The Railway Rapid Transit Network Construction Scheduling Problem

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Outline

1 Background

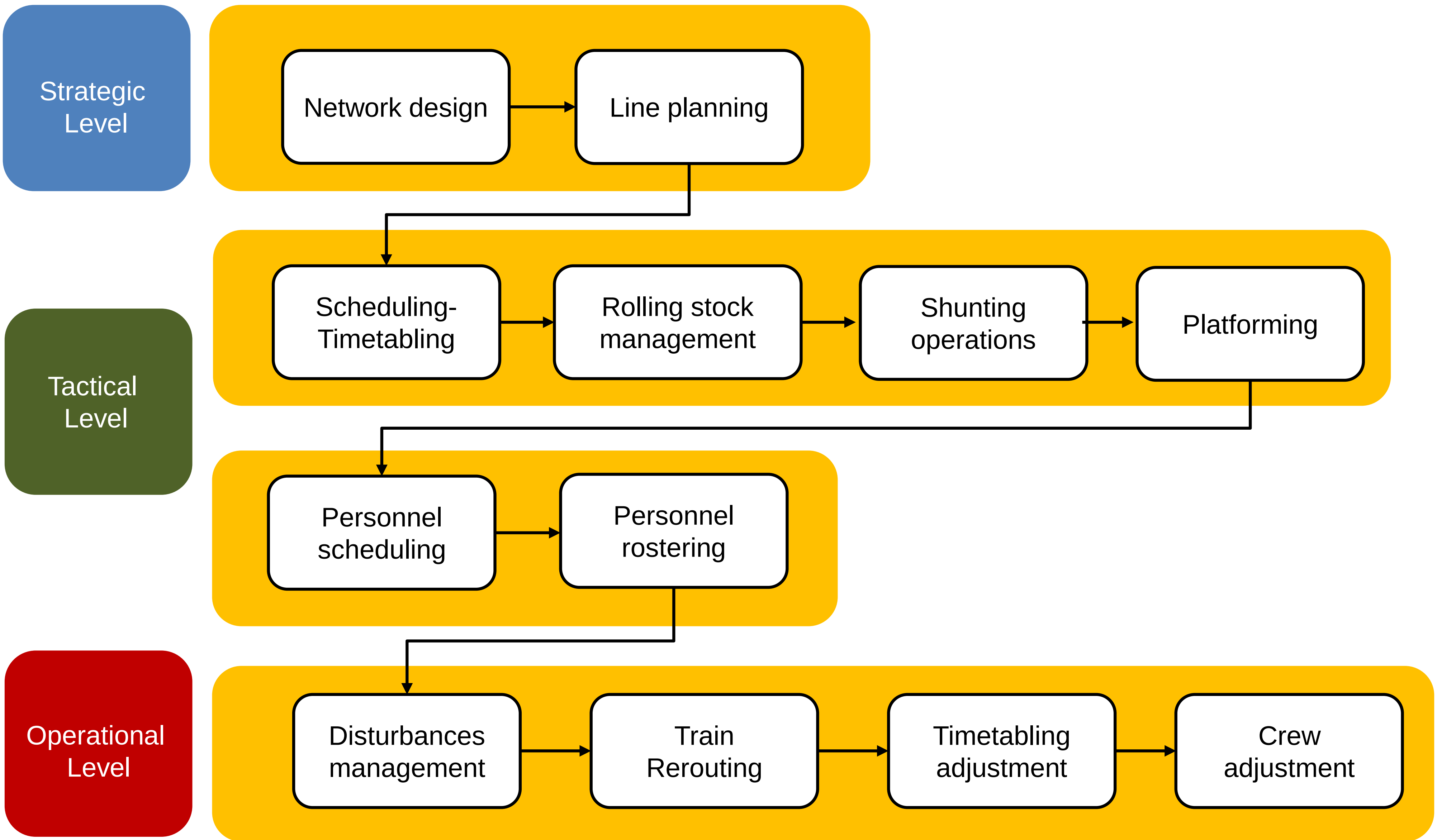
2 Problem description

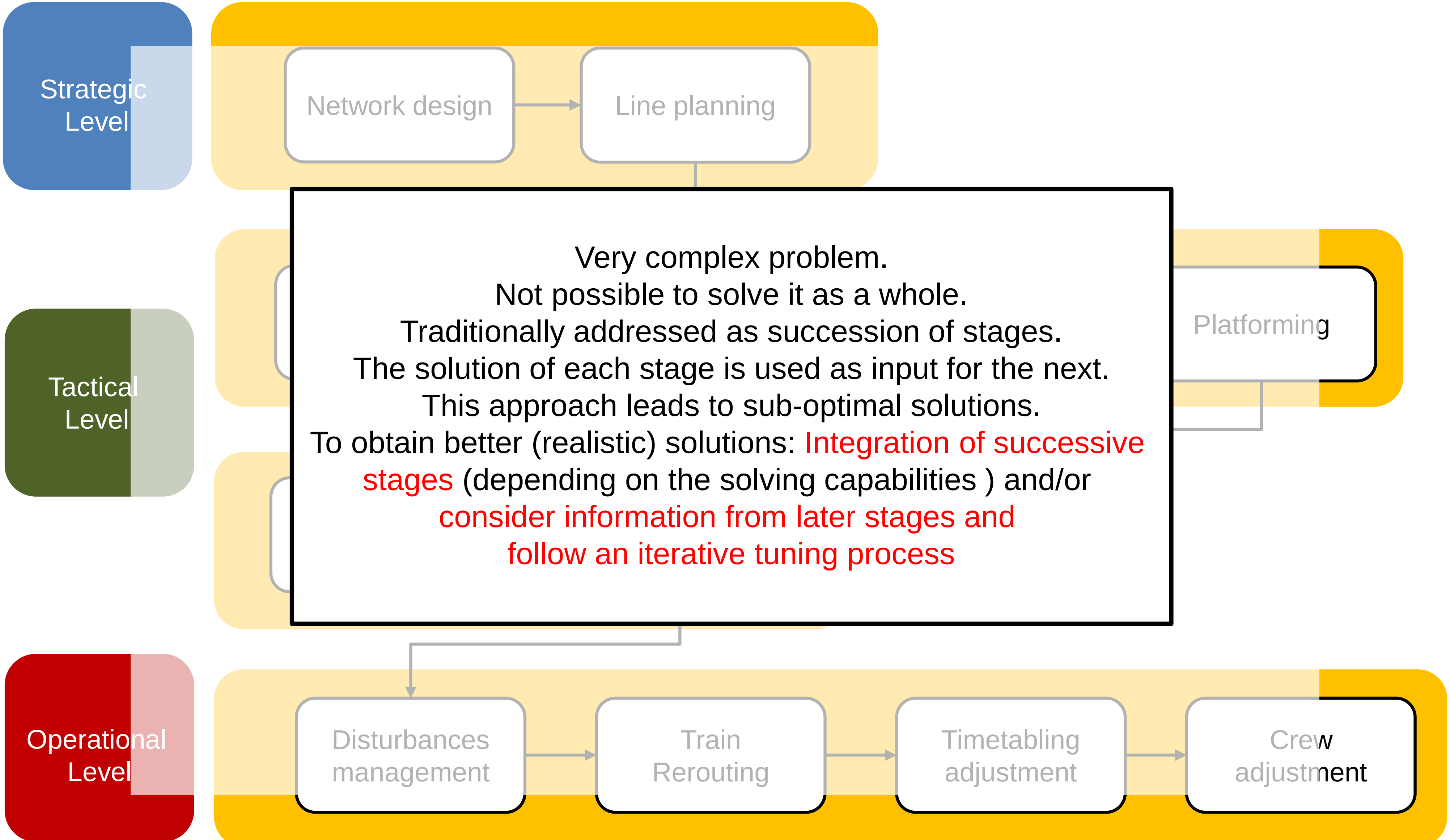
3 A quadratic MIP formulation

4 Illustration

5 Conclusions and further research

Introduction: The railway transportation planning process





Very complex problem.
Not possible to solve it as a whole.
Traditionally addressed as succession of stages.
The solution of each stage is used as input for the next.
This approach leads to sub-optimal solutions.
To obtain better (realistic) solutions: Integration of successive stages (depending on the solving capabilities) and/or consider information from later stages and follow an iterative tuning process

Strategic Level

Network design

From a specific scenario find the network (stations and stretches) that will be used to transport people using a public transportation mode

Topology of the city (scenario)
Potential stations places, potential stretches
Physical constraints (land characteristics)
Historical buildings……..

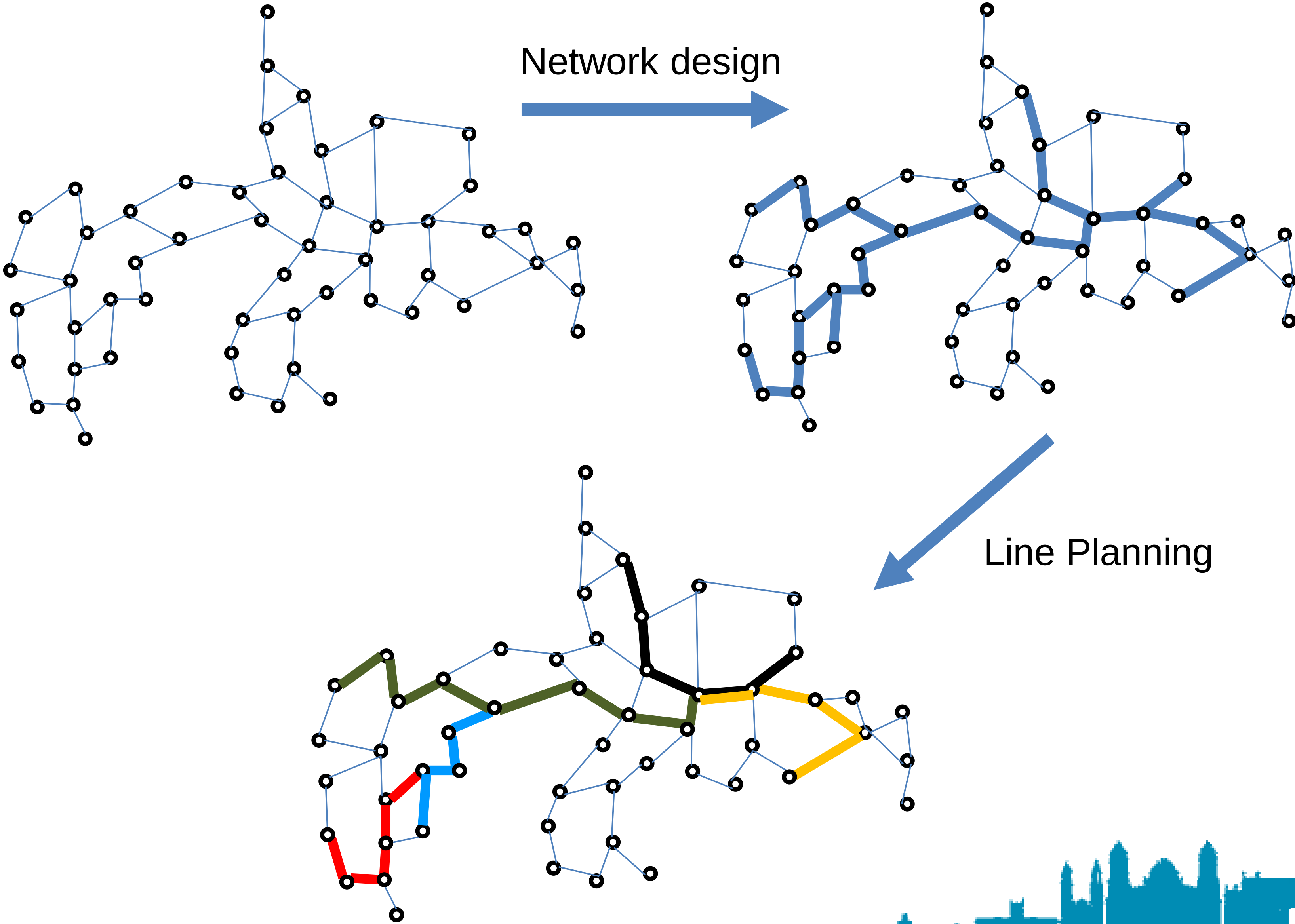
Demand mobility patterns
Alternative transportation modes
Population density……

Line planning

Given a certain network (obtained from the previous stage) define several corridors (lines) where trains will perform regular services to move people among stations according to one or several demand patterns

Origin and destination of trips
Line constraints (number, length, number of stations)
Construction cost

Distance between stations
Planning horizon
Alternative transportation modes
Demand coverage


Network design
Line Planning

Main Line planning approaches:

Survey: Schöbel, A. Line planning in public transportation: models and methods. OR Spectrum (2012) 34:491–510.

- **From a line pool:** Generating a set of plausible candidate lines. Combining a subset of lines to define the network (genetic algorithms and other heuristics) or generate new candidate lines from dual information (column generation) with different criteria (Maximizing trip coverage, minimum number of transfers, minimum costs, maximum social welfare…)

Fan W, Machemehl RB (2006) Optimal transit route network design problem with variable transit demand: genetic algorithm approach. J Transp Eng 132:40–51.

Borndörfer R, Grötschel M, Pfetsch ME (2007) A column generation approach to line planning in public transport. Transp Sci 41:123–132.

- **Constructive**: Selecting nodes and edges from an underlaying network while maintaining network structure and line constraints with different criteria (Trip coverage, Minimum transfers, minimum costs, maximum social welfare…)

Previous research

Network design and line planning

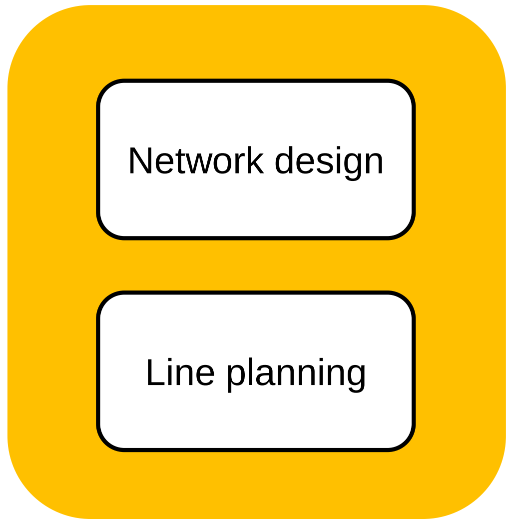
- Canca, D., De-Los-Santos, A., Laporte, G., Mesa, J.A. 2016. A general rapid network design,
  line planning and fleet investment integrated model. Annals of Operations Research. 246 (1–2), 127–144.

