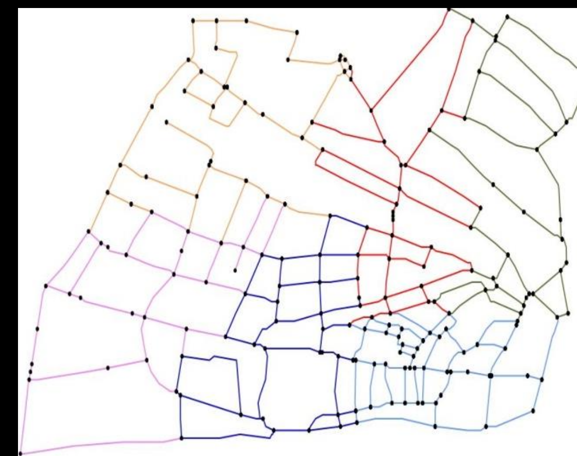
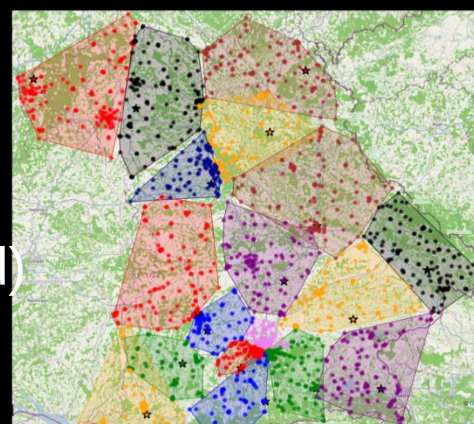
THE UNIVERSITY *of* EDINBURGH
School of Mathematics

Symmetry in Multi-period Sales Districting





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Outline

- The Multi-period Sales Districting Problem
- Symmetries in the Visit Scheduling Problem
- Conclusions

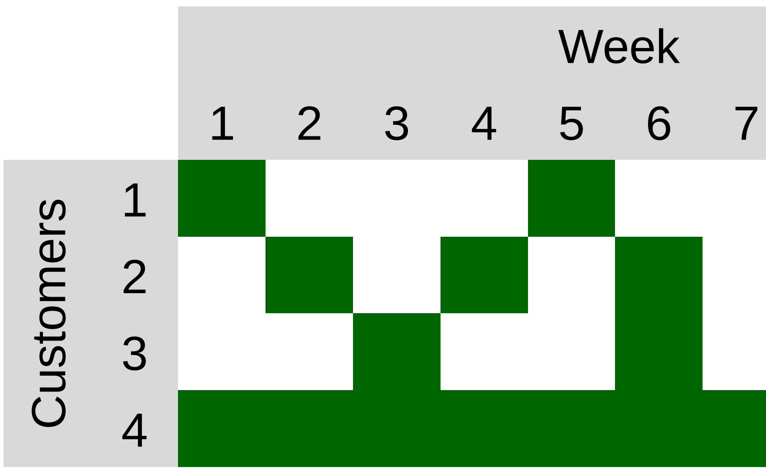
The Multi-period Sales Districting Problem

