Dynamic optimisation in solar power tower plants

T. Ashley[1] E. Carrizosa[2] E. Fernández-Cara [3]

[1]IMUS - Instituto de Matemáticas
Universidad de Sevilla

[2]IMUS - Instituto de Matemáticas
Universidad de Sevilla

[3]Dep. EDAN and IMUS
Universidad de Sevilla

IWOLOCA19 - Cádiz, España 31/01/19

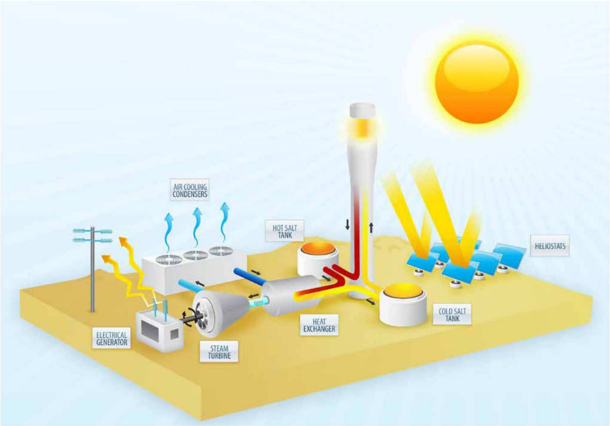
Outline

1. Introduction
2. Optimisation Problem
3. Algorithms
   - Penalisation
   - Augmented Lagrangian
   - Gradient Ascent
4. Numerical approximation
5. Illustrative Example

Introduction
Partners

Solar Power Tower (SPT) Plant

Solar Power Tower (SPT) Plant



Aiming Strategy

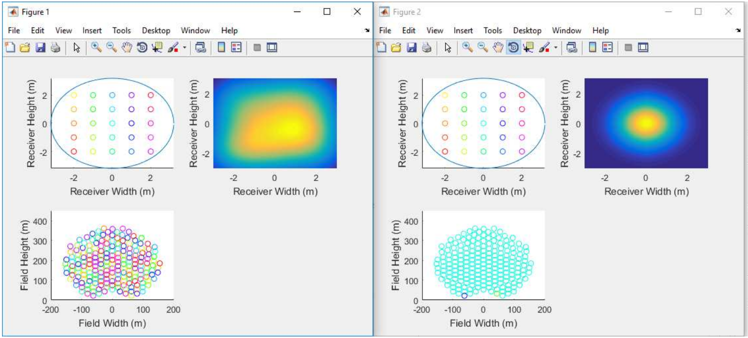
Publications:

"*Optimisation of aiming strategies in Solar Power Tower plants*". Ashley T., Carrizosa E., Fernández-Cara E.. Energy, 137C, pp. 285–291 (2017).

"*Continuous Optimisation Techniques for Optimal Aiming Strategies in Solar Power Tower plants*". Ashley T., Carrizosa E., Fernández-Cara E.. (Submitted)

"*Dynamic Continuous Optimisation Applied to Aiming Strategies in Solar Power Tower plants*". Ashley T., Carrizosa E., Fernández-Cara E.. (Not yet submitted)

Aiming Strategy
Target Distribution

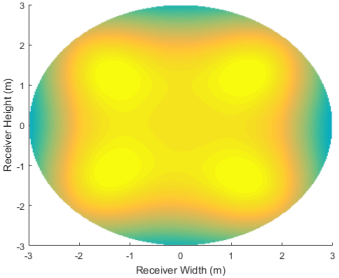


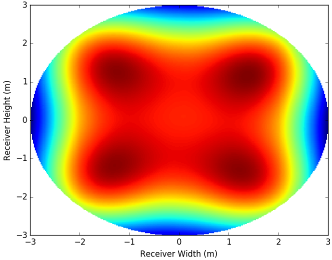

```
TotalInitial_E =

     1.123018463135011e+07


TotalOptEnergy =

     1.339351695305695e+07

Max

ans =

     2.408875343796182e+06

Min

ans =

     2.503208031699102e+04

Range

ans =

     2.383843263479191e+06
```

Aiming Strategy
Heliostat Rotation

Source:
http://theleadsouthaustralia.com.au/industries/resources-energy/solar-park-drive-plastic-heliostat-development/