- Canca, D., De-Los-Santos, A., Laporte, G., Mesa, J. A. 2017. An adaptive neighborhood search metaheuristic for the integrated railway rapid transit network design and line planning problem. Computers & Operations Research 78, 1-14

- Canca, D., De los Santos, A., Mesa, J. A., Laporte, G. 2017. The railway network design, line planning and capacity problem: An adaptive large neighborhood search metaheuristic. In Advanced Concepts, Methodologies and Technologies for Transportation and Logistics. Jacek Zak, Yuval Hadas, Riccardo Rossi (Eds.). Advances in Intelligent Systems and Computing.

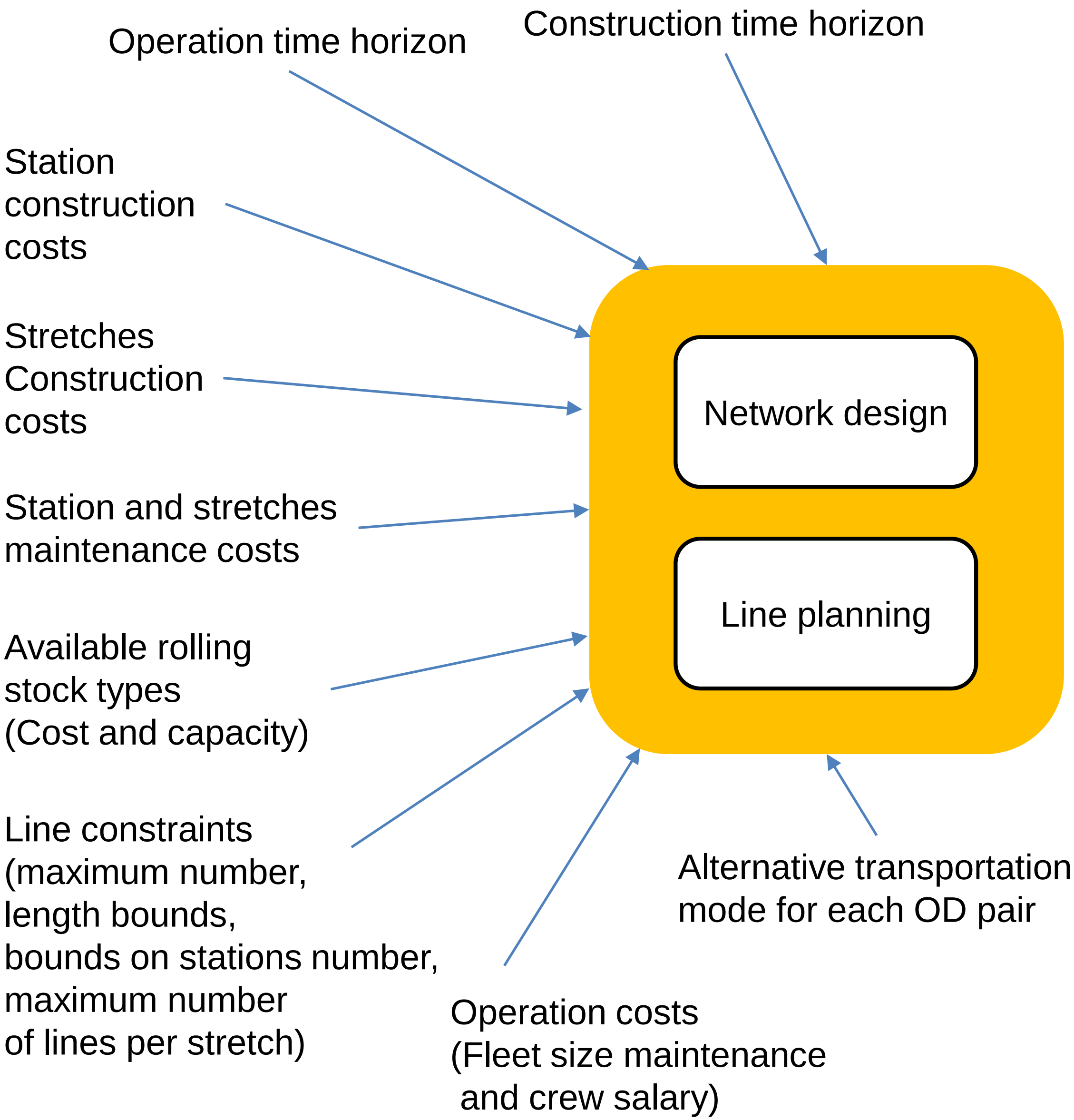
- Canca, D., De-Los-Santos, A., Laporte, G., Mesa, J.A. The Integrated Railway Rapid Transit Network Design and Line Planning Problem with Elastic Demand. Transportation Research E. Under review (2nd round).

Constructive approach



Our general approach I

Constructive approach



Our general approach I

Constructive approach

Operation time horizon

Construction time horizon

Station construction costs

Stretches Construction costs

Station and stretches maintenance costs

Available rolling stock types (Cost and capacity)

Line constraints (maximum number, length bounds, bounds on stations number, maximum number of lines per stretch)

Operation costs (Fleet size maintenance and crew salary)

Alternative transportation mode for each OD pair

Network design

Line planning

Maximize the Net Profit along the planning horizon
(balance between costs and revenue due to captured demand)

Output:

Definition of lines
Selection of rolling stock
Frequencies

Captured demand (depending on fares and competing modes)

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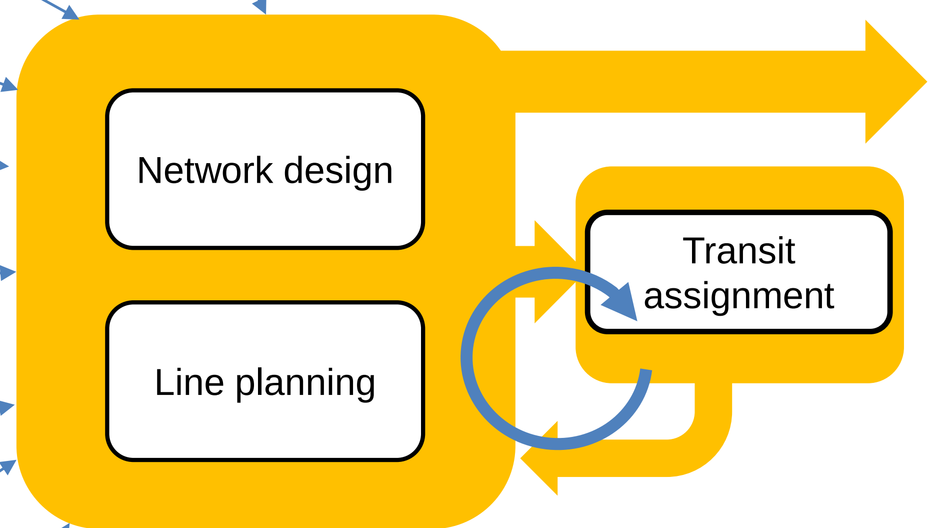

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The need of the temporal dimension:

The network will be constructed during certain time interval (10, 15 years). The passenger demand will change along time due to the inclusion of this new mode and other reasons (new transportation modes, changes in land use, etc.) . Moreover, the passenger demand is partially served during the construction period every time a facility (line or partial line) is put into operation.

The network will be exploited during a long time interval. It is necessary to consider a long time scenario in order to analyse the investment (30 - 40 - 50 years) using the Return of Investment (ROI) or the Net Present Value (NPV) .

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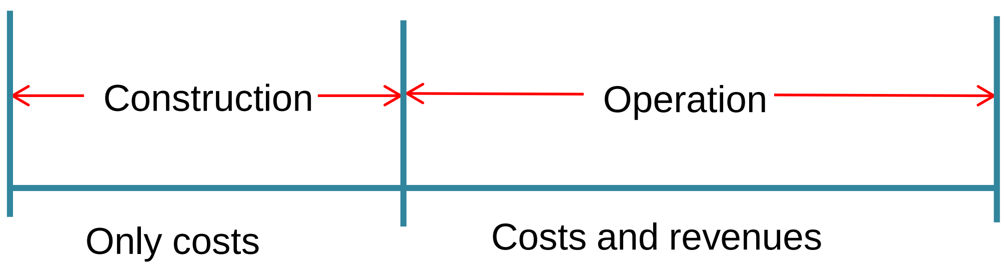
The inclusion of a detailed time management in our previous mathematical models, make them completely intractable.

The need of the temporal dimension:

The network will be constructed during certain time interval (10, 15 years). The

In our past works: The planning horizon (interval to analyse investments) is divided into two parts.

- **First part:** A certain time interval needed to construct the network.
- **Second part:** A time interval where the network is operated. This interval starts after the construction is finished.



NOT TOO MUCH REAL, BUT TRACTACBLE !!!!

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Suppose we have solved a network design problem following, for instance, one of our approaches and we have a “**good solution to be constructed**”.

QUESTION:

If we assume a certain behaviour in the transportation demand affecting this mode, Taking into account that the network can be partially operated during its construction

WHAT IS THE BEST WAY TO CONSTRUCT THE NETWORK, i.e.

WHAT IS THE MOST CONVENIENT ORDER IN THE

CONSTRUCTION PROJECT?

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A particular case of a
Double Resource-constrained scheduling problem

- The network is divided into segments (A succession of stations and links).
- Segments are put into operation when they are finished accordingly to certain rules imposed by the construction project plan (later).
- Demand is partially captured during the construction phase.
- We consider that passenger demand is no constant.
- We want to schedule the construction of segments in order to achieve the best **Net Present Value** considering a long planning time interval.
- The **construction order** affects the return of the investment.
- The construction of each segment requires certain inversion (according to the specified budget) and the usage of other resources (the most important, for subway segments: The need of BORING MACHINES).

Segments' Construction connectivity

Segment B can be started once segment A is started -> **Weak construction connectivity**

Segment B cannot be started until segment A is finished -> **Strong construction connectivity**

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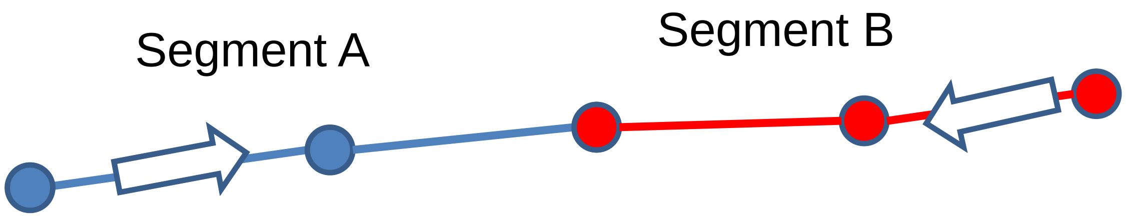

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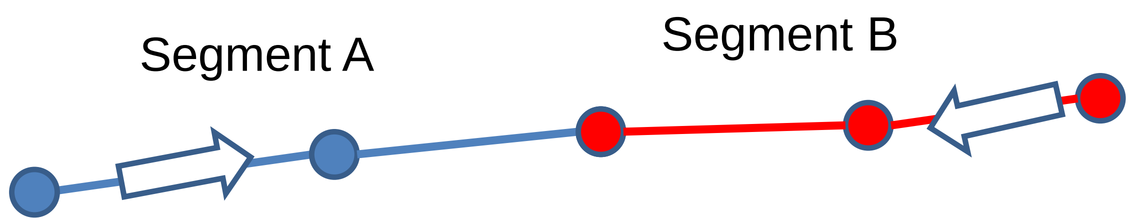

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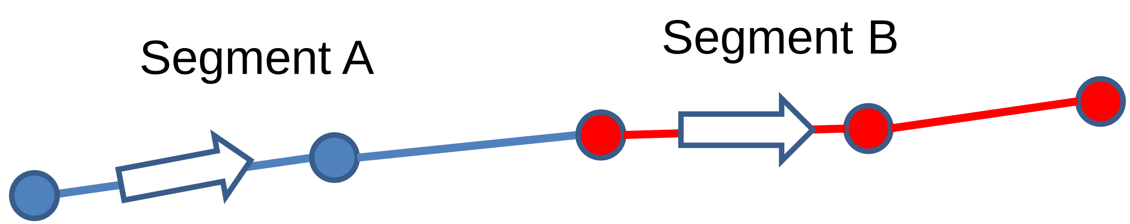
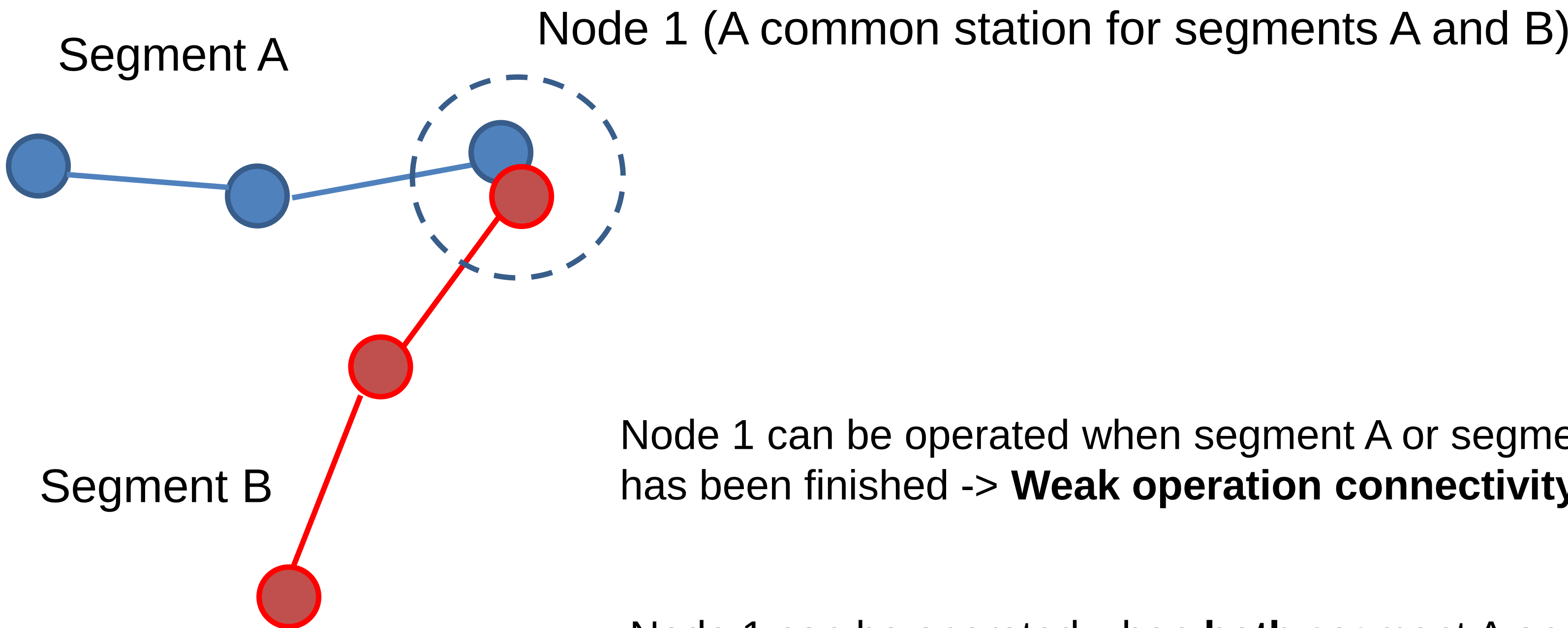


Nodes' Operation connectivity



Node 1 can be operated when segment A or segment B has been finished -> **Weak operation connectivity**

Node 1 can be operated when **both** segment A and segment B have been finished -> **Strong operation connectivity**

Outline

Sets

- $\mathcal{N} = \{1, \dots, N\}$, set of segments.
- $\mathcal{V} = \{1, \dots, K\}$, set of nodes representing stations.
- $\mathcal{T} = \{1, \dots, T\}$, set of periods.
- $\mathcal{L}$, set of lines to be constructed.
- $\mathcal{L}'$, mandatory set of lines to be constructed.
- $\mathcal{L}''$, non-mandatory, set of lines to be constructed.
- L, length of the planning horizon expressed in years.
- $\mathcal{A}$, adjacency matrix of segments in $\mathcal{N}$. Each element $a_{ij} \in \mathcal{A}$ is equal to 1 if segments $i, j \in \mathcal{N}$ share a station, zero otherwise.