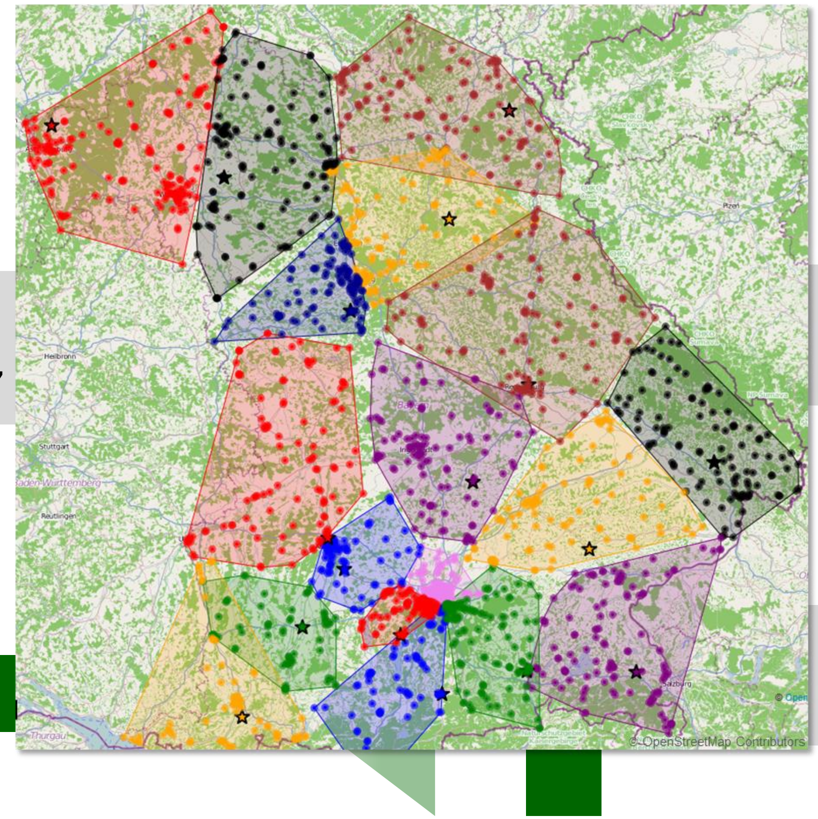
- **Customers**
  - Require on-site service by a sales person
  - Known average **on-site service time**
  - Have fixed but differing visiting frequencies
- **Sales persons**
  - Given number and locations
- **Planning horizon**
  - 3 – 12 months
- Typical problems size
  - $\geq 10{,}000$ customers & ≥ 50 sales persons

The Multi-period Sales Districting Problem

Decisions

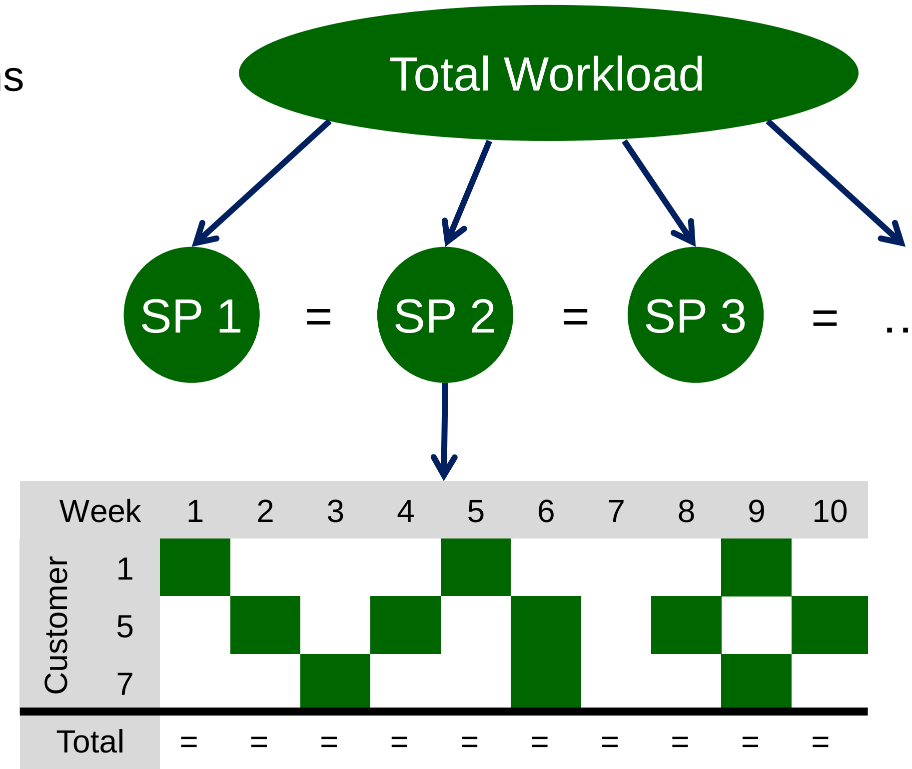
- Find assignment of customers to sales persons.
- Determine calendar for customers:
  - Assign visits to **weeks**
  - Assign visits to **weekdays**