Aiming Strategy
Radiation Change





Aiming Strategy

Summary

- Optimise aiming strategy of heliostats
- Maximise radiation
- Minimise deviation from target distribution
- Constrain heliostat rotation speed
- Constrain radiation change on receiver

Aiming Strategy
Summary

- Optimise aiming strategy of heliostats
- Maximise radiation
- Minimise deviation from target distribution
- Constrain heliostat rotation speed
- Constrain radiation change on receiver

Aiming Strategy

Summary

- Optimise aiming strategy of heliostats
- Maximise radiation
- Minimise deviation from target distribution
- Constrain heliostat rotation speed
- Constrain radiation change on receiver

Aiming Strategy
Summary

- Optimise aiming strategy of heliostats
- Maximise radiation
- Minimise deviation from target distribution
- Constrain heliostat rotation speed
- Constrain radiation change on receiver

Aiming Strategy
Summary

- Optimise aiming strategy of heliostats
- Maximise radiation
- Minimise deviation from target distribution
- Constrain heliostat rotation speed
- Constrain radiation change on receiver

Optimisation Problem
Model

The proportion of energy reaching the receiver for each heliostat within the field, given:

- Physical location (x, y)
- Aim point $(\rho\cos\phi, \rho\sin\phi)$

$$f_{sp}(x, y, \Theta) = f_1(x, y, \Theta) \iint_S exp\left(\frac{-f_2(u, v, x, y)}{2f_3^2(x, y, \Theta)}\right) du\, dv\ ,$$

$$\sum_{h \in H} f_{sp}(h, p, t).$$

Optimisation Problem
Model

The proportion of energy reaching the receiver for each heliostat within the field, given:

- Physical location (x, y)
- Aim point $(\rho \cos \phi, \rho \sin \phi)$

$$f_{sp}(x, y, \Theta) = f_1(x, y, \Theta) \iint_S exp\Big(\frac{-f_2(u, v, x, y)}{2f_3^2(x, y, \Theta)}\Big)\, du\, dv\ ,$$

$$\sum_{h \in H} f_{sp}(h, p, t).$$

Optimisation Problem
Model

The proportion of energy reaching the receiver for each heliostat within the field, given:

- Physical location (x, y)
- Aim point $(\rho \cos \phi, \rho \sin \phi)$

$$f_{sp}(x, y, \Theta) = f_1(x, y, \Theta) \iint_S exp\left(\frac{-f_2(u, v, x, y)}{2f_3^2(x, y, \Theta)}\right) du\, dv\ ,$$

$$\sum_{h \in H} f_{sp}(h, p, t).$$

Optimisation Problem
Model

The proportion of energy reaching the receiver for each heliostat within the field, given:

- Physical location (x, y)
- Aim point $(\rho \cos \phi, \rho \sin \phi)$

$$f_{sp}(x, y, \Theta) = f_1(x, y, \Theta) \iint_S exp\left(\frac{-f_2(u, v, x, y)}{2f_3^2(x, y, \Theta)}\right) du\, dv \ ,$$

$$\sum_{h \in H} f_{sp}(h, p, t).$$

Optimisation Problem
Model

The proportion of energy reaching the receiver for each heliostat within the field, given:

- Physical location (x, y)
- Aim point $(\rho \cos \phi, \rho \sin \phi)$

$$f_{sp}(x, y, \Theta) = f_1(x, y, \Theta) \iint_S exp\left(\frac{-f_2(u, v, x, y)}{2f_3^2(x, y, \Theta)} \right) du\, dv \;,$$

$$\sum_{h \in H} f_{sp}(h, p, t).$$

Optimisation Problem
Objective

Maximise Energy

$$\int_0^T \Big(\iint_R \sum_{h \in H} f(h, \mathbf{p}, t) dudv \Big) dt$$

Minimise target distribution deviation

$$\int_0^T \iint_R (\sum_{h \in H} F^{u,v}(h, \mathbf{p}, t) - E_{tar}^{u,v,t}(t))^2 dudvdt$$

Optimisation Problem
Objective

Penalised weighted objective

$$\text{Maximise} \quad A\int_0^T \Big(\iint_R \sum_{h\in H} f(h,\mathbf{p},t)\,dudv \Big)\,dt -$$

$$(1-A)\int_0^T \iint_R \Big(\sum_{h\in H} F^{u,v}(h,\mathbf{p},t) - E_{tar}^{u,v,t}(t)\Big)^2 dudvdt$$

with $A \in [0,1]$.