Auxiliary sets

- $\mathcal{N}(k)$, set of segments containing the node $k \in \mathcal{V}$.
- $\mathcal{N}_w$, set of segments with weak construction connectivity.
- $\mathcal{N}_s$, set of segments with strong construction connectivity..
- $\mathcal{V}(i)$, set of nodes belonging to segment $i \in \mathcal{N}$.
- $\mathcal{V}_w$, set of nodes with weak operation connectivity.
- $\mathcal{V}_s$, set of nodes with strong operation connectivity.
- $\mathcal{V}_w(i)$, set of nodes with weak operation connectivity in $\mathcal{V}(i)$.
- $\mathcal{V}_s$(i), set of nodes with strong operation connectivity in $\mathcal{V}(i)$.

Sets and parameters II

Segment's data

- $\mathcal{O}(t)$, origin-destination daily average matrix for period $t \in \mathcal{T}$. Each element of $\mathcal{O}(t)$ is denoted by $o_{kr}(t)$, $k, r \in \mathcal{V}$.
- C_i, cost of constructing segment $i \in \mathcal{N}$, including the construction cost of the facilities needed to operate the segment at stations $k \in \mathcal{V}(i)$.
- D_i, number of periods needed to construct segment $i \in \mathcal{N}$.

Budget

- R_t, total expenditure allowed at period $t \in \mathcal{T}$ if the construction budget is considered as a renewable resource.
- R, total expenditure allowed for the project if the construction budget is considered as a non-renewable resource.

Economical parameters

- σ, discount rate.
- ρ, initial ticket price.
- Δ_I, annual rate of interest.
- Δ_ρ, annual increase rate of ticket price ρ.
- Δ_C, annual increase rate of construction costs.
- p_{kr}, annual increase rate of the element $o_{kr}(t) \in \mathcal{O}(t)$ only for the case in which a linear increase rate of the demand matrix elements is considered.

Boring machines

- N_M set of segments whose construction requires a tunnel boring machine.
- M, number of tunnel boring machines.
- C_M, annual rental cost of tunnel boring machines.

Variables

- $x_i^t = 1$, if the segment $i \in \mathcal{N}$ is being built in period $t \in \mathcal{T}$, 0 otherwise.
- $y_i^t = 1$, if the construction of the segment $i \in \mathcal{N}$ starts in period $t \in \mathcal{T}$, 0 otherwise.
- $w_i^t = 1$, if the segment $i \in \mathcal{N}$ has been constructed before period $t \in \mathcal{T}$, 0 otherwise.
- $z_k^t = 1$, if the station $k \in \mathcal{V}$ is operational in period $t \in \mathcal{T}$, 0 otherwise.
- H_t, remaining construction budget at the end of period t.
- B_t, slack variables to determine the number of unused boring machines at period t.

Constraints I

Constraints I

From a construction point of view, we distinguish between two type of lines, mandatory and desirable. Mandatory lines must be completely constructed. In the case of desirable lines it is possible to construct only some segments of the line, depending on the available budget.

In mandatory lines, segments have to be constructed

$$\sum_{t=1}^{T} y_i^t = 1, \quad i \in \mathcal{N}_\ell,\ \ell \in \mathcal{L}'. \qquad (1)$$

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Additionally, since the construction duration of segment i is D_i, the num variables x_i^t must add up to the segment's duration:

The duration of segments is known

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For non-mandatory lines, constraints (1) must be relaxed:

In optional lines, constraints 1 are relaxed

$$\sum_{t=1}^{T} y_i^t \leq 1, \quad i \in \mathcal{N}_\ell,\ \ell \in \mathcal{L}''. \qquad (3)$$

Constraints II

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In this situation, the construction duration of a segment i could be D_i or (depending on whether the segment is built or not. Constraints (2) must b modified to capture this behavior:

$$\sum_{t=1}^{T} x_i^t = D_i \sum_{t=1}^{T} y_i^t, \quad i \in \mathcal{N}_\ell,\ \ell \in \mathcal{L}''. \tag{4}$$

Linking x and y variables for non mandatory segments

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Since the construction duration of a segment can exceed one period and we only consider non-preemptive tasks, it is necessary to ensure that once starte
a segment must be constructed without interruption. Then, if the constructi
of segment i starts at period t ($y_i^t = 1$) and the duration is D_i, the segme
must be in construction during periods $t, t+1, t+2, \ldots, t+D_i-1$:

$$D_i \cdot y_i^t \leq \sum_{s=t}^{t+D_i-1} x_i^s, \quad i \in \mathcal{N},\ t \in \mathcal{T}. \qquad (5)$$

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Consecutiveness constraints (non-preemptive tasks).

Since we do not impose an initial set of segments to be constructed, the fi
segments to initiate the network must be selected on the basis of the expect
profit. Then for period $t = 1$ we simply impose the need to start the construct
of several connected segments, no matter what they are. To this end, we imp
the following constraints:

$$y_i^1 + y_j^1 \leq a_{ij} + 1, \quad i, j \in \mathcal{N},\ j > i. \tag{6}$$

Connectivity constraints for the initial period

Constraints III

Here we have to distinguish between segments with weak construction co nectivity and those with strong connectivity. In the first case, at period t segment i can be initiated only if at least one of its connected segments h been previously initiated:

$$y_i^t \leq \sum_{s=1}^{t-1} \sum_{j=1}^{N} a_{ij} \cdot y_j^s, \quad i \in \mathcal{N}_w, \; t \in \mathcal{T} \backslash \{1\}. \tag{7}$$

Enforcing weak construction connectivity

In the second case, a segment i can be initiated at period t only if at least one of its connected segments has been completed:

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The next two families of constraints are imposed in order to know when segment is completed. For each segment $i \in \mathcal{N}$ in period $t \in \mathcal{T}$, if the sum c all the variables $x_i^s (s \leq t)$, which denotes the periods in which segment i ha been constructed, is precisely D_i, then segment i is completed and the binar variable w_i^t must be activated:

$$D_i \cdot w_i^t \leq \sum_{s=1}^{t-1} x_i^s, \quad i \in \mathcal{N},\ t \in \mathcal{T} \backslash \{1\}, \tag{9}$$

$$\sum_{s=1}^{t-1} x_i^s \leq D_i - 1 + w_i^t, \quad i \in \mathcal{N},\ t \in \mathcal{T} \backslash \{1\}. \tag{10}$$

Determining the completion of segments

In order to later compute the captured demand we need to known when a station k is active for operation (decisions which are represented by variables z_{kt}).

The first case allows the operation of a node if at least one of the segments containing it has been completed:

$$z_k^t \leq \sum_{j \in \mathcal{N}(k)} w_j^t, \qquad k \in \mathcal{V}_w,\ t \in \mathcal{T}. \tag{11}$$

If none of the segments containing node k is completed, then node k is not operational and its demand cannot be captured.

The second case, in its complete form (called full strong operation connectivity) enforces, for a given period t, the need of completing all segments containing the station before t:

$$|\mathcal{N}(k)| \cdot z_k^t \leq \sum_{j \in \mathcal{N}(k)} w_j^t, \quad k \in \mathcal{V}_s,\ t \in \mathcal{T}, \tag{12}$$

or alternatively:

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If we consider the construction budget as a renewable resource, then each period we will have an amount R_t to construct the network. Then, the total cost of all segments being constructed in period t cannot exceed R_t:

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t = R_t + (1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T}, \; H_0 = 0. \tag{13}$$

Note that the remaining budget of a period is added to the budget of the following period, which is modeled by using slacks variables. In contrast, if the budget is considered as a non-renewable resource the initial budget R will be exhausted as the construction project is executed. Therefore, for the first period

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot x_i^1 + H_1 = R, \tag{14}$$

and for the remaining periods

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t = (1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T} \setminus \{1\}. \tag{15}$$

If we consider the construction budget as a renewable resource, then eac period we will have an amount R_t to construct the network. Then, the tota cost of all segments being constructed in period t cannot exceed R_t:

Budget limitation for each period (renewable resource)

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t = R_t + (1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T}, \; H_0 = 0. \tag{13}$$

Note that the remaining budget of a period is added to the budget of the following period, which is modeled by using slacks variables. In contrast, if the budget is considered as a non-renewable resource the initial budget R will be exhausted as the construction project is executed. Therefore, for the first period

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot x_i^1 + H_1 = R, \tag{14}$$

and for the remaining periods

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t = (1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T} \backslash \{1\}. \tag{15}$$

If we consider the construction budget as a renewable resource, then eac period we will have an amount R_t to construct the network. Then, the tota cost of all segments being constructed in period t cannot exceed R_t:

Budget limitation for each period (renewable resource)

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t =$$
$$R_t + (1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T}, \; H_0 = 0. \tag{13}$$

Note that the remaining budget of a period is added to the budget of th following period, which is modeled by using slacks variables. In contrast, if th budget is considered as a non-renewable resource the initial budget R will b exhausted as the construction project is executed. Therefore, for the first perio

Or budget limitation in case of considering it as a global resource

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot x_i^1 + H_1 = R, \tag{14}$$

and for the remaining periods

$$\sum_{i=1}^{N} \frac{C_i}{D_i} \cdot (1 + \Delta_C L(t-1)) \cdot x_i^t + H_t =$$
$$(1 + \Delta_I L) \cdot H_{t-1}, \quad t \in \mathcal{T} \setminus \{1\}. \tag{15}$$

A second important resource for the construction of subway network the number of tunnel boring machines. If this is the case, we can control project deployment by incorporating constraints that take the number of m into account:

Boring machines number. Limitation per period (renewable resource)

$$\sum_{i=1}^{N_M} x_i^t + B_t = M, \quad t \in \mathcal{T}. \tag{16}$$

The objective function consists on maximizing the net profit, i.e. the t revenue achieved along certain planning horizon as a consequence of the c tured demand, minus the construction cost, both discounted to the begin of the planning horizon.

Capturing demand Capturing the demand as the stations are going to be active

Revenues

$$\sum_{t=1}^{T} \frac{\rho \cdot (1 + \Delta_\rho \cdot L \cdot (t-1)) \cdot 365L}{e^{\sigma L t}} \sum_{k=1}^{K} \sum_{r=1}^{K} o_{kr}(t) \, z_k^t \cdot z_r^t. \tag{17}$$

In the linear case, $o_{kr}(t) = o_{kr}(0) \cdot (1 + p_{kr}(t-1))$ and hence the net revenue will be

The linear case

$$\sum_{t=1}^{T} \frac{\rho \cdot (1 + \Delta_\rho \cdot L \cdot (t-1)) \cdot 365L}{e^{\sigma L t}} \sum_{k=1}^{K} \sum_{r=1}^{K} \boxed{o_{kr}(0) \cdot (1 + p_{kr}(t-1))} \cdot z_k^t \cdot z_r^t. \tag{18}$$

The net cost considers, period by period, the amount spent in the construction of the corresponding segments and the extra cost due to the rent of additional tunnel boring machines. Taking into account that C_M is the annual rental cost of additional boring machines, $M-B_t-1$ is the number of additional tunnel boring machines used in period t, C_i represents the total cost of segment $i \in N$ and D_i the number of periods needed to construct segment i, supposing a linear cost per period, the net cost of the project is given by

Costs

$$\sum_{t=1}^{T} \frac{(1+\Delta_C \cdot L \cdot (t-1))}{e^{\sigma L(t-1)}} \left(C_M \cdot L \cdot (M-B_t-1) + \sum_{i=1}^{N} \frac{C_i}{D_i} \cdot x_i^t \right). \qquad (19)$$

Objective function

$$\text{Maximize} \left[\sum_{t=1}^{T} \frac{\rho \cdot (1+\Delta_\rho \cdot L \cdot (t-1)) \cdot 365L}{e^{\sigma L t}} \sum_{k=1}^{K} \sum_{r=1}^{K} o_{kr}(t) \cdot z_k^t \cdot z_r^t \right.$$
$$\left. - \sum_{t=1}^{T} \frac{(1+\Delta_C \cdot L \cdot (t-1))}{e^{\sigma L(t-1)}} \left(C_M \cdot L \cdot (M-B_t-1) + \sum_{i=1}^{N} \frac{C_i}{D_i} \cdot x_i^t \right) \right]. \qquad (20)$$

The main differences with respect to a resource constrained scheduling problem:

- In a resource constrained scheduling problem there is a set of initial tasks (at least one). In our case, the initial task is unknown.
- The usual objective function in an scheduling problem consists in minimizing the total completion time instead of maximizing the net profit.
- Both cost and revenues depend on the order of tasks completion, which also differs from the usual formulation of resource constrained projects, especially in those related with revenues, that are not considered in this kind of problems.
- Finally, the residual budget of each period is also used as input for next periods, which is also a novelty with respect to the usual formulation.