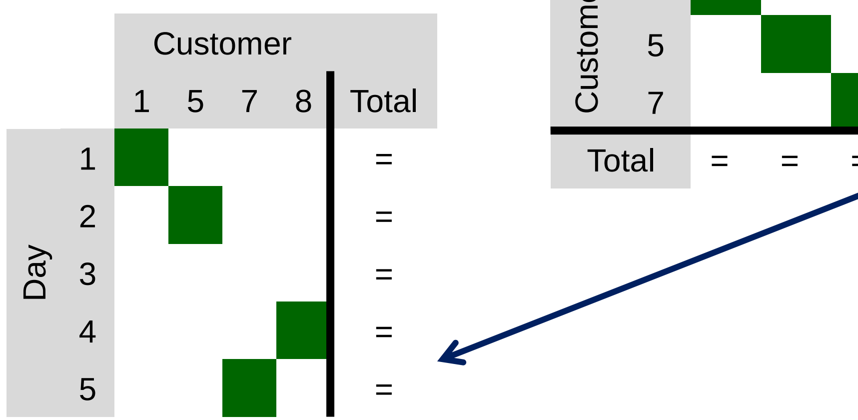




The Multi-period Sales Districting Problem

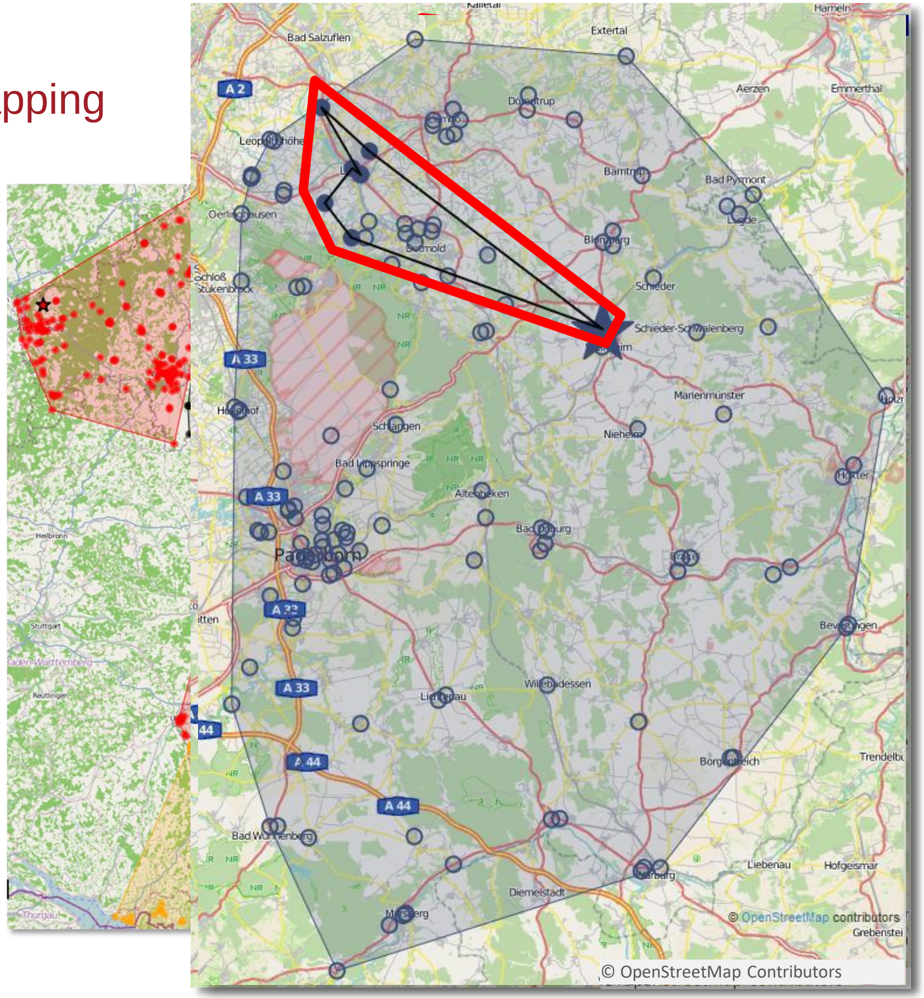
- **Planning goals**
  - Balance workload of sales persons
    - Over the planning horizon
    - Per week
    - Per day