Optimisation Problem
Constraints

Limit heliostat turn rate $\dot{p}_h(t)$

$$\int_0^T (||\dot{p}_h(t)|| - V_p)_+^2 \, dt \leq \tau_1 \quad \forall h \in H$$

Limit radiation change over time $\frac{\partial}{\partial t} F^{u,v}(h, \mathbf{p}, t)$

$$\int_0^T \iint_R \Big(\sum_{h \in H} \frac{\partial}{\partial t} F^{u,v}(h, \mathbf{p}, t) \Big) dudvdt \leq \tau_2$$

Optimisation Problem

$$\text{Maximise} \quad A\int_0^T \Big(\iint_R \sum_{h\in H} f(h,\mathbf{p},t)dudv\Big)dt -$$

$$(1-A)\int_0^T \iint_R (\sum_{h\in H} F^{u,v}(h,\mathbf{p},t) - E_{tar}^{u,v,t}(t))^2 dudvdt$$

with $A \in [0,1]$.

Subject to:

$$\int_0^T (||\dot{p}_h(t)|| - V_p)_+^2 dt \leq \tau_1 \quad \forall h \in H$$

$$\int_0^T \iint_R \Big(\sum_{h\in H} \frac{\partial}{\partial t} F^{u,v}(h,\mathbf{p},t)\Big) dudvdt \leq \tau_2$$

Penalisation

$$\text{Maximise}\quad A_1\int_0^T\Big(\iint_R\sum_{h\in H} f(h,\mathbf{p},t)dudv\Big)dt$$

$$-A_2\int_0^T\iint_R(\sum_{h\in H}F^{u,v}(h,\mathbf{p},t)-E_{tar}^{u,v,t}(t))^2dudvdt$$

$$-A_3\int_0^T(||\dot{p}_h(t)||-V_p)_+^2dt-\tau_1$$

$$-A_4\int_0^T\iint_R\Big(\sum_{h\in H}\frac{\partial}{\partial t}F^{u,v}(h,\mathbf{p},t)\Big)dudvdt-\tau_2$$

Augmented Lagrangian

$$\mathcal{L}_\mu(\mathbf{p};\lambda) := J(\mathbf{p}) + \psi(M(\mathbf{p}) - \sigma, \lambda; \mu),$$

$$\psi(z,\lambda;\mu) := \begin{cases} -z\cdot\lambda + \dfrac{1}{2\mu}|z|^2 & \text{if } \ z + \mu\lambda \leq 0 \\ -\dfrac{\mu}{2}|\lambda|^2 & \text{otherwise} \end{cases}$$

$\mu > 0$ small

Take $\lambda^{n+1} = (\lambda^n - \frac{1}{\mu}(M_i(p_N^n) - \sigma)_+$.

Gradient Ascent

Aiming points $\mathbf{p}_N^{n+1}$ found by gradient ascent:

$$\mathbf{p}_N^{n+1} = \mathbf{p}_N^n + \gamma_h^k \nabla G_N(\mathbf{p}_N^n).$$

Step size with *Armijo's Rule*

$$\gamma_h^k = \gamma_h^{k-1} \cdot \epsilon$$

with $\epsilon \in (0, 1)$

Numerical approximation
Objective

Maximise radiation

$$\int_0^T \iint_R \Big(\sum_{h=1}^{H} F^{(u,v)}(h, p_h(t), t)\Big)\, du\, dv\, dt \approx \sum_{n=1}^{T}\sum_{i=1}^{I}\sum_{h=1}^{H} F^{(u_i,v_i)}(h, p_h(t^n), t^n)$$

Minimise target distribution deviation

$$\int_0^T \iint_R \Big(\sum_{h=1}^{H} F^{u,v}(h, \mathbf{p}, t) - E_{tar}^{u,v}(t)\Big)^2 dudvdt \approx \sum_{n=1}^{T}\sum_{i=1}^{I}\Big(\sum_{h=1}^{H} F^{u_i,v_i}(h, \mathbf{p}, t^n) - E_{tar}^{u_i,v_i}(t^n)\Big)^2$$