Outline

Illustration I

Illustration I

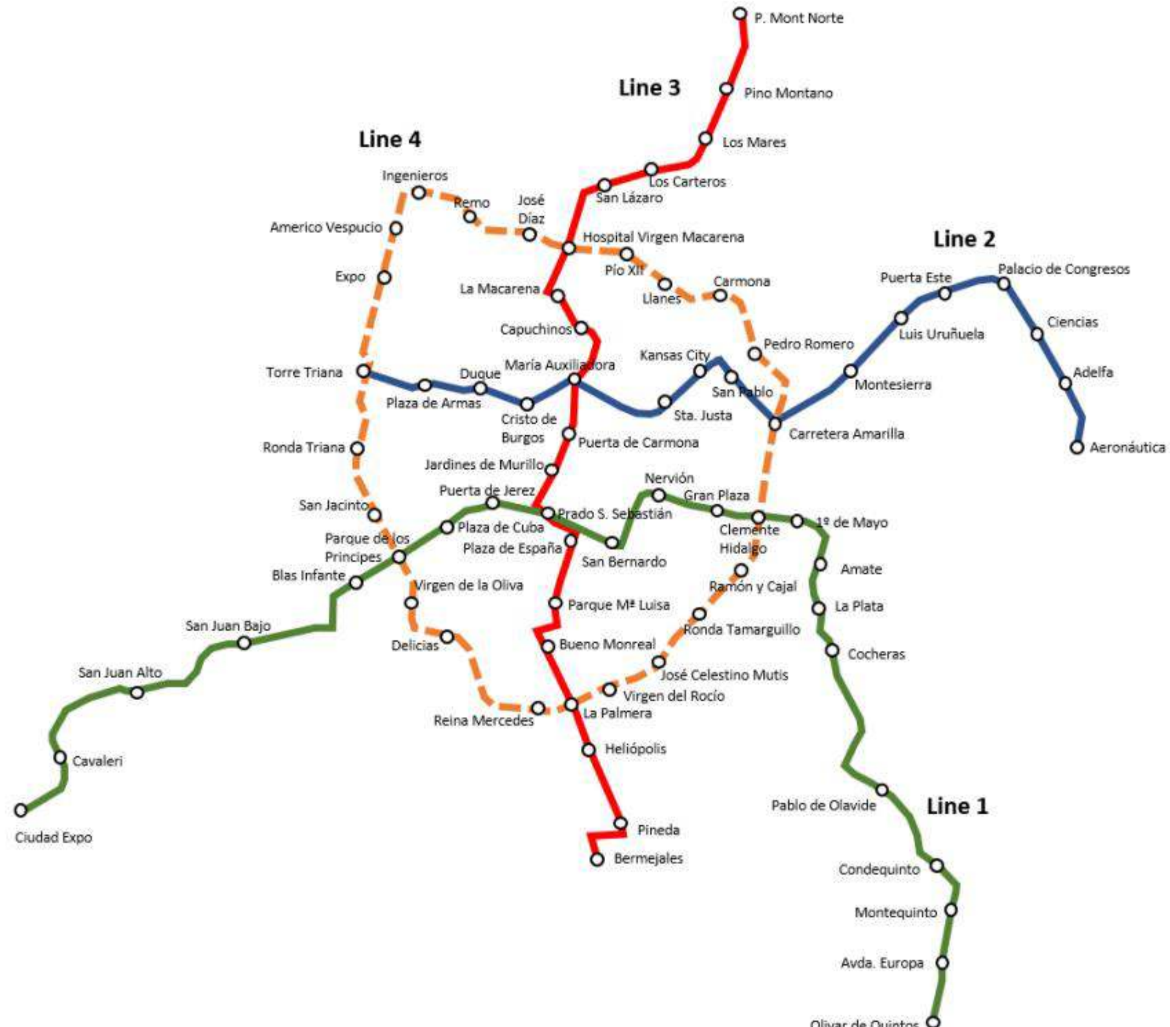
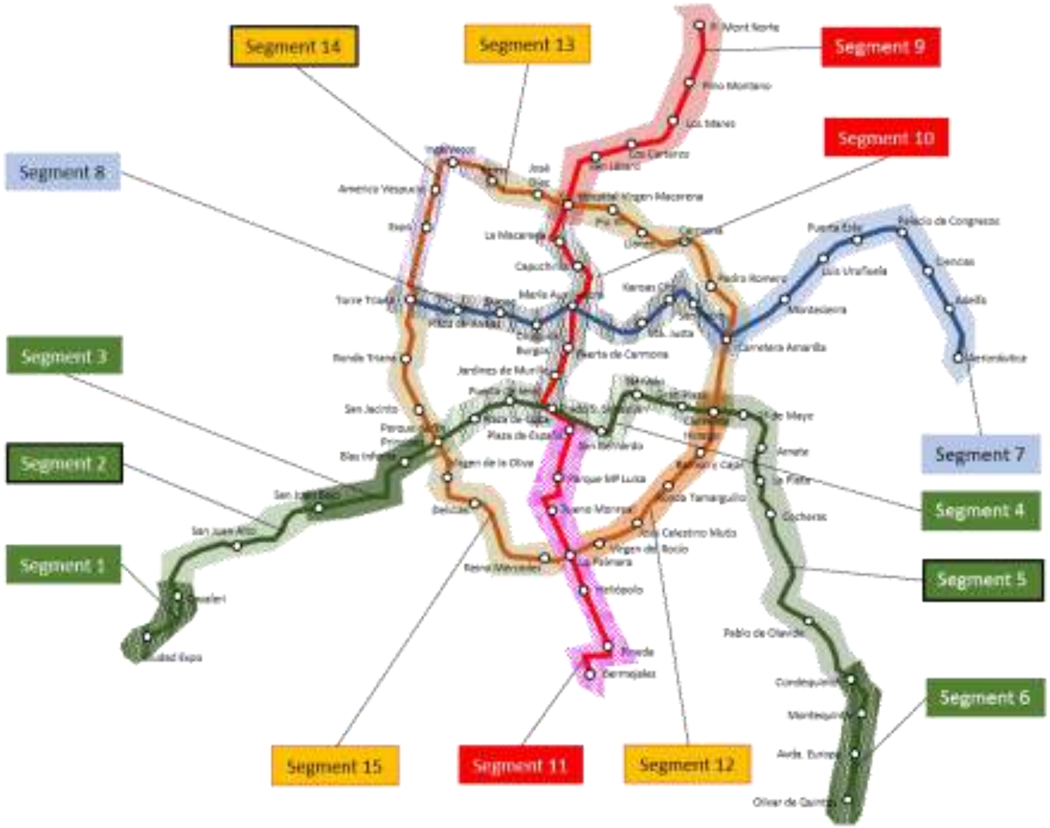


Illustration I

Lines							
1		2		3		4	
Length (m)							
18429		10769		11381		13810	
Number of stations							
22		16		17		18	
Stations							
N.	Name	N.	Name	N.	Name	N.	Name
1	Olivar de Quintos	23	Aeronáutica	39	P. Mont Norte	56	Virgen del Rocío
2	Avda. Europa	24	Adelfa	40	Pino Montano	57	José Celestino Mutis
3	Montequinto	25	Ciencias	41	Los Mares	58	Ronda Tamarguillo
4	Condequinto	26	Palacio de Congresos	42	Los Carteros	59	Ramón y Cajal
5	Pablo de Olavide	27	Puerta Este	43	San Lázaro	60	Pedro Romero
6	Cocheras	28	Luis Uruñuela	44	Hosp. Virgen Macarena	61	Carmona
7	La Plata	29	Montesierra	45	La Macarena	62	Llanes
8	Amate	30	Carretera Amarilla	46	Capuchinos	63	Pío XII
9	1° de Mayo	31	San Pablo	47	Puerta de Carmona	64	José Díaz
10	Clemente Hidalgo	32	Kansas City	48	Jardines de Murillo	65	Remo
11	Gran Plaza	33	Santa Justa	49	Plaza de España	66	Ingenieros
12	Nervión	34	María Auxiliadora	50	Parque María Luisa	67	Américo Vespucio
13	San Bernardo	35	Cristo de Burgos	51	Bueno Monreal	68	Expo
14	Prado S. Sebastián	36	Duque	52	La Palmera	69	Ronda Triana
15	Puerta de Jerez	37	Plaza de Armas	53	Heliópolis	70	San Jacinto
16	Plaza de Cuba	38	Torre Triana	54	Pineda	71	Virgen de la Oliva
17	P. de los Príncipes			55	Bermejales	72	Delicias
18	Blas Infante					73	Reina Mercedes
19	San Juan Bajo						
20	San Juan Alto						
21	Cavaleri						
22	Ciudad Expo						

Segments

Lines	N.	Nodes	Length (m)	Type Under. on Surface	N. Periods	Total cost (thousand €)	Cost per period (thousand € per period)
Line 1	1	22, 21	844.6	U	1	31356.11	31356.11
	2	21, 20, 19	4049.7	S	4	37407.58	9351.89
	3	19, 18, 17	2029.5	U	2	60899.54	30449.77
	4	17, 16, 15, 14, 13, 12, 11, 10	4602.3	U	4	147677.43	36919.36
	5	10, 9, 8, 7, 6, 5, 4	4844.3	S	4	53019.61	13254.90
	6	4, 3, 2, 1	2058.2	U	2	69422.51	34711.25
Line 2	7	23, 24, 25, 26, 27, 28, 29, 30	5479.8	U	5	163633.62	32726.72
	8	30, 31, 32, 33, 34, 35, 36, 37, 38	5288.9	U	5	168161.98	33632.40
Line 3	9	39, 40, 41, 42, 43, 44	3631.6	U	3	114029.08	38009.69
	10	44, 45, 46, 34, 47, 48, 14	3115.0	U	3	112636.20	37545.40
	11	14, 49, 50, 51, 52, 53, 54, 55	4614.4	U	4	147898.41	36974.60
Line 4	12	52, 56, 57, 58, 59, 10	3297.1	U	3	107947.32	35982.44
	13	10, 30, 60, 61, 62, 63, 44, 64, 65	5747.4	U	5	176498.31	35299.66
	14	65, 66, 67, 68, 38	2854.8	S	3	37976.32	12658.77
	15	38, 69, 70, 17, 71, 72, 73, 52	5207.9	U	5	158689.17	31737.83
						1587253.17	