The Multi-period Sales Districting Problem

- **Planning goals (cont'd)**
  - Determine compact and non-overlapping
    - **Overall** districts
    - **Weekly** sub-districts
    - **Daily** sub-districts



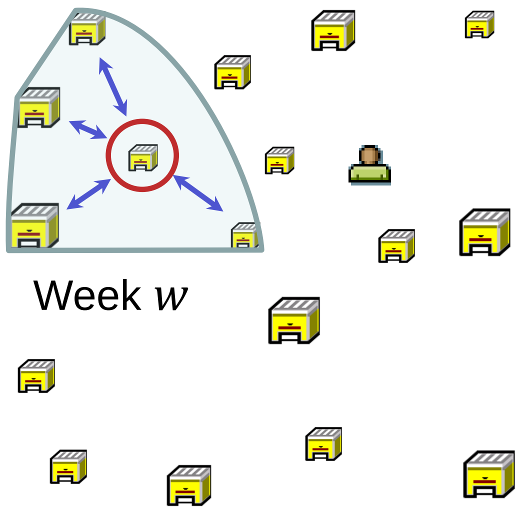
The Multi-period Sales Districting Problem

Planning goals (cont'd)

- Determine compact and non-overlapping
  - **Overall** districts
  - **Weekly** sub-districts
  - **Daily** sub-districts

Measuring compactness

- Weekly compactness
  - Determine a virtual centre in each week
  - Sum up the distances from all customers in the week to the centre
- Analogously for daily compactness



The Multi-period Sales Districting Problem

- Problem can be formulated as a **mixed-integer linear program** that
  - minimizes the sum of distances while ensuring
  - the district balance and
  - the visiting frequencies

Scheduling of Visits

- **Assumption**

  The districts have already been determined

- **“Remaining” problem: Visit scheduling**

  Schedule the visits for each sales person

Formulation for the Visit Scheduling Problem

Computational study

- Data sets
  - 5 random instances
  - # Customers: 30
  - CPLEX 12.6, max. runtime: 0.5 hours
  - Visiting frequencies

Type	Week rhythms	No. weeks	No. weekdays
1	{1,2,4}	4	5
2	{2,4}	4	5
3	{1,2,4,8}	8	5
4	{2,4,8}	8	5
5	{1,2,4,8,16}	16	3
6	{2,4,8,16}	16	3

Formulation for the Visit Scheduling Problem

Type	Rhythms	Weeks	Days
1	{1,2,4}	4	5
2	{2,4}	4	5
3	{1,2,4,8}	8	5
4	{2,4,8}	8	5
5	{1,2,4,8,16}	16	3
6	{2,4,8,16}	16	3

- **Results**

30 customers

Type	Gap	Opt	Time in sec.
1	0.01%	5	291.1
2	3.94%	1	1492
3	5.84%	0	1800
4	17.06%	0	1800
5	11.05%	0	1800
6	12.83%	0	1800

Outline

- The Multi-period Sales Districting Problem
- **Symmetries in the Visit Scheduling Problem**
- Conclusions

Symmetry in Multi-period Sales Districting

Symmetries in the Visit Scheduling Problem

- The visit scheduling problem is highly symmetric
- Let
  - $b \in B$ **Customers**
  - $w \in W$ **Weeks** in the planning horizon
  - $d \in D$ **Days** in the planning horizon
  - r_b Visit frequency or week rhythm of customer $b \in B$
  - B^w Customers visited in **week** $w \in W$ week cluster
  - $\tilde{B}^d$ Customers visited on **day** $d \in D$ day cluster
  - S A **solution**, $S = (B^1, \dots, B^{|W|})$

Symmetry in Multi-period Sales Districting

Symmetry on the level of days

- Given a specific week cluster $B^w = (\tilde{B}^{d_1}, \tilde{B}^{d_2}, \tilde{B}^{d_3}, \tilde{B}^{d_4}, \tilde{B}^{d_5})$.

