

Analysing the Impact of Capacity Fluctuations on the Design of a Supply Chain Network

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- ▶ Need of a better understanding of the impact of imperfect reliability on location patterns.

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- ▶ Imperfect information: customers visit a -probably unavailable facility- and are redirected to another one or lost.

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Facilities tend to locate closer to each other as the likelihood of failure increases. On the extreme, facilities are placed in the same location in order to provide effective back-up in case of failure.

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- ▶ Sensitivity of solution patterns to low levels of expected h-capacity (when the system is not working at planned capacity).

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- ▶ \mathcal{L} is the set of all feasible solutions (locations).

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and q_{ij} is the quantity shipped from a facility in $i \in \mathcal{I}$ to demand node $j \in \mathcal{J}$.

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- ▶ Improvement of **average gap** by facilities ordering may have a substantial positive impact on Z .

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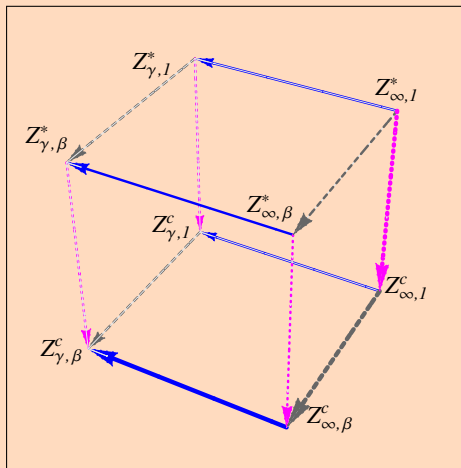
and define an ideal scenario

$$Z_{\infty,1}^*$$

where capacity is not a concern and the facility is perfectly reliable.

Analysis

Loss Cube



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- ▶ The more stable is the environment (larger α) the UCFLP will provide better solutions;
- ▶ Vulnerable facilities in volatile environments require investment in safety (larger β) or an increase in capacity (larger γ).

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- ▶ Analyse how spatial **location patterns** change with different combinations of resilience and capacity;
- ▶ Understand effects of **correlated disruptions** on objective and location patterns;

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Thank You!!!!