Segments

Lines	N.	Nodes	Length (m)	Type Under. on Surface	N. Periods	Total cost (thousand €)	Cost per period (thousand € per period)
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	2	21, 20, 19	4049.7	S	4	37407.58	9351.89
	3	19, 18, 17	2029.5	U	2	60899.54	30449.77
	4	17, 16, 15, 14, 13, 12, 11, 10	4602.3	U	4	147677.43	36919.36
	5	10, 9, 8, 7, 6, 5, 4	4844.3	S	4	53019.61	13254.90
	6	4, 3, 2, 1	2058.2	U	2	69422.51	34711.25
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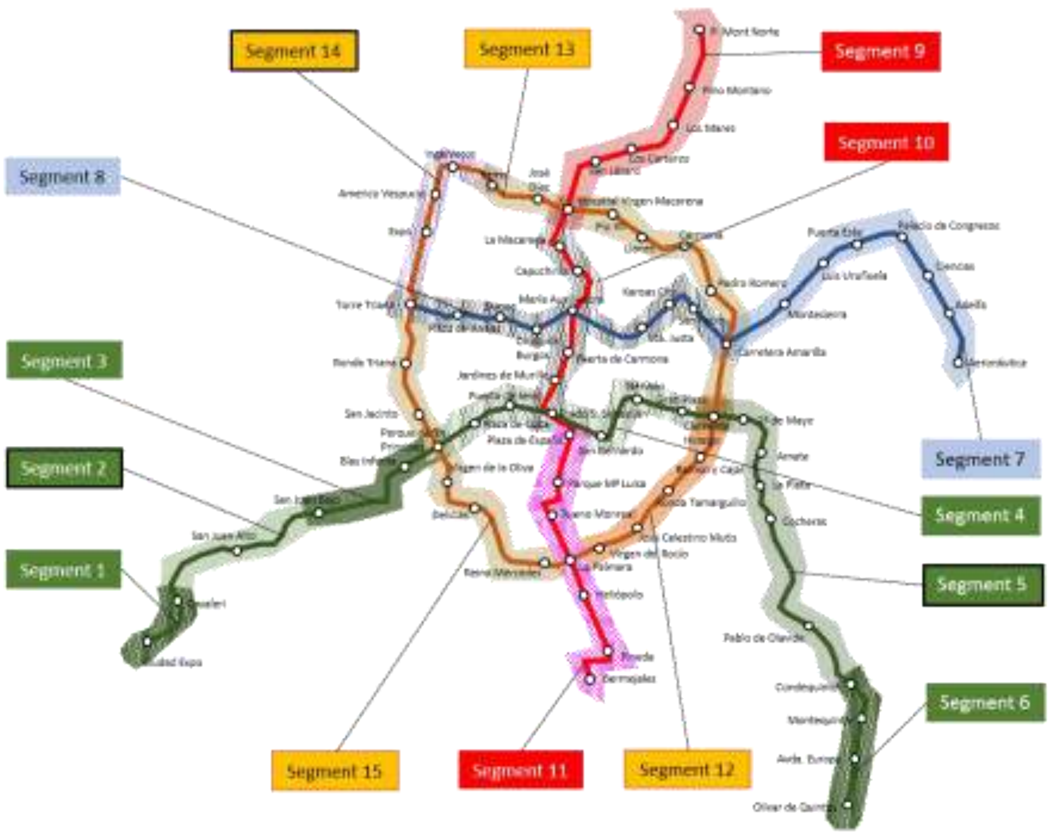
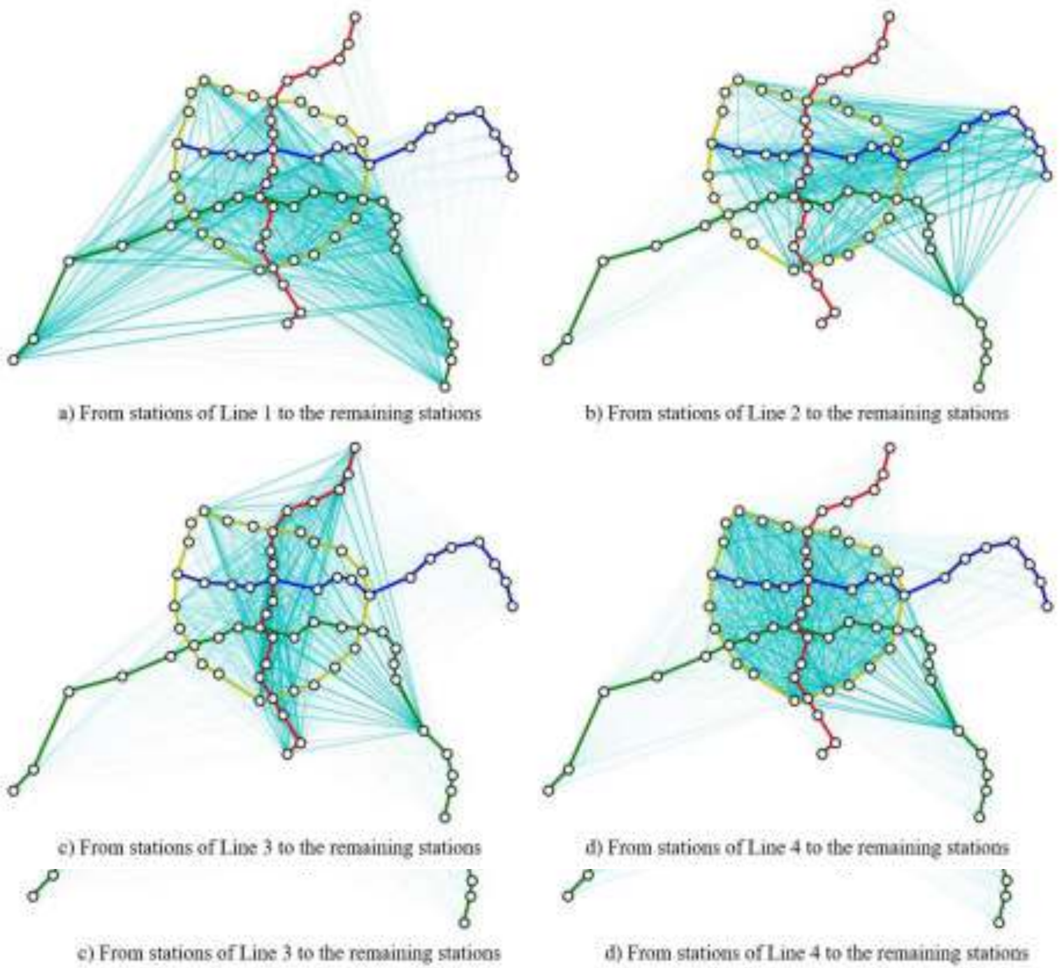
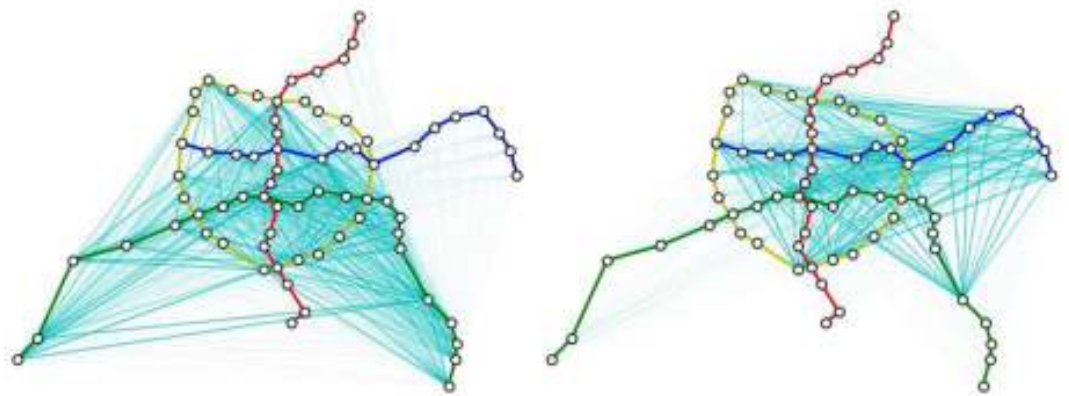
	15	38, 69, 70, 17, 71, 72, 73, 52	5207.9	U	5	158689.17	31737.83
						1587253.17	



- Boring machines speed: 15m/day
- Length of a time period: 3 months.
- All segments with the exception of segments 2, 5 and 14 are underground.
- Planning horizon is set to 25 years (100 time periods).
- Annual cost increment of 2%.
- Discount annual rate of 5%,
- Initial ticket price of 1 € with 2% of annual increment



- **Demand:**
  60 million passengers per year
  5256 OD pairs
  Annual increment of demand of 1%
- **Segment cost:**
  20,000 thousand € per tunnel km (double track)
  5,000 thousand € per km on surface double tracks.
- **Station cost:**
  8,000 thousand € per station
  3,000 thousand for on-surface station facilities.
- **Renewable budget** of 300 Millions € /year
- Boring machines
  3 tunnel boring machines
  Rental cost of 1,440 thousand €/year
  (120,000 € per month).
- **Weak construction connectivity**
- **Weak operation connectivity for all the stations.**



a) From stations of Line 1 to the remaining stations   b) From stations of Line 2 to the remaining stations

Instance main characteristics

- **20385 Constraints**
- **12301 Variables : 501 continuous, 11800 binary binary**
- **262800 Quadratic objective terms**
- **Standard Branch and Cut (Implemented using the python API of Gurobi)**
- **Approx: 15 min of computation time to reach the optimum.**

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  60 million passengers per year
  5256 OD pairs
  Annual increment of demand of 1%
- **Segment cost:**
  20,000 thousand € per tunnel km (double track)
  5,000 thousand € per km on surface double tracks.
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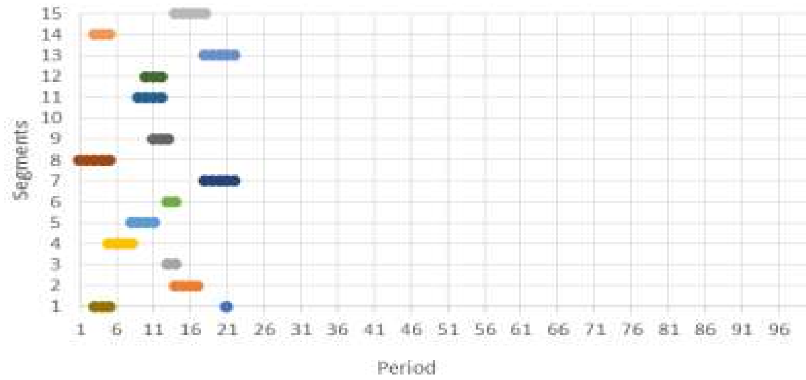
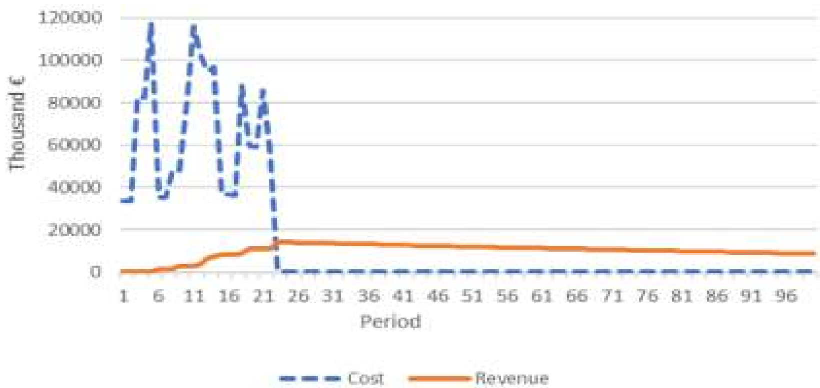
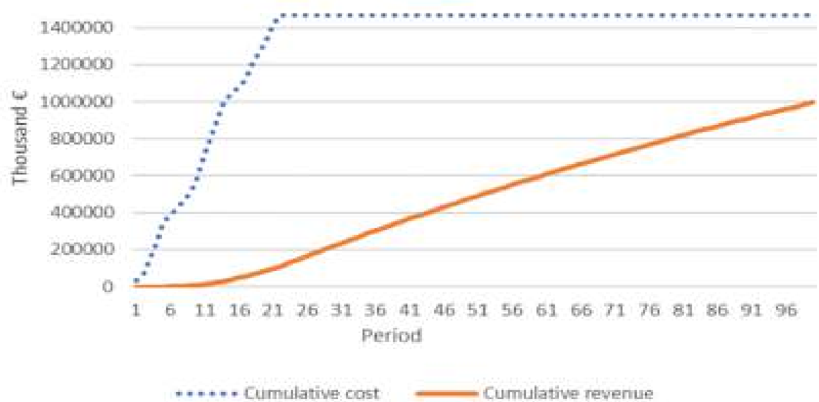


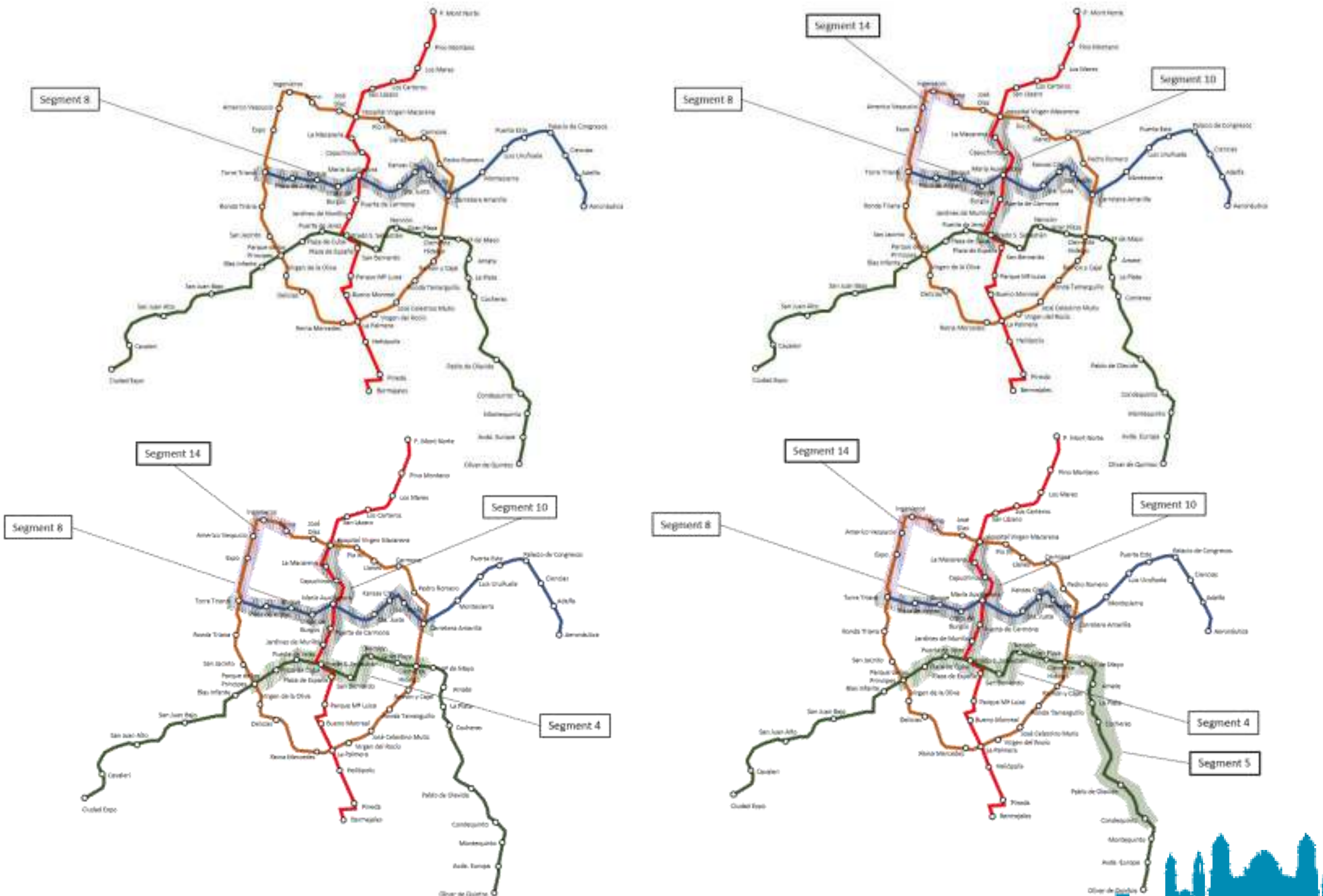
Segments schedule.
R = 300 Millions € /year.
M = 3 tunnel boring machines.



Temporal evolution of the discounted revenue and construction cost. R = Millions € /year.
M = 3 tunnel boring machines.



Temporal evolution of the cumulative discounted revenue and construction cost.
R= Millions € /year. .
M = 3 tunnel boring machines.