- Then, any permutation of the **five days** yields a symmetric solution for that week

- Each **week cluster** gives rise to $5! - 1 = 119$ **symmetric arrangements** of **day cluster** within a week

Symmetries in the Visit Scheduling Problem

Symmetry on the level of weeks

- Given a solution

$$S = \left(B^1, B^2, \dots, B^{|W|}\right)$$

- Assume
  - $r_b = 2^k, k \in \mathbb{N}$
  - $|W| = 2^m = \max_{b \in B} r_b$

- Then, every **solution** S has $2^{m(m+1)/2} - 1$ symmetric solutions with respect to the week cluster

m	$\lvert W \rvert$	# Sym.
1	2	1
2	4	7
3	8	63
4	16	1023
5	32	32,767

- **Case $m = 1$**: $|W| = 2$
  - $S = (B^1, B^2)$ is symmetric to $S = (B^2, B^1)$

Week Symmetries

- **Case $m = 2$**: $|W| = 4$
  - Let $C^w = \{b \in B^w \mid r_b < 4\}$
  - Then $(C^1, C^2) = (C^3, C^4)$ and $(C^1, C^2, C^3, C^4) = (C^1, C^2, C^1, C^2)$
  - (C^1, C^2) is symmetric to (C^2, C^1)

    Hence

    $$(C^1, C^2, C^3, C^4) \text{ is symmetric to } (C^2, C^1, C^4, C^3)$$
  - Thus

    $$(B^1, B^2, B^3, B^4) \text{ is symmetric to } (B^2, B^1, B^4, B^3)$$
  - Moreover, any **cyclic permutation** of (B^1, B^2, B^3, B^4) is also symmetric:

    $$(B^2, B^3, B^4, B^1), (B^3, B^4, B^1, B^2) \text{ and } (B^4, B^1, B^2, B^3)$$

Symmetries in the Visit Scheduling Problem

- **Breaking day symmetries**
  - Impose an "order" on the days of a week
  - Possibilities
    - Sort days by **increasing workload**
    - Sort days by **smallest customer index**
  - Will eliminate all symmetric solutions for days!

- **Breaking week symmetries**
  - Pick a customer $b \in B$ with maximal r_b and fix its visit to the **first day** of the **first week**
  - Will eliminate the number of symmetric solutions for weeks by a factor of $|W|$!

Symmetries in the Visit Scheduling Problem

- **Results for ordering days by indices**

  Add constraints

$$h_b^d \le \sum_{b'=1}^{b-1} h_{b'}^{d-1}$$

Type	Gap	Opt	Time
1	0.01%	5	291.1
2	3.94%	1	1492
3	5.84%	0	1800
4	17.06%	0	1800
5	11.05%	0	1800
6	12.83%	0	1800

30 customers

Type	Gap	Opt	Time in sec.
1	0.01%	3	1052
2	9.98%	0	1800
3	11.22%	0	1800
4	27.46%	0	1800
5	12.88%	0	1800
6	17.32%	0	1800

Symmetries in the Visit Scheduling Problem

- **Results for fixing a customer visit**

30 customers

Type	Gap	Opt	Time in sec.
1	0.01%	5	146.4
2	2.29%	3	1198
3	3.73%	0	1800
4	14.70%	0	1800
5	9.00%	0	1800
6	12.39%	0	1800

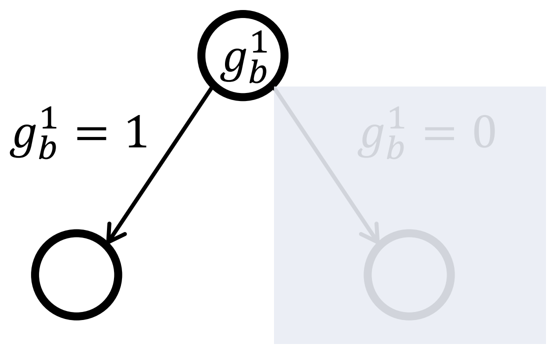
Type	Gap	Opt	Time
1	0.01%	5	291.1
2	3.94%	1	1492
3	5.84%	0	1800
4	17.06%	0	1800
5	11.05%	0	1800
6	12.83%	0	1800

Symmetry-Reduced Branching

- Add additional variable fixations to **eliminate symmetric solutions** with respect to weeks when branching on **fractional variables** in the branch & bound tree.

