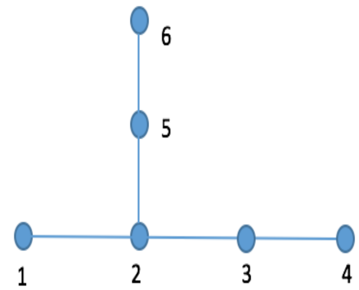
- The Min-max hypergraph partition problem

$$\min \max\{L_1(x), L_2(x), \dots, L_K(x)\}$$
$$\textstyle\sum_{k\in K} x_{i,k} = 1, i \in N$$
$$\textstyle\sum_{i=1}^{n} A_k^{i-1} > 0, k \in \{1, 2, \dots, K\}$$
$$x_{i,k} = \textstyle\sum_{j=1}^{|N_k|} i_{j,i}^{(k)}$$
$$A_k = I_k \times A \times I_k^T$$
$$X \in \{0, 1\}^{n\times K}$$

Algorithms and experiments

- Naive (complete enumeration)
- Partitions (based on toolbox Matlab)
- Iterative bipartition

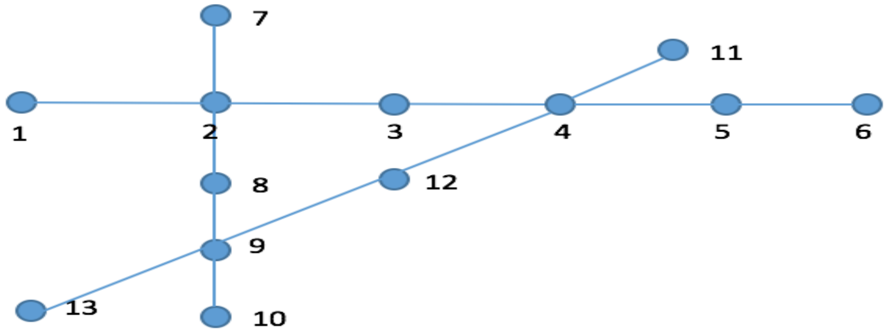
Experiments



- Exact algorithms and their performance (N=6, K=3)
- Method Solution Optimal value Time
- Naive partition N1 = {5, 6}, N2 = {3, 4}, N3 = {1, 2} 2 0.05468s
- B-partition N1 = {5, 6}, N2 = {3, 4}, N3 = {1, 2} 2 0.1123s
- Par-partition N1 = {1;,2}, N2 = {3, 4}, N3 = {5, 6} 2 0.03888s

Experiments

- 13 node graph



- Method Solution Optimal value Time
- Naive partition N1 ={9,10,13}, N2 = {4,5,6,11,12}, N3 = {1,2,3,8,9} 2 87.85s
- B-partition N1 = {3,4,5,6,11,12}, N2 = {9,10,13}, N3 = {1,2,7;,8} 2 9.027s
- Par-partition N1 = {1,2,7}, N2 = {3,4,5,6,11}, N3 = {8,9,10,12,13} 2 37.06s

Thank you

for your

attention!