Segment 8
Segment 14
Segment 10
Segment 8
Segment 14
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Segment 8
Segment 4
Segment 14
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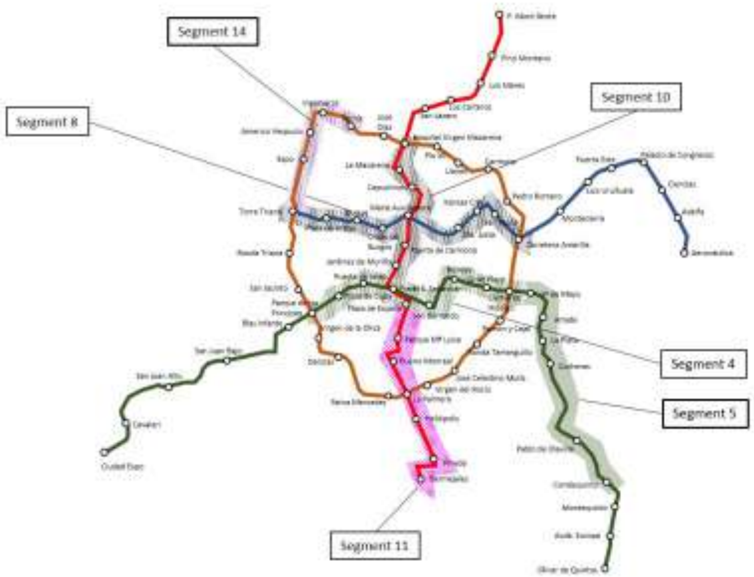
Segment 8
Segment 4
Segment 5


Segment 14
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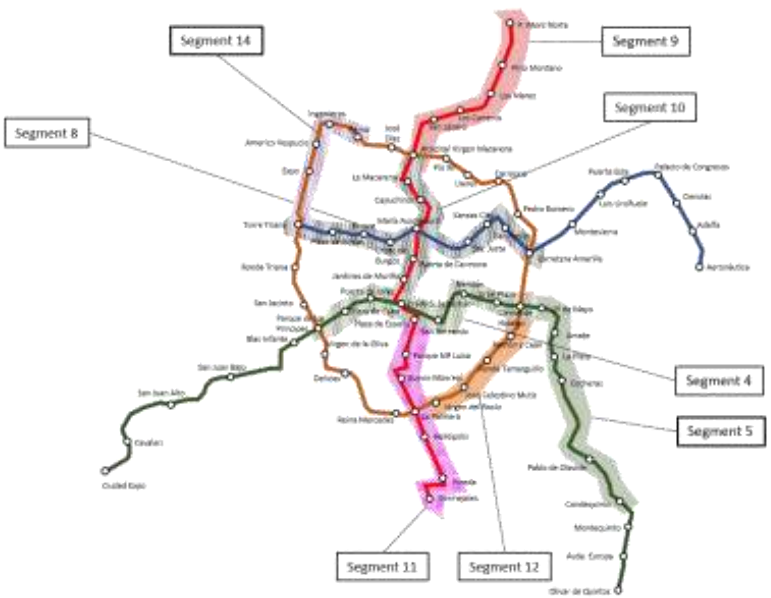
Segment 5
Segment 11


Segment 14
Segment 8
Segment 10
Segment 4
Segment 5
Segment 11
Segment 12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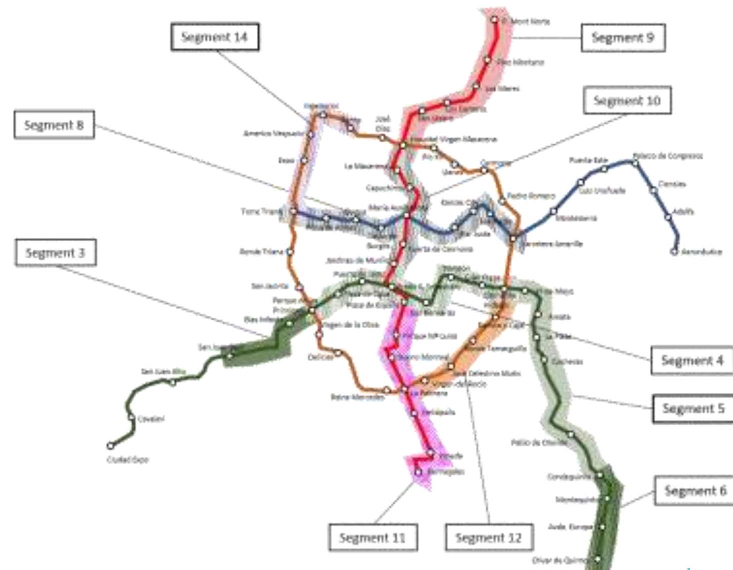
Segment 11
Segment 12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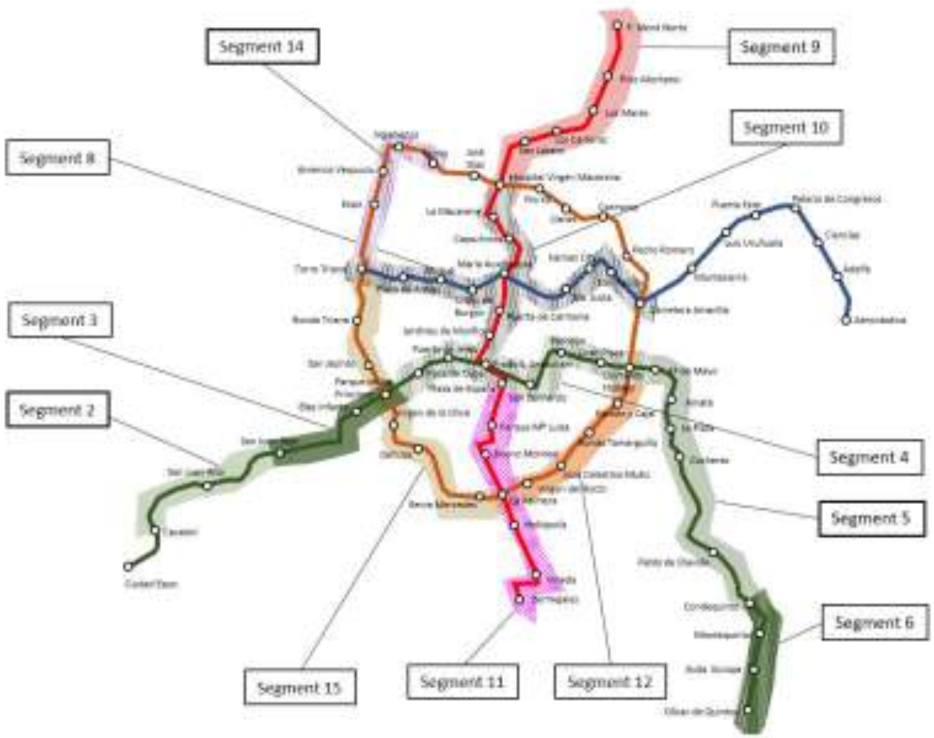
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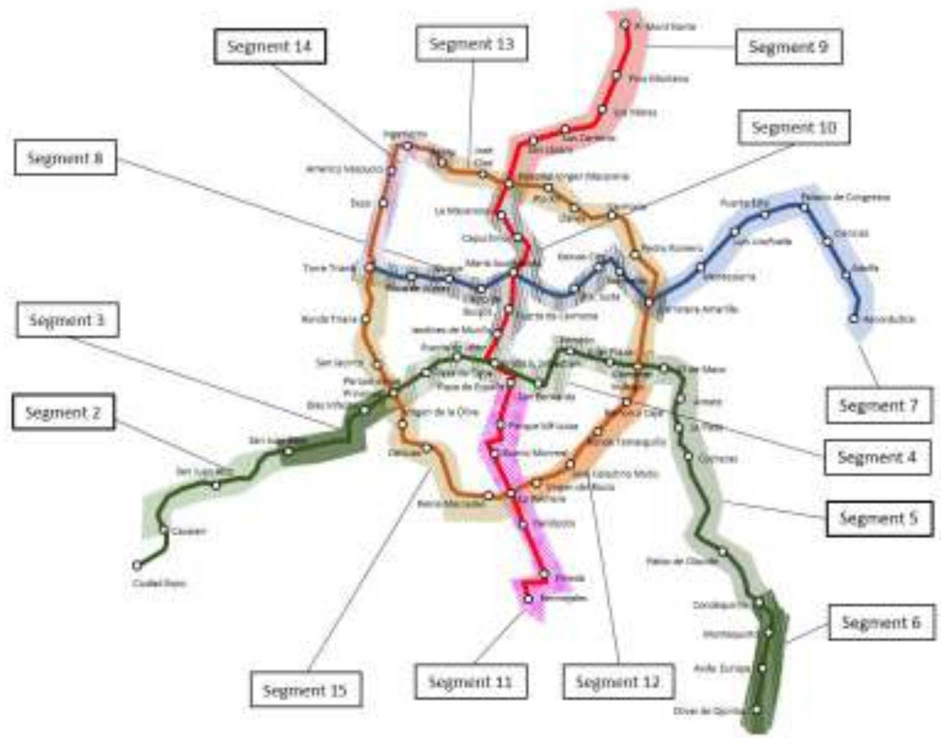
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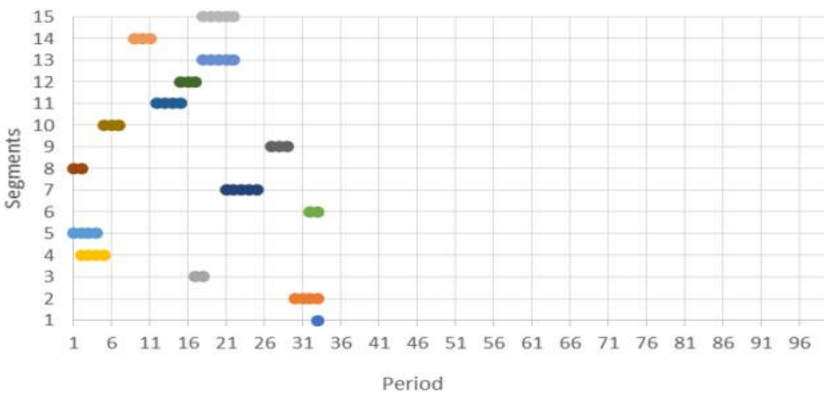
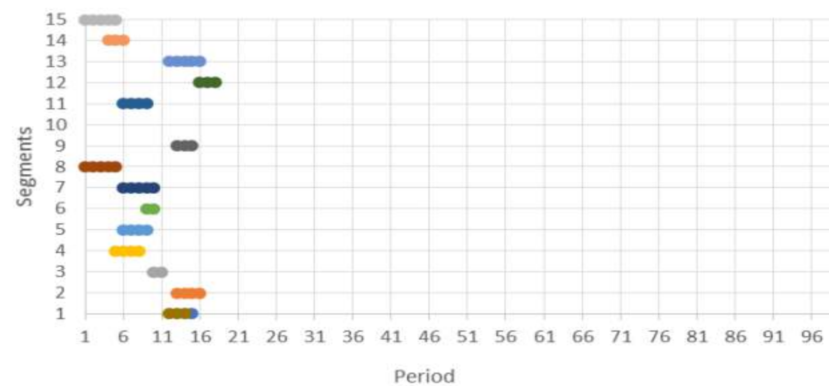
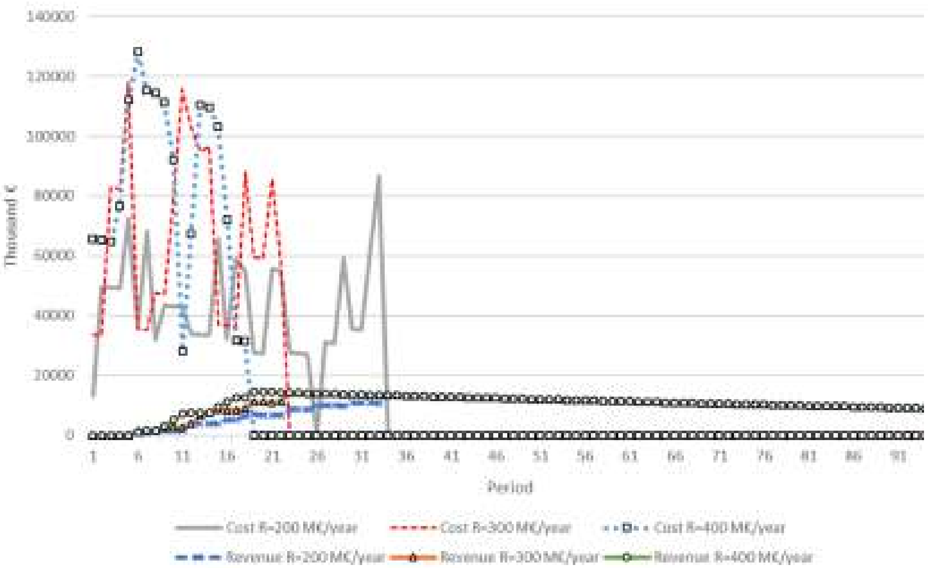
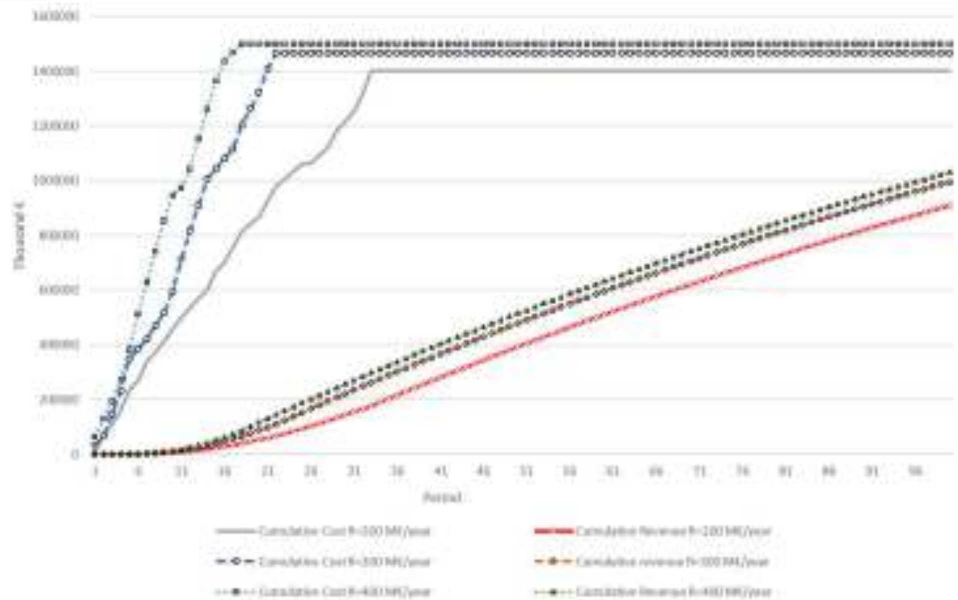