- Assume
  - $r_b = 2^k, k \in \mathbb{N}$
  - $|W| = 2^m = \max_{b \in B} r_b$

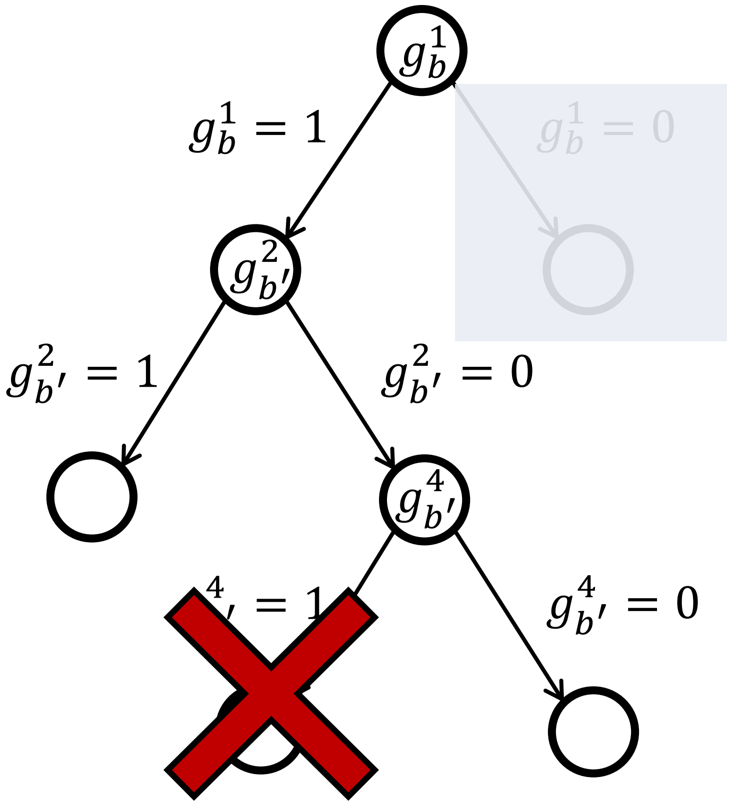
- Let
  - $m = 2$
  - $b \in B: r_b = 4$



#	Solution
1	(B^1, B^2, B^3, B^4)
2	(B^1, B^4, B^3, B^2)
3	(B^2, B^3, B^4, B^1)
4	(B^2, B^1, B^4, B^3)
5	(B^3, B^4, B^1, B^2)
6	(B^3, B^2, B^1, B^4)
7	(B^4, B^1, B^2, B^3)
8	(B^4, B^3, B^2, B^1)

Symmetry-Reduced Branching

- Let
  - $b' \in B\colon r_{b'} = 4$



#	Solution
1	(B^1, B^2, B^3, B^4)
2	(B^1, B^4, B^3, B^2)
3	(B^2, B^3, B^4, B^1)
4	(B^2, B^1, B^4, B^3)
5	(B^3, B^4, B^1, B^2)
6	(B^3, B^2, B^1, B^4)
7	(B^4, B^1, B^2, B^3)
8	(B^4, B^3, B^2, B^1)

Symmetry-Reduced Branching

- **Computational Results for a Branch-and-Price Algorithm**
  - 5 real-world data sets provided by PTV Group
  - # Weeks in planning horizon: 4
  - # Days per week: 5
  - Week rhythms: $r_b \in \{1, 2, 4\}$
  - **Benchmark:** Gurobi 7.0.1, 10 hours

	#Customers	w/o symmetry reduction		Fixing a customer		Fixing customer + sym. red. branching	
Inst	(#Visits)	Time in s	#Nodes	Time in s	#Nodes	Time in s	#Nodes
1	31 (80)	606	2,251	130	660	54	167
2	26 (74)	8	37	2	6	2	7
3	32 (76)	4,284	12,799	504	2,172	114	429
4	25 (71)	36,000	669,752	36,000	661,801	458	6,181
5	35 (84)	33,995	140,657	5768	22,862	2,113	8,231
Avg		14,979	165,099	8,480	137,500	552	3,003

Outline

- The Multi-period Sales Districting Problem
- Symmetries in the Visit Scheduling Problem
- **Conclusions**

Conclusions

- Visit scheduling problem is highly symmetric
- Limited success in reducing the **number** of **symmetric solutions** a priori
- Some success in reducing the number of **symmetric solutions** while solving the problem