Numerical approximation
Constraints

Limit heliostat turn rate

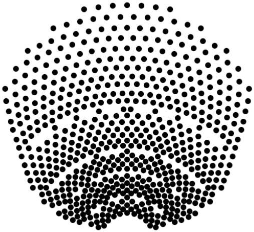
$$\int_0^T \|(\dot{\mathbf{p}}(t) - \mathbf{V}_p)_+\|^2 \, dt \approx \int_0^T \sum_{n=1}^{N} (|\mathbf{P}^n - \mathbf{P}^{n-1}| - V_p)_+^2 \, dt,$$

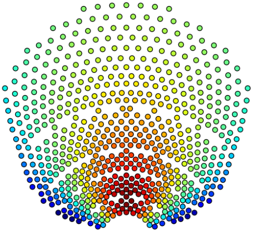
Limit radiation change on receiver

$$\int_0^T \left(\iint_R \Big(\sum_{h=1}^{H} \frac{d}{dt} F^{u,v}(h, \mathbf{p}(t), t) \Big)^2 \, du \, dv \right) dt$$
$$\approx \int_0^T \sum_{i=1}^{I} |\sum_{h=1}^{H} F^{u_i,v_i}(h, \mathbf{P}_h^n, t^n) - F^{u_i,v_i}(h, \mathbf{P}_h^{n-1}, t^{n-1})|^2 \, dt.$$