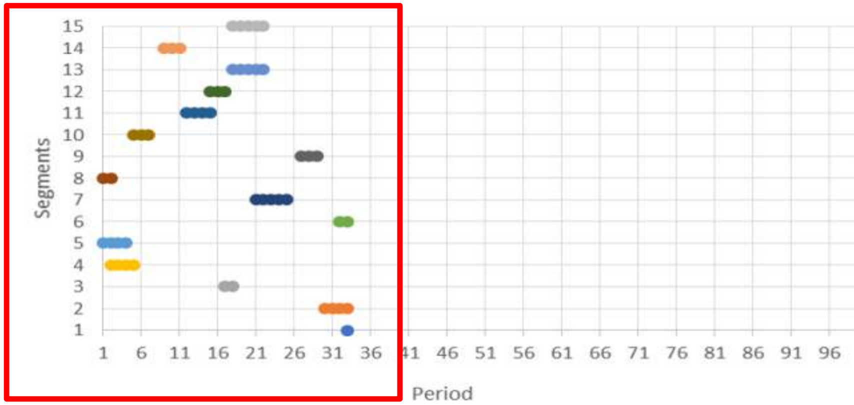
(a) $R = 200{,}000$ thousand € per year



(b) $R = 400{,}000$ thousand € per year







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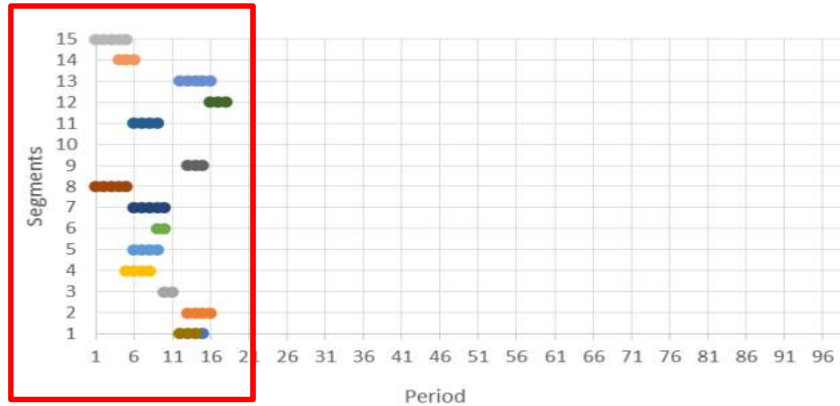
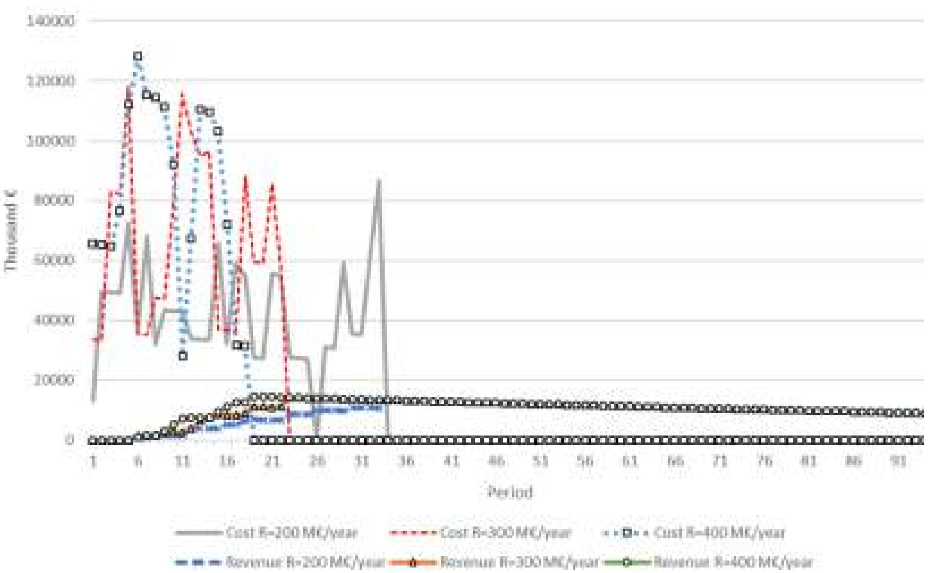
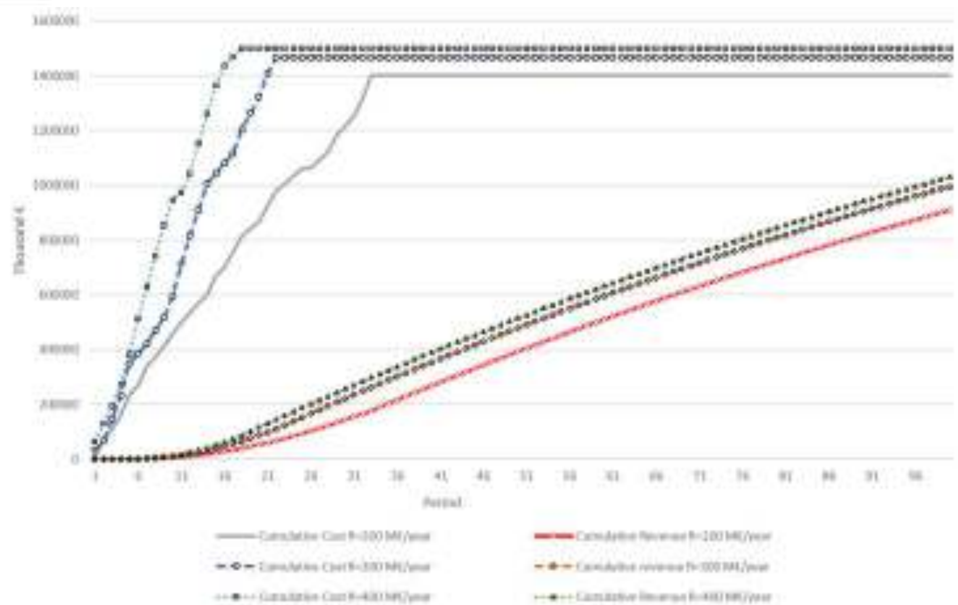
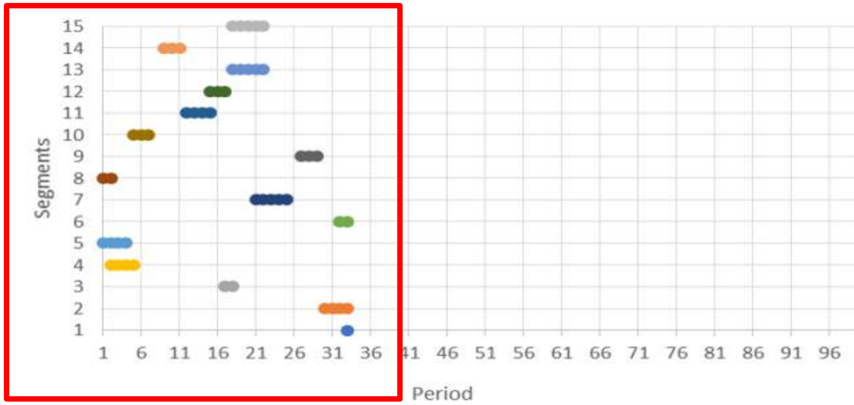


(b) $R = 400{,}000$ thousand € per year







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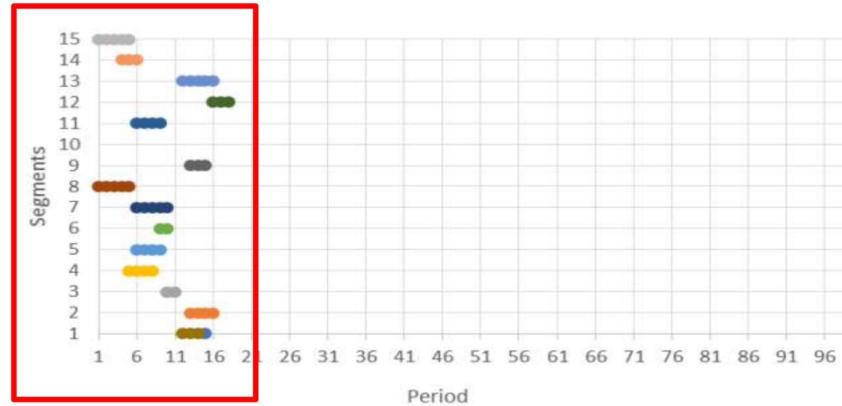

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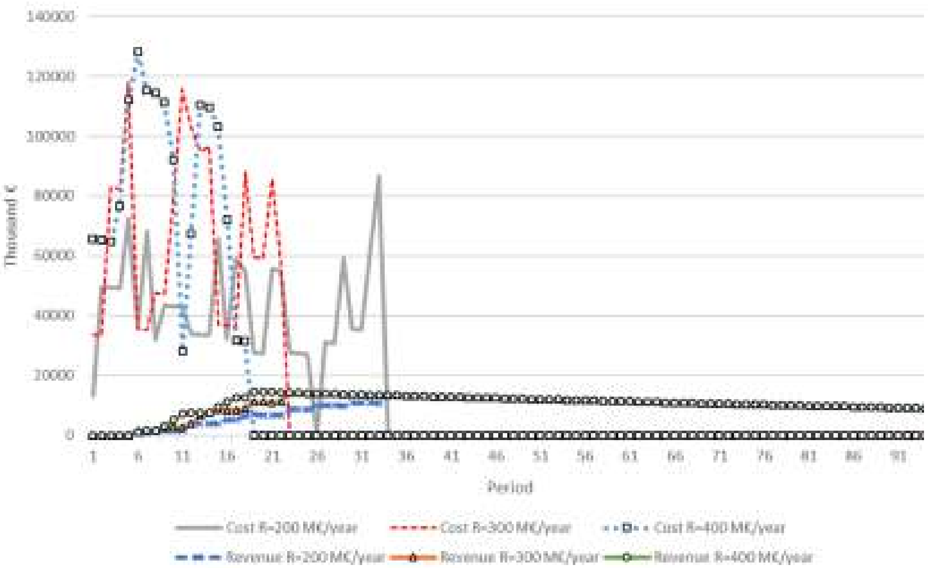
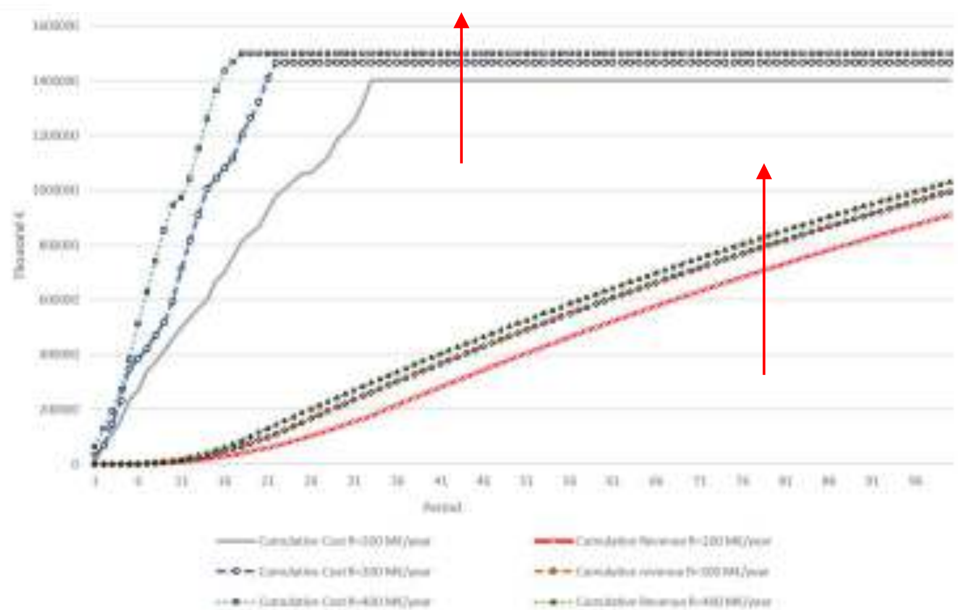
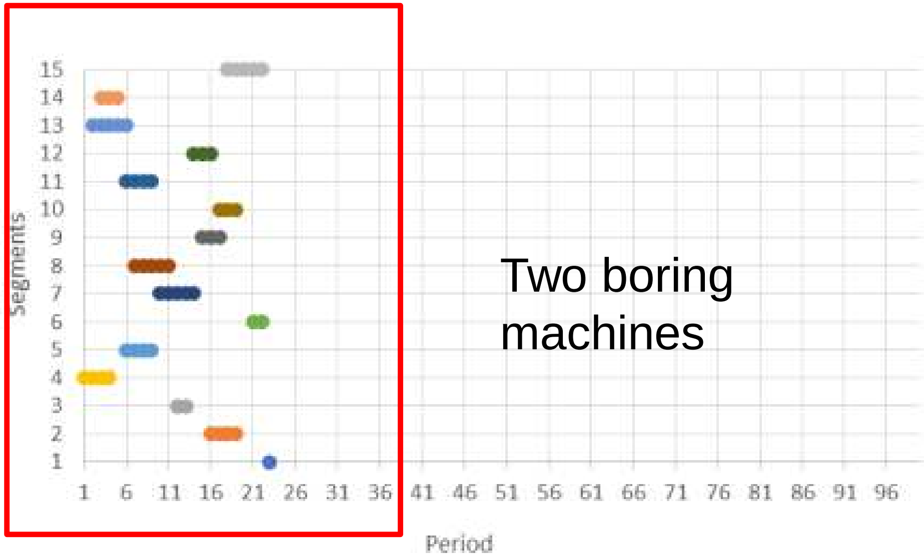
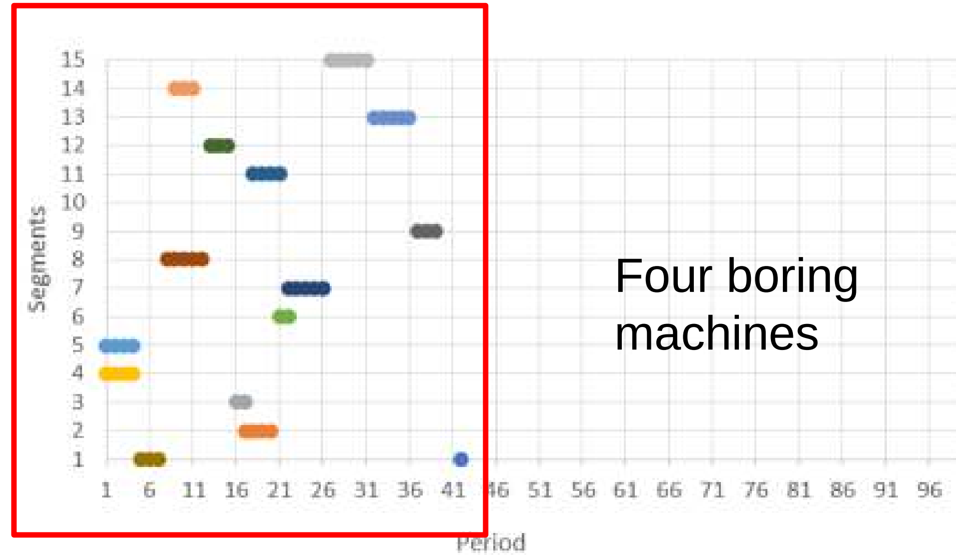




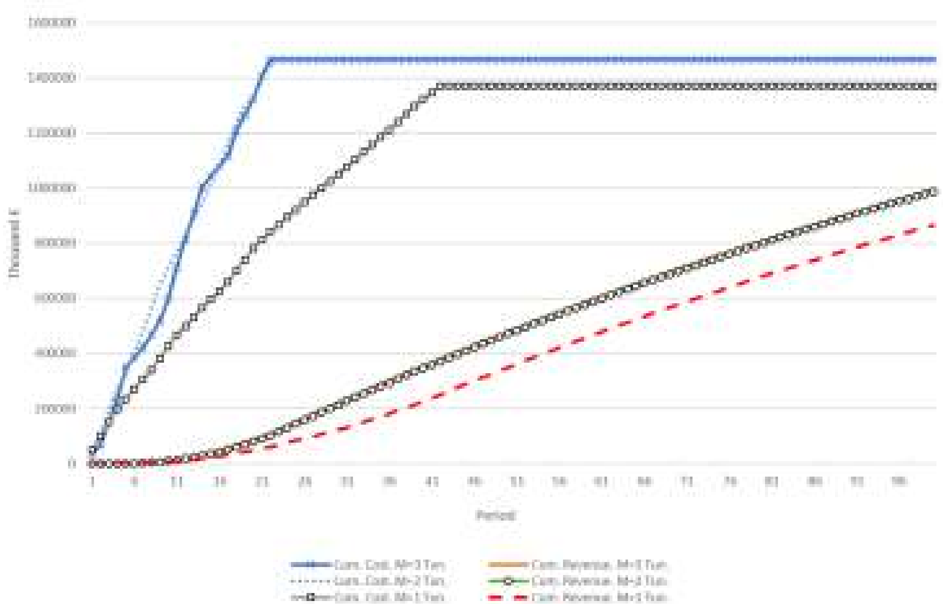
Results VI

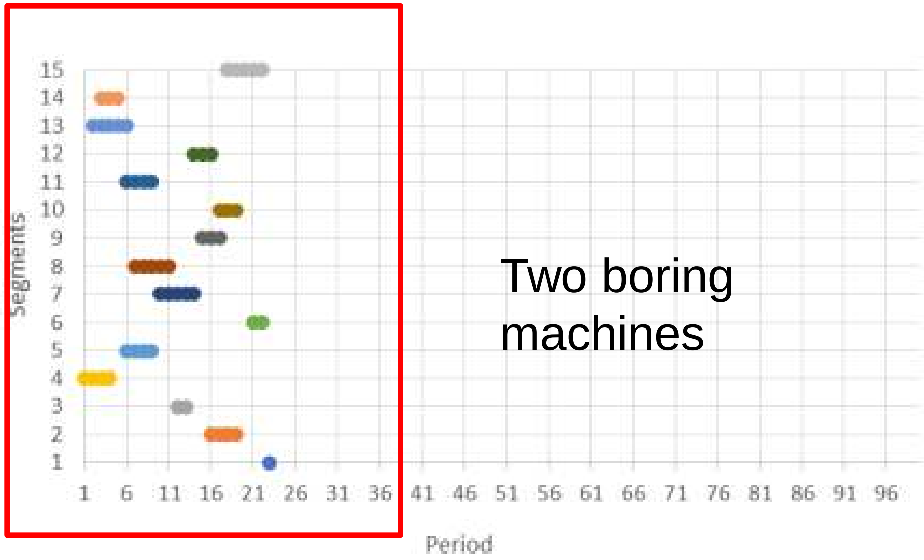


Two boring machines



Four boring machines




15
14
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5
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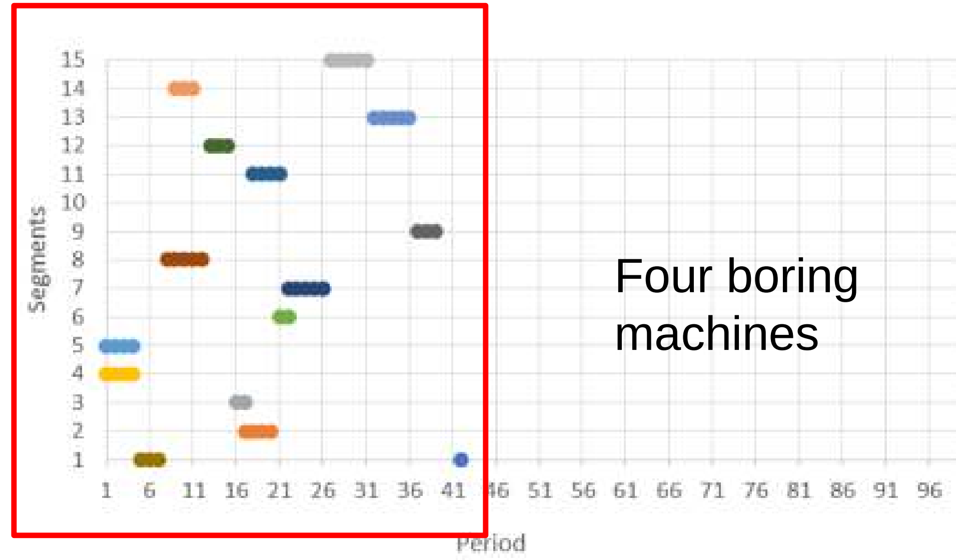
3
2
1
Segments
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
Period
Two boring machines


15
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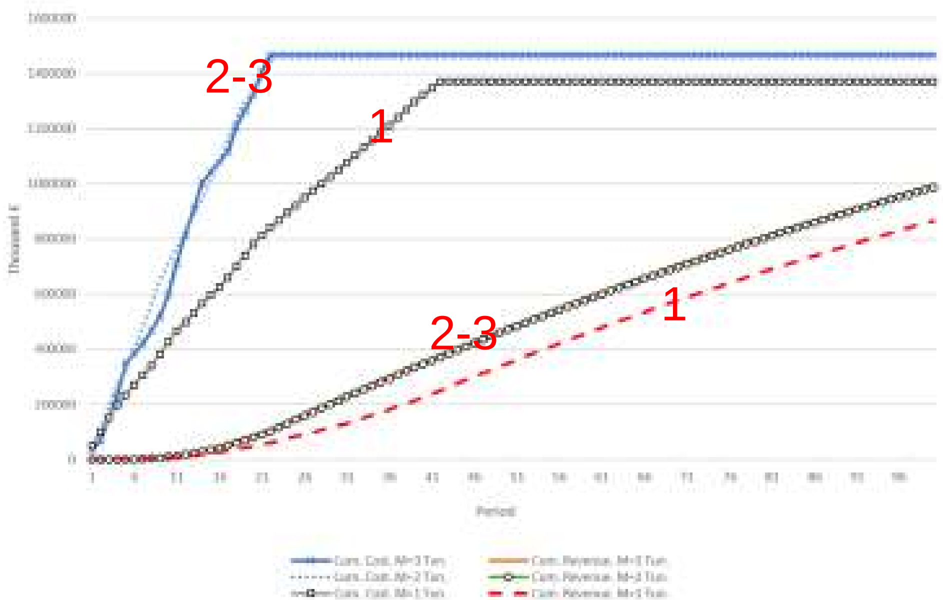
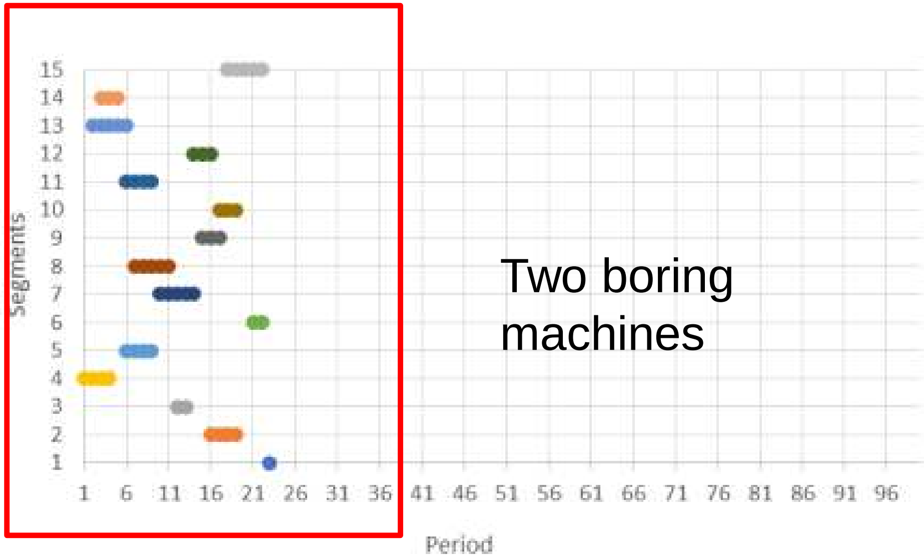
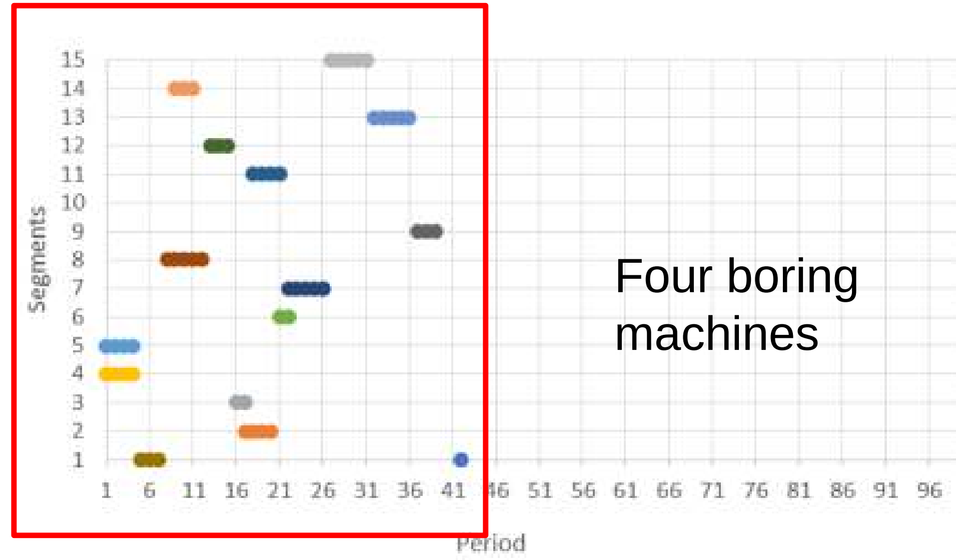
2
1
Segments
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76 81 86 91 96
Period
Four boring machines


2-3
1
2-3
1
Period

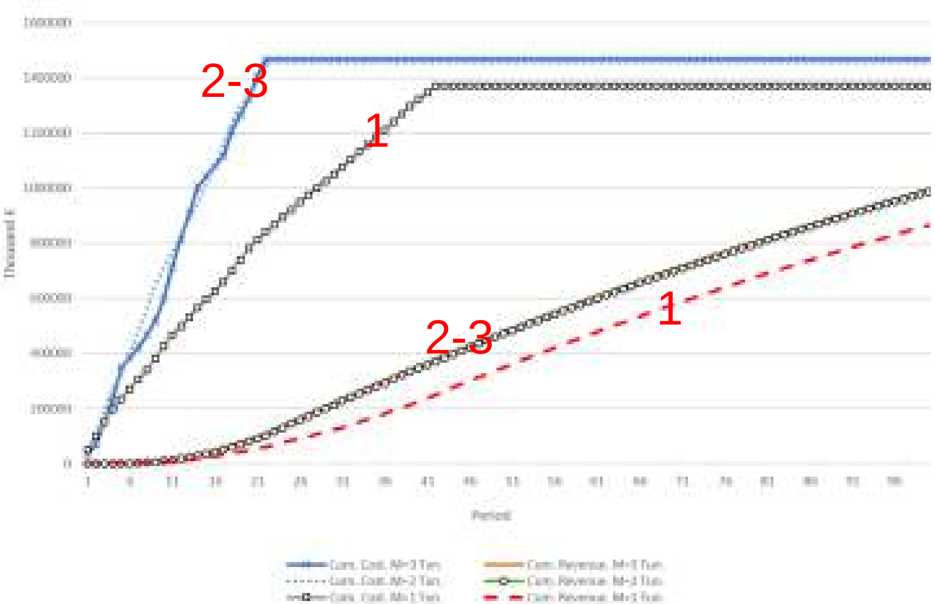
Results VI

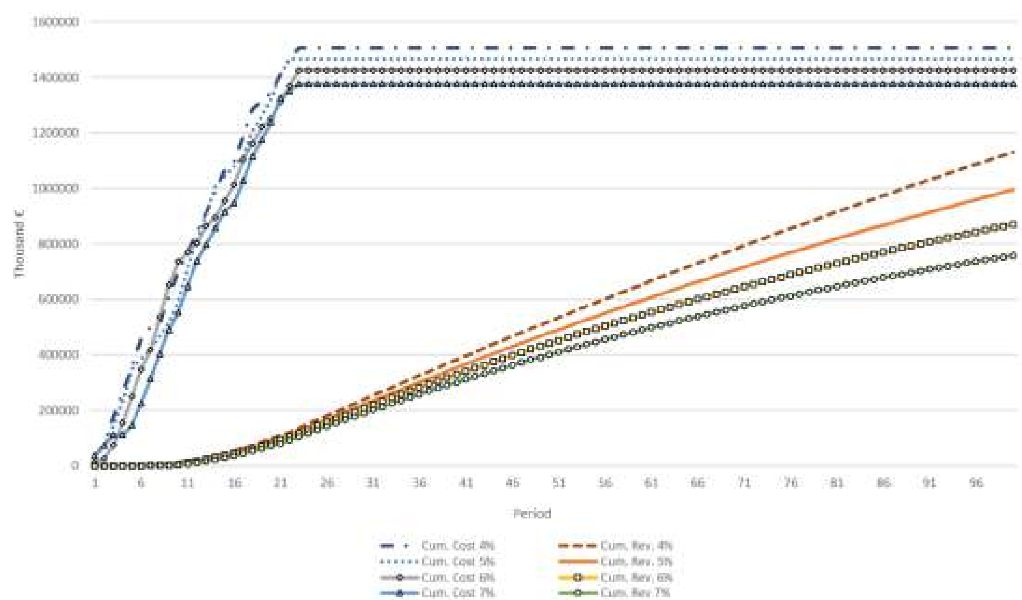


Two boring machines



Four boring machines





Outline

We have proposed a methodology to practically analyse the construction of a transportation network.

We can deal with different technological constraints regarding construction Issues.

The actual formulation allows us to solve relatively big scenarios in reasonable computing times.

Future topics

- Consideration of non-deterministic segment construction times.
- Inclusion of operational costs (in order to determine possible subsidies to transportation service providers)

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THANK YOU VERY MUCH!!

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