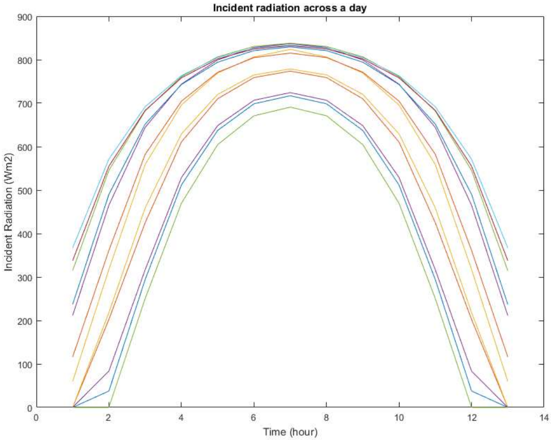
Illustrative Example

- Results - *PS10, Sanlúcar la Mayor*

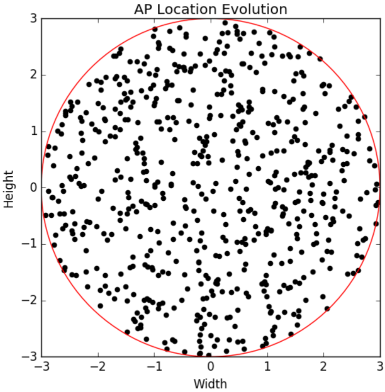


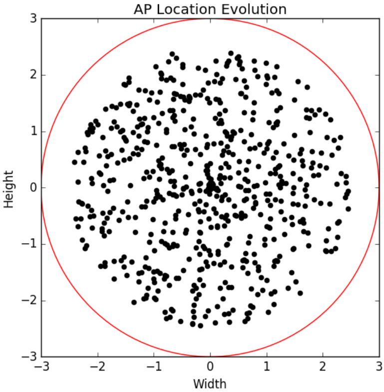


Illustrative Example

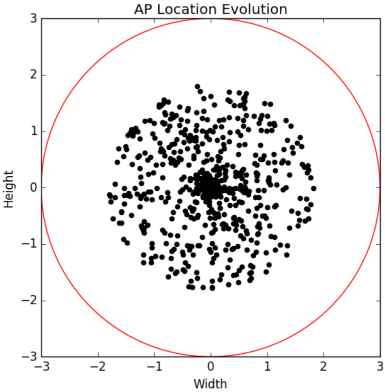


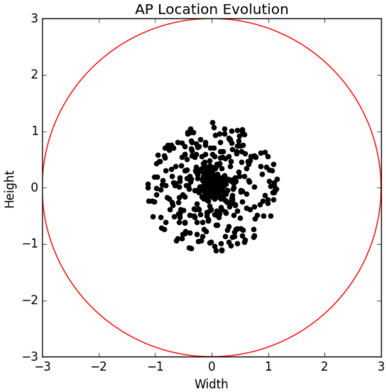
Illustrative Example



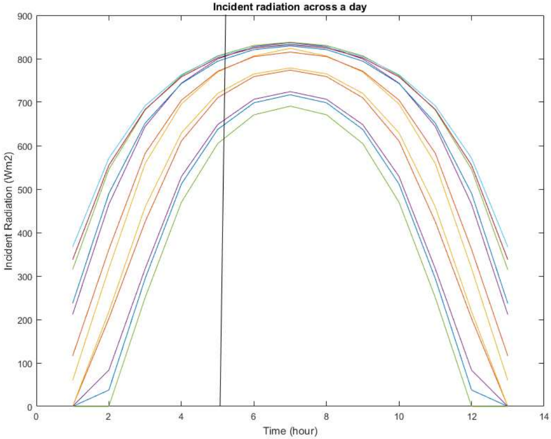


Illustrative Example

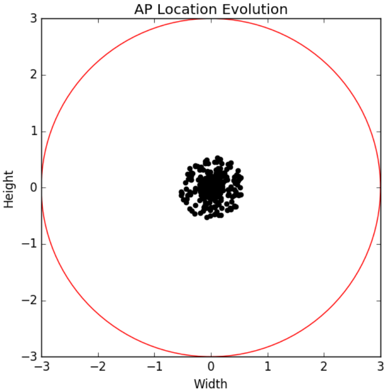


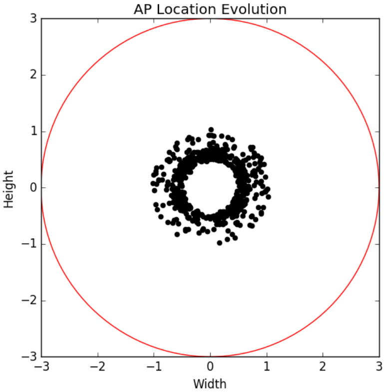


Illustrative Example

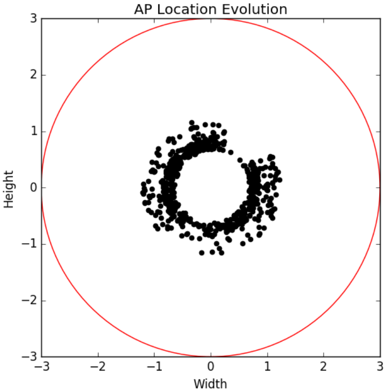


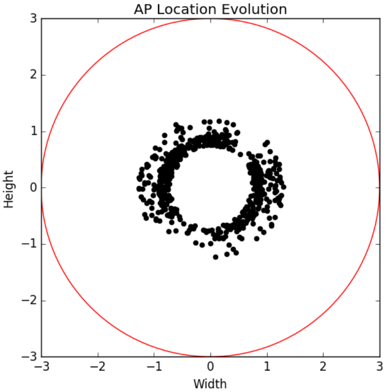
Illustrative Example



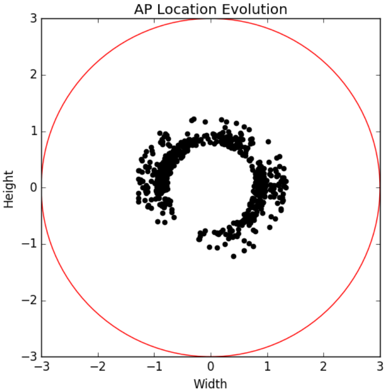


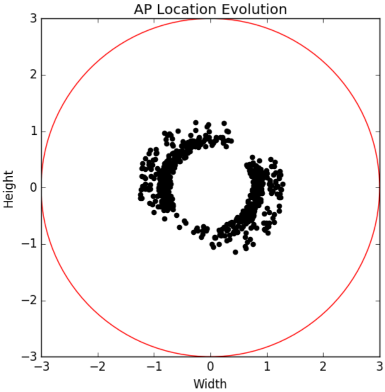
Illustrative Example





Illustrative Example





Illustrative Example

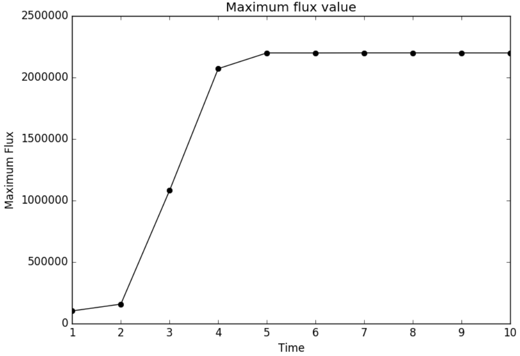


Figure: Maximum flux on receiver over time

Questions

Questions?

tashley@us.es